ACCOUNTING & TAXATION ♦ Volume 3♦ Number 1 ♦ 2011

HIGHER ORDER MOMENTS RESAMPLING

Giuseppe Galloppo, Università di Roma Tor Vergata

ABSTRACT

This paper develops a set of portfolio optimization models that involve a resampling approach of the higher order moments of financial assets return distributions. Specifically, the first four moments are examined. The Resampled Efficiency (RE) techniques introduce Monte Carlo methods to properly represent investment information uncertainty in computing minimum variance (MV) portfolio optimality. Notwithstanding the central limit theorem, for both the academic and financial communities it is a well known fact that stock market returns exhibit latent higher moment risk in the form of negative skewness and high kurtosis. Taking cue from these considerations we have added higher-order moments to the resampling rule. We discuss the solution of the higher order moments resampling approach by replaying an investment game. The game compares the performance of a player using four portfolio schemes for determining portfolio weights using a Monte Carlo based resampling approach. Extensive computational results are obtained on a real-world dataset with two different resampling approaches. Surprisingly, when higher moments of stock return distributions are accounted for in the resampling optimisation algorithm success is mixed.

JEL: G11

KEYWORDS: higher-order moments, resampled efficiency (RE), Monte Carlo, MV portfolio optimality

INTRODUCTION

Optimal asset allocation has generated considerable interest in finance since the seminal papers by Merton (1969) and Samuelson (1969) but important caveats remain. One of these caveats is estimation error in parameters. Empirical evidence indicates that asset returns are partially predictable. Three methods are currently available to address estimation errors. One deals with changing the objective function to explicitly include estimation risk. One form of this approach is often called "robust" optimization and aims at explicitly incorporating estimation error into the portfolio optimisation process (Täutäuncäu and Käoenig, 2004, Ceria and Stubbs, 2005). According to Täutäuncäu and Käoenig (2004) robust optimization consists of finding solutions to optimization problems with uncertain input parameters. Uncertainty is described using an uncertainty set which includes all, or most, possible realizations of the uncertain input parameters. On this issue Sherer (2006) show that the optimality of robust optimisation critically depends on the complicated interplay between risk aversion and uncertainty aversion.

An alternate is Bayesian methods which have a very strong rooting in decision theory. This approach involves rescaling the input parameters to certainty equivalent values. This latter approach consist of using of quadrature methods (Ang and Bekaert (2001), Lynch (2001)) or resampling methods based on Monte Carlo simulations (Barberis (2000)) to find a range of optimal portfolios. The user picks the one preferred according to a certain objective function. Quadrature methods may not be very precise when the underlying asset return distributions are not Gaussian, as is strongly suggested by empirical research, (see Bollerslev et al., 1992 and Gallant and Tauchen, 1989). While Monte Carlo methods do not suffer from this problem, they can be computationally expensive to use as they rely on discretization of the state space and use grid methods. Besides with regard to other approaches, resampling methods have additional benefits related to trading costs. In this article we focus on the resampling approach. Resampling is based on a stochastic simulation procedure where resampled returns and standard deviations are derived stochastically using the original historical optimiser inputs (New Frontier Advisors, 2001).

Markowitz and Usmen (2003) show resampled efficiency optimized portfolios exhibited superior performance on average and in each of their 30 individual tests. Previous works have focused only on resampling the mean and variance ignoring higher order moments. They also focus their research effort on mean-variance approaches. This article enriches the previous literature on both fronts by using the higher moments resampling approach with regard to model portfolios. Specifically, we examine Markowitz Mean-Variance, Tracking Error Minimization (TEM), Mean Absolute Deviation Minimization (MADM) and Shortfall Probability Minimization Models (SPM). The remainder of the paper is organized as follows. In Section 2, we present the empirical methodology that we use in Section 3. In Section 3 we describe the data source and portfolio strategies. Next empirical results obtained from a dataset consisting of equity returns are presented. Section 4 summarizes the findings and provides some concluding remarks.

