STABILITY OF RENTS AND RETURNS AS A SOURCE OF INTERNAL FINANCING: EVIDENCE FROM APPALACHIAN COAL PRODUCERS

C. W. Yang, Clarion University of Pennsylvania
Ken Hung, Texas A&M International University

ABSTRACT

The role of steam coal was constrained by the Kyoto Protocol and the Copenhagen Climate Summit which call for reduced emissions of green house gases and related measures. These agreements increase the importance in properly managing emissions. Coupled with rapidly increasing demand from China and India, a study on the rents of the Appalachian coal mines is as important as it is timely. In this paper, we show that response surfaces of producers' surpluses are nonlinear with respect to changes in any parameters. They are closely related to a given flow pattern in which only m+n-l positive coal flows prevail. Only when the flow patterns change does the response surfaces of the producers' surplus undergo structural changes. Production taxes decrease supplier's welfare. Furthermore the result of the Friedman test indicates that the relative welfare position, measured in terms of producers' rents, differs significantly in our simulation.

JEL: B41; C15; H2; M48

KEYWORDS: Appalachian coal producers, rents and returns, internal financing, simulation technique, non-liner response function

INTRODUCTION

A major source of internal finance for producing firms in the Appalachian coal market is the generated rents or producer surplus. Some of the Appalachian areas were economically depressed especially before the era of skyrocketing oil price. Some had bounced back when crude oil price hit close to $150 a barrel. On the other hand, steam coal burning does generate substantial amount of pollution and has been considered one of the culprits of global warming. The main thrust of the Kyoto Protocol and Copenhagen Climate Summit is on the reduction of carbon dioxide emission and imposition of emission fees in terms of carbon tax or related measures. On the global level, a computable general equilibrium model may shed a light on the issue. On national level, however, such a simulation both mathematically consistent and price responsive, is lacking. One approach found successful for analyzing rents on a regional basis is that of spatial allocation modeling. The impact on location rents of Appalachian coal producers, due to changes in taxation policies or economic parameters (environments) in the framework of the spatial equilibrium model (Takayama and Judge 1964, 1971) has not been studied thus far due to the complex nature of the problem. Most of the applications were on shipment pattern of the commodity especially steam coal. In this paper, we first analytically investigate this problem; then we perform some simulations on the stability of rents or returns of the Appalachian steam coal producing regions. The analysis is based on an estimated spatial equilibrium model (Labys and Yang, 1980) from which "optimum" shipments between Appalachian producers and eastern utilities are determined. It is capable of generating a set of optimal coal productions, consumptions, coal flows, and prices which, in turn, permit a calculation of producers' surplus. With the computational aid of a software package (Cutler and Pass, 1971), such simulation can be made conveniently. It is encouraging that we discover some anomalies which contradict the long recognized classical result in the space-less models. Further, in view of current financial stringency at regional levels, the relationship between different taxes and interconnected spatial rents is worthy of a careful evaluation.

The paper is composed of three parts: (1) Mathematical Analysis of the Sensitivity of the Rent, (2) Nonparametric Analysis of the Rent Response Surface via Friedman's Test (M. Friedman, 1937), and (3)

Policy Implications. To the best of our knowledge, the results of the paper being previously unknown are to fill a void in the literature.

LITERATURE REVIEW AND BACKGROUND

Historical developments of the spatial equilibrium model have been along the line of Enke (1951), Samuelson (1952), and Takayama and Judge (1964, 1971). Further extensions and applications based on Cottle-Dantzig complementarity pivot theories (1968) or variation inequalities are witnessed in the works by Pang and Chan (1981), Irwin and Yang (1982, 1983), Friesz et a1. (1983), Takayama and Uri (1983), Dafermos and Nagurney (1983, 1984, 1989) Nagurney (1986), Yang and Labys (1985), Takayama and Hashimoto (1989), and Yang and Page (1993). Applications of the spatial equilibrium models have proliferated since the live stock feed model by Fox (1951) and egg model by Judge (1956). The continued interest can be witnessed via the model applications by Uri (1989) and Peeters (1990). Readers are referred to Labys and Yang (1991) for a discussion of advances of the spatial equilibrium models; and to Thompson (1984) and Labys (1989) for model applications.

Despite these advances, a study of the spatial rent of a production region has thus far evaded the literature. Our modest purpose of the paper is to fill a void in this regard.