LITERATURE REVIEW

Many researchers in empirical and theoretical articles have argued that the higher moments of return distributions, such as skewness and Kurtosis, cannot be neglected unless there is reason to believe that the equity returns have a normal (symmetrical) probability distribution. When a set of asset returns has a multivariate normal distribution, the correlation matrix contains all the information about the statistical dependence among them. Unfortunately, as it has been observed in various recent papers (e.g. Embrechts et al., 2002), there is ample evidence that the behaviour of stock market returns does not agree with the frequently assumed normal distribution. Moreover, it is well known that stock market returns have negative skewness and excess kurtosis. This stylized fact has been supported by a huge collection of empirical studies. Some papers on this issue include Ibbotson, (1975), Prakash et al., (2001), Bates (1996), Jorion (1988), Hwang and Satchell (1999), and Harvey and Siddique (1999, 2000).

The role of higher moments has become increasingly important in the literature mainly because the traditional measure of risk, variance, has failed to fully capture the "true risk" of stock market returns. Homogeneous and severely asymmetric distributions show that the mean-variance criterion does not correctly approximate expected utility. In this case an higher moment optimization better approximates the expected utility (Athayde and Flôres, 2004). However analytical closed solutions are available only if marginal distributions have defined functions such as the multivariate skewed Student’s t (Jondeau and Rockinger, 2005). Other marginal distributions do not have closed formulas to be applied yet.

Recently, elegant non-parametric solutions to the optimization problem with co-skewness and co-kurtosis matrix have been proposed by Jondeau and Rockinger (2006). However they propose an approximation of the utility function given by Taylor expansion up to order four and thus they rely on a defined utility function (CARA) for the investor. A recent work by Harvey, Liechty et al., (2004) propose a method to address both estimation risk and the inclusion of higher moments in the portfolio selection. They document that the multivariate normal distribution is not useful for modelling portfolio returns because it does not allow for skewness of returns. Also they suggest specifying a Bayesian probability model for the joint distribution of the asset returns when these returns are driven by a Skew Normal distribution. This allows us to capture the asymmetry of the returns and include it in the portfolio selection task. In such a Bayesian framework the expected utilities are then maximized using predictive returns.

Konno et al. (1993) consider the problem where the portfolio’s skewness is maximized under constraints on expected return and variance. In the presence of higher order moments, optimizing with respect to mean and variance only can lead to highly undesirable effects, as the mean-variance optimization problem is oblivious to skewness and kurtosis. Trying to circumvent some of the failures of the MV approach, several researchers have proposed advances to the traditional mean variance theory in order to include higher moments in the portfolio optimisation task (see Athayde and Flores (2001), Adcock (2002), Jondeau and Rockinger (2004) among others). For example, Harvey and Siddique (1999, 2000) pointed out that the skewness of stock returns is relevant to portfolio selection. Their argument is if asset returns have no diversifiable co-skewness, expected returns must reward for it. Lai (1991), Chunhachinda et al. (1997), Prakash et al. (2003) and Sun and Yan (2003) have applied the polynomial goal programming method (PGP), introduced in financial research by Tayi and Leonard (1988), to the portfolio selection with skewness. In the hedge fund context, recent research has proposed new methods to include higher moments in the hedge fund portfolio selection. A work by Bacmann and Bosshard (2003) suggests using an asymmetric risk measure in order to penalise fat negative tailed investments and reward investments with fat positive tails. The role of skewness and kurtosis has also been remarked by Niu and Cui (2002), and Sun and Yan (2003). This suggests that true risk may be a multi-dimensional concept and that other measures of distributional shape such as higher moments can be useful in obtaining a better description of multi-dimensional risk.

Resampled Efficiency™ (RE) optimization and rebalancing, first proposed in Michaud (1998), introduces a multivariate normal Monte Carlo simulation for asset returns whose parameters are calibrated on the historical vectors of average returns, average standard deviations and the correlation coefficient matrix, to more realistically reflect the uncertainty in investment information. The process of stochastic simulations, otherwise known as Monte Carlo simulations, is a mathematical technique that factors in randomness (Kautt, 2001). Parametric Resampling converts all input parameters into a multivariate normal distribution and takes random draws from the multivariate normal to generate new scenarios. An optimal portfolio for each scenario formed and a method to average across all optimal portfolios to find a good compromise is developed.

RE technology also includes statistically rigorous portfolio trading and monitoring rules and tests for assets avoiding the often ineffective and costly rebalancings typical of the MV optimization asset management process making resampled portfolios more stable and have the added benefits of simplifying the management of a portfolio. In this regard Michaud (1998) suggests that data input resampling leads to asset allocations that are more robust and intuitive relative to classic mean-variance analysis using historical data. It is worth noting that Michaud's approach does not consider tail dependences and extreme (negative) returns (tail risk), not assumed in the classical multinormality assumption.

To cover this gap in literature, we replay an investment game that compares the performance of a player using Monte Carlo based resampling approach advocated in Michaud (1998), with a player that uses a resampling approach in which also high moments namely skewness and kurtosis are taken into account. Moreover, we perform high order resampling with regard to several model portfolios. Specifically, Mean-Variance, Tracking Error Minimization (TEM), Mean Absolute Deviation Minimization (MADM) and Shortfall Probability Minimization Models (SPM). The proposed heuristic method can be analytically divided in four steps that we perform for each portfolio model.

Step 1. Sample a mean vector and covariance matrix of returns from a distribution of both cantered at the original (point estimate) values normally used in portfolio optimization. Unlike all previous applications found in literature we consider the first four moments for each sample distribution namely mean, variance Kurtosis and Skewness. The kurtosis is a function of both the second and fourth central moments of the underlying distribution; that is, the kurtosis is a multi-dimensional measure of risk. It then follows, in general, that risk is multidimensional and depends not only on the scale but also the shape of the underlying distribution of returns.

THE MODEL

We consider every stock return, as a process $\{y_t\}_{t=1,2,..,N}$ assuming that the y_t's are independent and identically distributed with a cumulative distribution function *F*. The conventional coefficients of skewness and kurtosis for y_t are given by:

$$SK_t = E\left(\frac{y_t-\mu}{\sigma}\right)^3 \qquad KR_t = E\left(\frac{y_t-\mu}{\sigma}\right)^4 - 3 \tag{1}$$

where $\mu = E(y_t)$ and $\sigma^2 = E(y_t - \mu)^2$, and expectation E is taken with respect to F. Given the data $\{y_t\}_{t=1,2,..,N}$ SK_t and KR_t are usually estimated by the sample averages

$$\widehat{SK}_t = T^{-1}\sum_{t=1}^{N}\left(\frac{y_t-\hat{\mu}}{\hat{\sigma}}\right)^3 \qquad \widehat{KR}_t = T^{-1}\sum_{t=1}^{N}\left(\frac{y_t-\hat{\mu}}{\hat{\sigma}}\right)^4 - 3 \tag{2}$$

where $\hat{\mu}$=$T^{-1}\sum_{t=1}^{N} y_t$, $\widehat{\sigma^2}$=$T^{-1}\sum_{t=1}^{N}(y_t - \hat{\mu})^2$

Step 2. The simulated resampled data were used as data inputs for the optimizer, in other words these stochastically derived inputs are, in turn, used as inputs into a portfolio optimization algorithms. Step 3, the simulation (repetition of Step 1 and Step 2) was subjected to 500 trials. We get 500 mean vectors and covariance matrices. Given the level of uncertainty inherent in determining inputs, the resampling process leads to many alternative outcomes based on the original inputs. The number of simulated observations is a free parameter of the RE optimization process and is a natural way to model the amount of confidence an investor has in their risk-return estimates. So the number of simulated observations is a mechanism for tuning the optimization process according to the level of certainty and time horizon associated with estimates. Step 4, the listed asset allocation percentages were averaged for the respective portfolios.

It is possible to divide portfolio models, at least chronologically, into two families: the 'traditional models' (CAPM, Markowitz), which constitute the modern theory of portfolio choices, and the so-called 'post-modern' models (MADM, TEV, SPM).

The Tracking Error Minimization Model (TEM) is a parametric model based on two factors: the expected return and the variance of the differential between the performance of the portfolio and the performance of the benchmark, which is the square of the Tracking Error Volatility. The objective is to seek a weight to assign to each asset in the portfolio, in order to obtain the minimum portfolio tracking error, with the constraints that the expected returns to be achieved, are equal to or below a preset level, and that the weightings of the activities are positive and have sums equal to 1.

A generalization of the structure of the constraints is also permitted, in the sense that the presence of arbitrary linear constraints on the structure of the portfolio or lower (upper) bound is permitted.

The objective function to minimize is:

$$\textit{Min variance } \sum(\omega_i \cdot r_i) - \sum(\chi_i \cdot r_i) \tag{3}$$

where:

χ_i = fraction of the benchmark portfolio held in asset i.