The objective function of the original model is to maximize the "net social payoff" or NSP, which is the sum of the consumer's surplus of n demand regions and the producer's surplus (returns or rents) of m supply regions. In 1964, Takayama and Judge formally converted such problems into the operationally efficient quadratic programming model as shown below:

Maximize

$$NSP(x_i, y_j, z_{ij}) = \sum_{j\in J} a_j y_j - \frac{1}{2}\sum_{j\in J} b_j y_j^2 - \sum_{i\in I} c_i x_i - \frac{1}{2}\sum_{i\in I} d_i x_i^2 \sum_{i\in I}\sum_{j\in J} t_{ij} z_{ij} \qquad (1)$$

Subject to

$$y_{ij} - \sum z_{ij} \le 0 \qquad \forall j \in J$$

$$x_{ij} - \sum z_{ij} \ge 0 \qquad \forall i \in I$$

$$z_{ij} \ge 0 \qquad \forall ij \in (I \times J)$$

where I, J and (IXJ) are sets of finite positive integer sets and their corresponding Cartesian product. The inverse demand and supply relations are linear functional mapping or $R_+ \rightarrow R_+$ in which a_j and c_i denote intercepts of demand and supply equations in region j and i respectively; b_j and d_i denote slopes of the demand and supply equations; x_i and y_j denote output and consumption of region i and j; t_{ij} and z_{ij} are the unit transportation rate and commodity flow from supply region i to demand region j respectively. For the detail of the model, one can find excellent sources in Takayama and Judge (1971). In order to analyze the responses of the producer's rents, we need to form the Lagrange of the above problem:

$$L(x_i, y_j, z_{ij}, \alpha_j, \beta_j) = \sum_{j\in J} a_j y_j - \tfrac{1}{2}\sum_{j\in J} b_j y_j^2 - \sum_{i\in I} c_i x_i - \tfrac{1}{2}\sum_{i\in I} d_i x_i^2 - \sum_{i\in I}\sum_{j\in J} t_{ij} z_{ij} +$$

$$\sum_{j\in J} \alpha_j (\sum_{i\in I} z_{ij} - y_i) + \sum_{i\in I} \beta_i \left(-\sum_{j\in J} z_{ij} + x_i\right) \qquad (2)$$

Where α_i and $ß_j$ are Lagrange multipliers (imputed steam coal prices) for the jth demand and ith supply region. The corresponding Kuhn-Tucker necessary (also sufficient) conditions take the form as follows:

$$\frac{\partial L}{\partial y_j} = a_j - b_j\overline{y_j} - \alpha_j \leq 0 \qquad and \qquad \frac{\partial L}{\partial y_j}\cdot\overline{y_j} = 0 \tag{3}$$

$$\frac{\partial L}{\partial x_i} = -c_i - d_i\overline{x_i} + \beta_i \leq 0 \qquad and \qquad \frac{\partial L}{\partial x_i}\cdot\overline{x_i} = 0 \tag{4}$$

$$\frac{\partial L}{\partial z_{ij}} = \overline{\alpha_j} - \overline{\beta_i} - t_{ij} \leq 0 \qquad and \qquad \frac{\partial L}{\partial z_{ij}}\cdot\overline{z_{ij}} = 0 \tag{5}$$

$$\frac{\partial L}{\partial \alpha_j} = \sum_{i\in I}\overline{z_{ij}} - \overline{y_j} \geq 0 \qquad and \qquad \frac{\partial L}{\partial \alpha_j}\cdot\overline{\alpha_j} = 0 \tag{6}$$

$$\frac{\partial L}{\partial \beta_i} = x_i - \sum_{j\in J} z_{ij} \geq 0 \qquad and \qquad \frac{\partial L}{\partial \beta_i}\cdot\overline{\beta_i} = 0 \tag{7}$$

Where the barred variables are optimum values.

The knowledge of equations (3), (4). (5), (6) and (7) is not adequate for the analysis since not all steam coal flows are positive in equation (5). We need to borrow a theorem by Silberberg (1970) and Gass (1985) that no more than m+n-l positive flows can appear in the base as optimal solutions. Therefore, the knowledge of m+n-l steam coal flow patterns must be known before conducting the impact analysis. This is the major difficulty in conducting a general sensitivity analysis in any mathematical programming model of this type. Hence, a suitable statistical test is necessary to complete such a stability test.

Given known flow patterns, we substitute equations (3) and (4) into equation (5) for $x_i > 0$ and $y_j > O$. In addition, by adding equation (6) to (7) for $\alpha_j > 0$ and $\beta_i > 0$, we have a system of m+n equations for the non-degeneracy case as shown below:

$$a_j - b_j\overline{y_j} - c_i - d_i\overline{x_i} = t_{ij} \qquad \forall\overline{z_{ij}} > 0 \tag{8}$$

$$\sum_{j\in J} y_j = \sum_{i\in I} x_i \tag{9}$$

Relation (8) and (9) form the base for our analysis and x_i's and $y_{j's}$ are proven to be uniquely solvable (Irwin and Yang 1982, 1983) since, in a single commodity spatial equilibrium model, the Kuhn-Tucker conditions are both necessary and sufficient for the sensitivity analysis. Note in the case of degeneracy which generates isolated trade patterns we have less than m+n-1 flows. In such a case we shall have more of equation (9) or information on "isolated trade patterns" to make up the loss in numbers of equation (8). This is what Samuelson (1952) called "degeneracy" in the sense that one block of steam coal flows are completely independent of that of the other block. In the case of zero y_j or x_i , we shall discard the variable until they become positive, since zero demand and supply have little economic meaning.

Rewriting equations (8) and (9) in matrix form and assuming the case of non-degenerate flows, we have

$$\begin{bmatrix} -b_j & -d_i \\ -1 \ldots -1 & +1 \ldots +1 \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \vdots \\ \overline{y_n} \\ \overline{x_1} \\ \vdots \\ \overline{x_m} \end{bmatrix} = \begin{bmatrix} -a_j + c_i + t_{ij} \\ 0 \end{bmatrix} \quad (10)$$

or J K = L and hence K = $J^{-1}L$ where $JxR^{(m+n)x(m+n)}$, KxR^{m+n}, LxR^{m+n}. where R^{m+n} denotes Euclidean mn dimensional space. Differentiating equation (10) with respect to all c_i's (C) and d_i's (D) takes the form

$$\begin{vmatrix} \frac{\partial \overline{y_1}}{\partial C} \\ \vdots \\ \frac{\partial \overline{y_n}}{\partial C} \\ \frac{\partial \overline{x_1}}{\partial C} \\ \vdots \\ \frac{\partial \overline{x_m}}{\partial C} \end{vmatrix} = J^{-1} \begin{vmatrix} 1 \\ \vdots \\ \vdots \\ \vdots \\ 1 \\ 0 \end{vmatrix} \quad (11)$$

$$\begin{vmatrix} \frac{\partial \overline{y_1}}{\partial D} \\ \vdots \\ \frac{\partial \overline{y_n}}{\partial D} \\ \frac{\partial \overline{x_1}}{\partial D} \\ \vdots \\ \frac{\partial \overline{x_m}}{\partial D} \end{vmatrix} = J^{-1} \begin{vmatrix} 0 \\ \vdots \\ 0 \\ \bar{x}_1 \\ \vdots \\ \bar{x}_m \end{vmatrix} \quad (12)$$

Note equation (11) is linear for a given set of slope parameters but equation (12) is not linear for a given set of intercept parameters. These relations are verified through separate computer runs.

MATHEMATICAL PROPERTY OF THE PRODUCER'S RENTS UNDER TAXATION AND CHANGING DEMAND AND SUPPLY ENVIRONMENTS

The net social payoff can be derived by subtracting equation (1) from the corresponding parts of complementary slackness of equations (3) through (7) or

$$NSP(a_j, b_j, c_i, d_i, t_{ij}, \bar{x}_i, \bar{y}_j, \bar{z}_{ij}) = \frac{1}{2}\sum_{i \in J} b_j \bar{y}_j^2 + \frac{1}{2}\sum_{i \in I} d_i\, \bar{x}_i^2 \quad (13)$$

The second term on the right hand side of equation (13) is the producer's surpluses or rents (PS) of the Appalachian coal producers at a set of optimal solutions which correspond exactly to the well-known triangles above the supply curves. Evidently, the steeper the slope of the inverse supply curve (d_i) and/or the larger the optimum output ($\overline{x_\iota}$), the greater the value of rent ($\frac{1}{2}\sum_{i\in J} d_i x_i^2$) will be.

A coal-producing site with cost advantages due to production characteristics (seam thickness, less amount of sulfur dioxide contents, cheap production costs) and/or location advantages (strong demand from a nearby market) will enjoy a significant amount of economic rent; and hence is a source of tax revenues.

Within a given set of commodity flows, a federal specific tax is equivalent to changing all the intercepts of supply equations (C) as shown in equation (11). Hence, its impact on the producer's surplus can be evaluated as

$$\frac{\partial PS_i(a_j, b_j, c_i, d_i, t_{ij}, \bar{x}_i, \bar{y}_j)}{\partial C} = d_i \bar{x}_i \left(\frac{\partial \bar{x}_i}{\partial C}\right) \tag{14}$$

By the similar line of reasoning, the impact of an ad valorem tax (i.e., increasing the value of c_i and d_i by v/(l-v) for all i∊I) on the producer's surplus for a very small v is (1)

$$\frac{\partial PS_i}{\partial v} = \bar{d}_i \bar{x}_i \left(\frac{\partial \bar{x}_i}{\partial d_i}\right)\left(\frac{\partial d_i}{\partial v}\right) + \bar{d}_i \bar{x}_i \left(\frac{\partial \bar{x}_i}{\partial c_i}\right)\left(\frac{\partial c_i}{\partial v}\right) + \bar{x}_i^2 \left(\frac{\partial d_i}{\partial v}\right) \tag{15}$$

Where $\frac{\partial d_i}{\partial v} = \frac{v}{(1-v)}\overline{d_\iota}$ and $\frac{\partial c_i}{\partial v} = \frac{v}{(1-v)}\overline{c_\iota}$, in which c_i and d_i are the original unperturbed parameters.