$\sum(\chi_i \cdot r_i) = r_t$ (benchmark return)

ω_i = asset i's weight by optimization process

The Mean Absolute Deviation Minimization Model (MADM) is a non-parametric model, based on the idea of finding a benchmark, against which a predetermined over-performance is required. It seeks therefore to achieve a certain return trying, at the same time, not to depart too much from the chosen benchmark. As the risk measure, the distance from the benchmark is adopted, represented by the absolute median difference, calculated over a predetermined period of time. The goal is to find the weight to assign to each security in the portfolio, with the condition of minimum absolute mean deviation, and the security return, or better the portfolio return, expected to be achieved, is equal to or less than a value set in advance. Moreover, the weights of all activities must be positive and of sum equal to 1. It also permits the presence of arbitrary linear constraints in the structure of the portfolio. The model does not take into account hypotheses on the shape of the distribution returns, the only implicit assumption is that the return distribution, observed in the past, remains in the future. The objective function to minimize is:

$$Min_{\omega} \sum \left| \sum (\omega_i \cdot r_i) - \sum (\chi_i \cdot r_{it}) \right| \tag{4}$$

where:

r_i = asset i's return

ω_i = asset i's weight by optimization process

χ_i = fraction of the benchmark portfolio held in asset i.

$\sum (\chi_i \cdot r_{it}) = r_{bt}$ = benchmark return

Remaining with models designed to optimize performance against a benchmark, the Shortfall Probability Minimization (SPM) model aims to reduce the probability of occurrence of an underperformance of the portfolio against a benchmark. The probability of shortfall is estimated over time, by relating the number of periods in which there was a shortfall to the total over the time periods preselected.

The aim of the model of minimizing the shortfall probability is, therefore, to find the weight assigned to each financial instrument so that in a given timeframe, the shortfall frequency is minimal, with the constraint that the expected return is equal to or less than a value to be assigned, and the sum of the weightings is equal to 1. Other restrictions can also be imposed on the linear weights of financial assets.
In this model, as with the previous, the only assumptions on the distribution of returns made are that, they are the product of a stationary process (in this way the past contains useful information for future distribution). The objective function to minimize is:

$$\min_{\omega} \sum_{t=1}^{M} \frac{I_t}{m} \tag{5}$$

where:
It = dichotomic variable, which assumes value equal to 1, in the event that at time t, shortfall occurs, otherwise it assumes value equal to 0.
m = number of sample periods in the time domain considered.

For all portfolio we assume constraints as follow:

The sum of all weights in the portfolio is unity:

$$\sum_{i=1}^{N} w_i = 1$$

And all the weights are positive (no short selling):

$$w_i \geq 0$$

For *Markowitz Portfolio Optimization* we have

$$w_{mv}^{*} = arg_{w\epsilon C} max\, w^{T}\bar{\mu}\text{-}\frac{\lambda}{2}w^{T}\Omega w \tag{6}$$

The traditional optimization problem is given by

$$L(w,\theta) = w^{T}\bar{\mu} - \frac{\lambda}{2}w^{T}\Omega w + \theta(w^{T}I - 1) \tag{7}$$

where θ denotes the multiplier associated with the full investment constraint ($w^{T}I = 1$).

After taking first-order derivatives with respect to the Lagrange multiplier and the vector of portfolio weights, solving for the Lagrange multiplier and substituting this back into the derivative with respect to portfolio weights we arrive at the familiar solution:

$$w_{mv}^{*} = \frac{1}{\lambda}\Omega^{-1}\left(\bar{\mu} - \frac{\mu^{T}\Omega^{-1}1}{1^{T}\Omega^{-1}1}1\right) + \frac{\Omega^{-1}1}{1^{T}\Omega^{-1}1} \tag{8}$$

The Resampled Efficient Frontier is the collection of all possible RE optimal portfolios with risk aversion parameters from expected utility curves ranging from total risk aversion to total risk indifference. The REF plots below the classical efficient frontier because it expects less return and restricts risk to a narrower range.