The effect of changing slopes of demand and supply equation(s) in our case can be evaluated for given flow patterns:

$$\frac{\partial PS_i}{\partial b_j} = \bar{d}_i \bar{x}_i \left(\frac{\partial \bar{x}_i}{\partial b_j}\right) \tag{16}$$

$$\frac{\partial PS_i}{\partial d_i} = \bar{d}_i \bar{x}_i \left(\frac{\partial \bar{x}_i}{\partial d_i}\right) + \frac{1}{2}\overline{x_\iota^2} \qquad \forall\, i \in M \tag{17}$$

It is important to know that relations in equations (14), (15), (16) and (17) are neither deterministic nor linear; and they hold only in a given set of positive flow patterns. As is the case of analyzing response surface in flows, consumptions, and productions (Yang and Labys 1981, 1982; Page and Yang 1984) a deterministic conclusion is not feasible. However, once the directions of flow patterns are known, these relations can be predicted locally. In a later section, we shall employ a nonparametric test to perform the analysis on the producers' rents.

THE IMPACT ANALYSES OF SPATIAL RENTS

Based on the estimated equations shown in Tables 1A and 1B we will first investigate the response surface of the producer's rents for each supply region. The responses of the producer's rents under federal specific or unit taxes (dollars per ton of coal produced) are shown in Table 2. One wishes to ask if the producer's rents decrease monotonically as the federal tax rates are increased as is expected in classical welfare economics. Surprisingly enough, we observe a welfare anomaly as can be seen from Table II: the producers' surpluses in Pennsylvania and Maryland coal mines increase from 2.3455 units (in 1973 10^7 dollars) in a no tax case to 2.38579 units with the imposition of fifty cents per ton of federal unit tax at the expenses of other producing regions.(2) As the tax rates are increased from fifty cents to one dollar per ton of coal produced, the producer's surpluses in the Pennsylvania and Maryland area drop rapidly to 1.71207 as compared with the fifty cent tax case (2.38579). As we arbitrarily increase the tax rate, the producers' rent in the same region bounces back again. Hence we observe an oscillating welfare position for Pennsylvania and Maryland coal mines. This observation contradicts the classical space-less model in which an increase in the unit tax is expected to decrease producer's rents. In light of this anomaly, an analytical and deterministic approach is not possible in evaluating welfare positions of producers in the Appalachian steam coal market.

Table 1A: Estimated Regression Equations

	Dependent Variable (Prices)	Adjusted Intercept *	Slopes
	P^d_1	69.8 (0.80)	-462.73 (-0.95)
	P^d_2	86.4 (19.33)***	-33.03 (-17.79)***
DEMAND EQUATION	P^d_3	57.5 (23.38)***	-20.5 (-16.50)***
	P^d_4	49.7 (2.68)**	-40.08 (-16.01)***
	P^d_5	61.3 (16.19)***	-12.71 (-21.46)***
	P^d_6	47.4 (7.9)***	-17.79 (-8.73)***
	P^d_7	61.6 (2.73)**	-22.13 (-9.28)***
	P^d_1	27 (29.28)***	4.46 (1.52)
DEMAND EQUATION	P^d_2	26.3 (24.85)***	4.221 (4.09)***
	P^d_3	25 (6.38)***	14.259 (4.24)***
	P^d_4	30.4 (15.22)***	25.07 (9.25)***
	P^d_5	23 (24.73)***	24.35 (9.28)***
	P^d_6	27 (118.02)***	2.6028 (4.01)***
	P^d_7	28.2 (9.21)***	59.62 (2.92)**

Intercept includes adjustment for exogenous variable influence. Values within parenthesis are t-va1ues.* *The original demand functions were estimated in the linear form of P = a û By + fz where z is a set of exogenous variables, which are emerged into the intercept term shown in the table. Values within parentheses are t-va1ues. Note that *,** and * denote significant at 1%,5% and 10% significance level *SOURCE: C. W. Yang, "A Critical Analysis of Spatial Commodity Modeling: The Case of Coal," Unpublished Ph.D. Dissertation (1979), Department of Economics, West Virginia University. W. C. Labys and C. W. Yang, "A Quadratic Programming Model of the Appalachian Steam Coal Market," Energy Economics, Vol. 2, No.2 (April 1980), pp. 86-95. Also see references (9) and (16).*

Table 1B: Transportation Cost (cents per million BTU)

FROM	1	2	3	4	5	6	7
TO							
1	19.1	22.2	20.2	20.5	21.2	24.3	26.8
2	14.5	18.3	13.5	16.9	17.6	20.4	23.3
3	13.2	13.8	11.8	16.6	20.3	16.7	19.8
4	18.8	19.8	19.6	22.5	21.8	17.1	18.1
5	15.1	13.3	13.7	12.6	17.2	13.4	15.7
6	21.1	22.5	21.8	19.2	14.1	16.9	7
7	21	24	20.9	16.5	14.7	13.6	14.3

Appalachian coal production states include: l=Pennsylvania, Maryland; 2=Ohio; 3=Northern West Virginia; 4=Southern West Virginia; 5=Virginia; 6=East Kentucky, Tennessee; and 7=A1abama. Eastern utilities coal consumption states include: 1 = Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island, Vermont; 2 = New Jersey, New York, Pennsylvania, Washington DC, Maryland, Delaware; 3 = Indiana, Michigan; 4 = Illinois, Wisconsin, Minnesota; 5 = West Virginia, Ohio, Kentucky; 6 = Tennessee, Alabama, Mississippi; 7 = Virginia, North Carolina, Southern Carolina, Georgia, and Florida. The unit transportation cost from supply region 1 to demand region 1 for instance is 19.1 cents per million BTU. SOURCE: P. H. Mutschler, R. J. Evans and G. M. Larwood, Comparative Transportation Costs of Supplying Low-sulfur Fuels to Midwestern and Eastern Coal Markets, IC 8614, US Bureau of Mines, Washington, DC, 1972.

Table 2: Producer's Rents with Federal per Unit Taxes (in 1973 107 dollars and dollars per ton unit tax)

		Tax Rates					
Coal Supply Region	No Tax Imposed	$.50/tax	$1/tax	$1.50/tax	$2/tax	$2.50/tax	$3/tax
PA	2.34550	2.38578	1.71207	1.82547	1.25514	1.34560	0.86370
MD		+*	-*	+	-	+	-
OH	2.16387	1.55580	1.23575	0.84335	0.62140	0.35024	0.21322
		-	-	-	-	-	-
Northern	2.01157	1.76127	1.64820	1.4621	1.36641	1.19368	1.10738
WV		-	-	-	-	-	-
Southern	0.01524	0.00393	0.00144	0	0	0	0
WV		-	-				
VA	0.83426	0.72650	0.68550	0.60773	0.60094	0.50100	0.47027
		-	-	-	-	-	-
Eastern	2.31845	1.71586	1.46218	1.07151	0.88665	0.58294	0.44889
KY & TN		-	-	-	-	-	-
AL	0.57414	0.52196	0.36682	0.22871	0.12308	0.04991	0.00922
		-	-	-	-	-	-

** Rents are in 1973 dollars (millions); + denotes the amount of producer's rent that increases (compared with the zero tax case) as an additional tax is imposed; - denotes it decreases as an additional tax is levied. For instance, one dollar per ton federal tax is expected to cause Pennsylvania and Maryland supply region to reduce rent to $1.71207 million in 1973 dollars.*

Such oscillating patterns, however, are not observed in the cases of imposing federal ad valorem taxes (supply) in our example. The changing producer's surpluses are shown in Table 3. As is evident from Table 3, the welfare positions under such taxations deteriorate rapidly for Southern West Virginia and Alabama coal mines. Such a deterioration, especially in West Virginia, would have had profound impacts on local and state economies which depend heavily on the steam coal revenues.

The impacts on producer's rents are shown in Table 4 as slopes of all demand equations (B) are varied. Such variations in slopes may reflect the changing demand conditions for the Appalachian steam coal. The producer's rents in all supply regions of the Appalachian market would increase with the decreasing slopes of the demand schedules. This trend would help internal finance of the coal mine companies, especially in Southern WV, PA, MD, OH, Eastern KY and TN. However, producers in Northern WV, AL and VA would experience only relatively smaller increases in producers' rents.

Table 3: Producer's Rents with Federal Ad Valorem Taxes

	SUPPLY REGION						
TAX RATE	**PA & MD**	**OH**	**Northern WV**	**Southern WV**	**VA**	**Eastern KY & TN**	**AL**
NO TAX	2.3455	2.16387	2.01157	0.01524	0.83426	2.31845	0.57414
1%	2.29236	2.11999	1.98666	0.01297	0.92979	2.25668	0.56271
2%	2.2383	2.0751	1.96117	0.01083	0.82304	2.19382	0.55117
3%	2.18336	2.02926	1.9351	0.00883	0.817	2.12986	0.5395
4%	2.1275	1.98245	1.90845	0.007	0.81065	2.06484	0.52772
5%	2.07076	1.93464	1.88118	0.00533	0.80401	1.99879	0.51582
6%	2.01313	1.88586	1.8533	0.00386	0.79705	1.93173	0.50379
7%	1.95751	1.83897	1.82634	0.00264	0.79055	1.86738	0.48533
8%	1.90823	1.79832	1.80269	0.00172	0.78572	1.81127	0.45133
9%	1.86275	1.76136	1.781	0.00102	0.78188	1.76004	0.41805
10%	1.81735	1.72442	1.75927	0.0005	0.77804	1.70903	0.38557
15%	1.57108	1.51887	1.63807	0	0.75255	1.43105	0.23668
20%	1.29911	1.28398	1.49734	0	0.71685	1.12526	0.11597

Numbers in the table represent producer surplus (rent) in 1973 dollars (millions). For instance, a 5% ad valorem coal or carbon tax is expected to decrease the rent in Pennsylvania and Maryland area from $2.3455 million to $2.07076 million.

Table 4: Producer's Rents with Changing Slopes of All Demand Equations

	SUPPLY REGION						
% Change in all Demand Slopes	PA & MD	OH	Northern WV	Southern WV	VA	Eastern KY & TN	AL
-20%	3.22722	3.03928	2.45385	0.05531	1.05425	3.49491	0.6892
-15%	2.96235	2.77629	2.32413	0.04143	0.98933	3.13729	0.65555
-10%	2.73146	2.54596	2.20847	0.03047	0.93172	2.82682	0.62548
-5%	2.5271	2.34341	2.10494	0.0219	0.88037	2.5562	0.5985
0%	2.3455	2.16387	2.00157	0.01524	0.83426	2.31845	0.5744
5%	2.1835	2.00413	1.92708	0.01015	0.7927	2.10882	0.55204
10%	2.03842	1.86144	1.85032	0.00635	0.75511	1.92324	0.53193
15%	1.90812	1.73361	1.78042	0.00361	0.72099	1.75852	0.51359
20%	1.79035	1.61836	1.71637	0.00175	0.68987	1.61138	0.49675

**If the slopes of all demand functions in the Appalachian market increases by 10% due to a change in the nature of substitution between oil and steam coal, it can cause the rent in Pennsylvania and Maryland area to decrease from $2.3455 million to $2.03842 million in 1973 dollars.*

The last simulation involves changing the slopes of supply schedules. An increasing supply slope of a coal producing region may indicate the increasing cost in extracting an additional ton of coal from a deeper mine deposit. The results are reported in Table 5. With changes made in supply slopes, coal mines in Northern West Virginia and Alabama would experience declining producers' rents as slopes of supply schedules are graduate1y increased, while the rest of supply regions would gain producer-'s surpluses with their slopes getting steeper by the same percentage. Hence, we have observed from Table 5 that response surfaces of supply region's revenues are not monotonic. Losses of revenues in certain regions may be made at the expenses of other regions and there is no single way to tell these directions.

Table 5: Producer's Rents under Changing Slopes of All Supply Equations

	REGION / SUPPLY						
% Change in all Demand Slopes	PA & MD	OH	Northern WV	Southern WV	VA	Eastern KY & TN	AL
-20%	2.10433	1.89299	2.07167	0.00076	0.82667	1.84948	0.60162
-15%	2.16929 +	1.966 +	2.05396 -	0.00298 +	0.82843 +	1.97521 +	0.59352 -
-10%	2.23115 +	2.03559 +	2.03775 -	0.00628 +	0.83016 +	2.09454 +	0.58631 -
-5%	2.28942 +	2.10125 +	2.02406 -	0.01044 +	0.83226 +	2.20952 +	0.57992 -
0%	2.3455 +	2.16387 +	2.00157 -	0.01524 +	0.83426 +	2.31845 +	0.57414 -
5%	2.39733 +	2.22301 +	2.00014 -	0.02051 +	0.83618 +	2.42357 +	0.56874 -
10%	2.44829 +	2.27949 +	1.99026 -	0.02616 +	0.83827 +	2.52266 +	0.56404 -
15%	2.49486 +	2.3333 +	1.98053 -	0.03205 +	0.84014 +	2.61801 +	0.55953 -
20%	2.53989 +	2.38339 +	1.9717 -	0.038101 +	0.84185 +	2.70901 +	0.55533 -

*For instance, a 15% increase in all supply functions due to perhaps a stricter pollution standard is expected to increase the rent of Pennsylvania and Maryland region from $2.3455 million to 2.49486 million (1973 dollars). Note this is one of the paradoxical results, which differs from the classical space-less models: a stricter pollution standard is expected to decrease the rent of a supplier.

THE FRIEDMAN TEST ON RENTS OF THE APPALACHIAN COAL PRODUCING REGIONS

The welfare position in terms of locationa1 rents was analyzed mathematically in the previous section. That is, the responses of rents under federal taxes or from changes in general economic environments are typically mathematically intractable. Hence, a statistical procedure is needed to test the overall stability of relative welfare positions (in terms of rankings) for the seven Appalachian coal-producing regions. The producers' rent for the ith region (PS_i) is $\frac{1}{2}d_i x_i^2$ where d_i (regression slope coefficient of the ith supply region) is essentially normally distributed and $\overline{x_i}$ represents the optimal coal production from the concave quadratic programming model. As the result, the probability distribution for the producer's rent PS_i may well not be normally distributed. Hence, the conventional analysis of variance cannot be used to test the stability of producers' rents. Instead, the Friedman Test (M. Friedman, 1937 and 1940) is used to perform the analysis of variance with the assumption that normality of the rent is violated.

The null hypothesis of the Friedman Test is that each ranking of the producers' rent within each block (row) is equally likely (i.e., the relative welfare positions in terms of the producers' rents is the same for each of seven Appalachian coal-producing regions). To avoid the violation on the assumption of the Friedman Test (Iman and Conover, 1989), we chose only those policies that generate independent rankings within each block (i.e., policy changes must be significant enough to avoid the identical ranking). The rankings of rents on seven Appalachian coal-producing regions are reported in Table 6 with the sum of ranking and average ranking for each coal producing region R_j and $\overline{R_j}$ shown in bottom lines. To test the null hypothesis, the Friedman F statistic with k-1 and (k-l)*(b-l) degrees of freedom are shown below:

$$F = \frac{(b-1)[B - bk(k+1)^2/4]}{A - B} \quad (18)$$

$$where\ A = \frac{bk(k+1)(2k+1)}{6} \quad (19)$$

$$B = \frac{1}{b}\sum_{j=1}^{k} R_j^2 \qquad (20)$$

b = # of blocks or rows = 18

k = # of treatment or columns = 7

Rj = sum of rank for the j th column

The sample F from equation [18] of the Appalachian coal model is 134.81 and is significantly greater than the critical F = 2.809 at α = 1%. Consequently, the null hypothesis is rejected in favor of the claim that there exists a significantly unequal welfare position among seven coal-producing regions. To perform the multiple comparisons between each pair of coal- producing regions, we adopt the following rule (Iman and Conover, 1989):

$$IF \quad |\bar{R}_i - \bar{R}_j| > t(\frac{2(A-B)}{b(b-1)(k-1)})^{\frac{1}{2}} \qquad (21)$$

then there exists significant difference in relative welfare positions between region *i* and *j*. Note that *R1* is the average ranking for the *i*th region and *t* is evaluated with the significant level of *α/2* and the degree of freedom of *(b-l)(k-l)*. In our simulation, the right hand side of equation (21) equals 0.65 and it indicates that there exist significant differences in producers' rents between each pair of coal producing regions except Northern West Virginia and Eastern Kentucky-Tennessee in which the difference is insignificant. An examination of R's in Table 6 reveals that sizes of rents of seven coal producing regions can be ranked as shown in the last row: Pennsylvania and Maryland coal mines would receive highest location rent while Southern West Virginia coal mines remain in the least advantageous position.

CONCLUDING REMARKS

In the midst of green energy and in a carbon-constrained world, advocates for a permanent and increasing carbon tax seems to carry the day around the world. Steam coal is no doubt the primary source to generate electricity at utility companies in the U.S. The abundance of supply has its flip side: it is responsible for rapid increases in carbon dioxides and related pollutants. The close substitution between the coal and crude oil makes its price go hand-in-hand with volatile oil prices. For instance, the spot price was $57.40 per short ton during the recession (December, 2009).Over one hundred dollars per ton is entirely possible in the future when the demand is strong.

The role of steam coal constrained by the Kyoto Protocol and the Copenhagen Climate Summit, which calls for reduced emissions of green house gases and related measures, begins to become more important than ever. Coupled with rapidly increasing demand from China and India, a study on the rents of the Appalachian coal mines is as important as it is timely. Stricter pollution controls may be viewed as increasing slopes of all supply functions in the region. A federal ad valorem carbon tax is equivalent to shifting the supply functions proportionately. And a switch from high crude oil price can be modeled as decreasing the slopes of all the demand functions. Coal is known as a bulky commodity and as such the spatial equilibrium model is an ideal candidate for this purpose.

Due to the nonlinear nature of the rent, we expect the results to be different from the space-less economic models that have limited predictive power. The nonlinearity begs the use of Milton Friedman's nonparametric model developed 70 years ago. By using the spatial equilibrium model (Labys and Yang, 1980; Yang and Labys, 1985), we have performed simulations by calibrating parameters in demand and supply functions. We have shown that response surfaces of the producers' surpluses are in general nonlinear with respect to changes in any parameters. Also, they are closely related to a given flow pattern in which only m+n-l positive coal flows prevail. Only when the flow patterns change would the response surfaces of the producers' surplus undergo structural changes. In this light, changing welfare positions of coal-producing regions are analytically unpredictable and in some cases do not follow the same direction.

Table 6: Rankings of Welfare Positions of Appalachian Coal Production Regions

	PA & MD	OH	Northern WV	Southern WV	VA	Eastern KY & TN	AL
No tax imposed	7	5	4	1	3	6	2
$1/ton tax(")	7	4	6	1	3	5	2
$2/ton tax	6	4	7	1	3	5	2
$2.5/ton tax	7	3	6	1	4	5	2
$3/ton tax	6	3	7	1	5	4	2
5% sales tax (supply)	7	5	4	1	3	6	2
8% sales tax (supply)	7	4	5	1	3	6	2
10% sales tax (supply)	7	5	6	1	3	4	2
15% sales tax (supply)	6	5	7	1	3	4	2
B(Change in all demand slopes)= -10%	6	5	4	1	3	7	2
B=5%	7	5	4	1	3	6	2
B=15%	7	4	6	1	3	5	2
B=20%	7	5	6	1	3	4	2
D(Change in all supply slopes)= -20%	7	5	6	1	3	4	2
D=-15%	7	4	6	1	3	5	2
D=-15%	7	4	5	1	3	6	2
D=-15%	7	5	4	1	3	6	2
D=-15%	6	5	4	1	3	7	2
Sum	121	80	97	18	57	95	36
$\bar{R}_t$	6.72	4.44	5.39	1	3.167	5.27	2

**A rank score 7 is considered the best (largest) rent revenue scenario whereas a rank score 1 denotes the worst (smallest) rent revenue scenario. For instance, the financial situation in Southern West Virginia did not improve for sometimes as can be seen from its score of 1.*

Consequently, one must be cautious in implementing the policy in the spatial allocation model in which transportation cost constitutes a good portion of the commodity price. While the taxation or other impact analyses on a space-less market in which transportation cost is zero have been long known in classical economic theory, the same cannot be said in the model of the spatial separated markets which are far more empirically relevant. The intractability of the spatial model suggests a proper use of simulation analyses. For instance, some federal carbon taxes may lead to the improvement of the financial positions of some regions at the expense of other coal supply regions. This is surprising and contradicts the long-established result in the classical space-less model.

That is, production taxes decrease producers' welfare (rent). Furthermore the result of the Friedman test indicates that the relative welfare position in terms of producers' rents differs significantly in our simulation. While coal mines in Pennsylvania, Maryland, Northern West Virginia, Tennessee, and Eastern Kentucky continue to enjoy better revenues from location rents, Southern West Virginia and Alabama coal mines remain in the disadvantageous positions within all reasonable parameter values in our simulation. Therefore, a good policy (e.g., a differential federal coal tax) is not possible without taking into consideration comprehensive considerations of welfare positions of the entire steam coal markets in the Appalachian steam coal markets.

Our paper has some limitations as well. First, the empirically estimated demand and supply functions are old and have limited predictive power. Second, an important vehicle in the carbon-constrained world cap and trade is not directly modeled. They remain, however, interesting avenue for future research in the field.

ENDNOTES

(1) A supply ad valorem coal or carbon tax at rate v is equivalent to dividing both intercepts and slopes by l/(l/v). Hence, the increase in both intercept and slope is l/(l-v) - 1 = v/(l-v), see Henderson and Quandt (1971).

(2) The producer's surpluses are computed as follows. First, 7 estimated regional demand and supply coal equations coupled with 49 transportation costs were fed into the quadratic programming subroutine (Cutler and Pass, 1971) to obtain optimum xi's, yj's and zij's.

Second, the producer's surplus can then be computed as PS = $\frac{1}{2} d_i \overline{x_i^2}$.

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BIOGRAPHY

Dr. Chin-Wei Yang is a professor of Economics at Clarion University of Pennsylvania and National Chung Chen University. He can be contacted at College of Business, Clarion University of Pennsylvania, Clarion, PA 16214. Telephone: (814) 393 2609 office and (814) 393 1910 fax. Email: yang@clarion.edu

Dr. Ken Hung, is a professor of Finance at Texas A&M International University. He can be contacted at Sanchez School of Business, Texas A&M International University, 5201 University Boulevard, Laredo, Texas 78041. Telephone: (956) 326 2541 office and (956) 326 2479 fax. Email: ken.hung@tamiu.edu