RESULTS

In order to compare the performance of robust optimization approaches detailed in the previous section with traditional mean-variance and minimum-variance portfolios, we applied a "rolling horizon" procedure similar as in DeMiguel and Nogales (2006). First, the sample estimates of mean returns and covariances are made using an estimation window of *T*=52 weekly observations, which for weekly data corresponds to 1 year. Two, using these samples estimates we compute the optimal portfolio policies according to each strategy. Three, we repeat this procedure for the next period, by including data for the new date and dropping the data for the earliest period. We continue doing this until the end of the data set is reached. At the end of this process, we have generated *L* - *T* portfolio weight vectors for each strategy, where *L* is the total number of samples The out-of-sample performance of each strategy is evaluated according to the following statistics: Total Return, Average Return, Standard Deviation, Downside Risk, Tem, Tev, Information Ratio, Sharpe, Sortino, Beta and Treynor.

The stocks were selected according to market capitalization (large cap stocks) for the top Blue Chip equities for each stock index. We collect weekly data on 3 international indices from yahoo finance from 12/01/2001 to 04/05/2007. The price series for each stock index are subsequently converted to return series. So we define the one-period rate of return during the interval (*j-1*) to *j* as:

$$r_{j} = \frac{P_{j} - P_{j-1}}{P_{j-1}} \tag{9}$$

Not surprisingly, the assumption of a Gaussian normal distribution can be rejected for all of the assets both with a Jarque–Bera, Kolmogorov–Smirnov test and Mardia's test of multivariate skewness and kurtosis. Specifically Mardia's test is based on the Mahalanobis distance of data vector from its sample mean and it allows to reject the hypothesis of the normality if the sample has no significant skew and the measure of kurtosis deviates from expectancy only randomly.

Starting from introduced models, we examine eight investment strategies for each index, for a period of 330 weeks (from 12/01/2001 to 04/05/2007). In this application four models were considered : Madm, Spm, Tev, Markowiztz for each of these two variants were proposed: 1) the application, to the reference model, of the technique of Resampling using Gaussian distributions where one considers only the mean and standard deviation for formulating hypotheses on the distribution of asset returns in the index, (these models will be called with the following code: "model name+ res", eg. Madmres) and 2) the application, to the reference model, of High Order Resampling in which not only we take account of mean and the standard deviation to make assumptions on the distribution of asset returns, but also of Skewness and the Kurtosis (these models will be called with the following code: "model name + resdd such as Madmresdd)

<u>Sp Mib Results</u>

Results for the Madm model, in Table1 show how the application of the Resampling technique has brought a benefit to the reference model in terms of return (average return of +0.04% on a weekly basis compared to the model) and in terms of risk. With reference to this last, it must be noted that this improvement is measured not only in terms of lower standard deviation (-0.04% on a weekly basis) but also in terms of lower Downside Risk (-0.18% on a weekly basis) and Tev (-0.27% on a weekly basis). In light the above discussion, a logical consequence emerges from Rap measures in this regard. Analyzing the Sharpe ratio, Sortino and Information Ratio it can be concluded that the technique of Resampling, model "res", have improved the original model (Madm) in terms not only of risk and return, but also in terms of Rap measures that benefits in terms of risk-adjusted profitability. The same considerations made with respect to the Madmres model in terms of Rap measures can be made to the Madmresdd model. For this model it shows an improvement compared to model Madm in terms of performance (+0.10% on a weekly basis) and SD (-0.29% on a weekly basis) and in terms of the Sharpe Ratio (+4.56%). However there is a deterioration in terms of down-side risk (+0.27% on a weekly basis). Starting from this very last finding it becomes necessary to understand if the rising performance produced by the model is associated with an increase in the acceptable down-side risk than characterized in the Madm model.

To answer this question you need to compare Information Ratio indices. The measure reports an increase of 2.95%. This result allows us to conclude that with the increase in the down-side risk, there is an increase of excess return (relative to the benchmark model Madmresdd) more than proportional to the Madm model or an improvement in terms of adjusted risk return calculated by Information Ratio. Analyzing Spmres and Spmresdd models, a lower performance of the latter emerges in terms of average weekly return (respectively -0.06% and -0.07%,) compared to the Spm model. With regard to risk (Standard Deviation, Down Side risk, Tev), models based on resampling improved the reference model. So it becomes essential to analyze the Rap measures to highlight any improvements made by the Spmresdd and Spmres to the Spm model. In this case, and for these models, in all cases a benefit is shown by using the Resampling techniques on the Reference Model (Spm). It can be concluded with reference to the Spm model that Resampling techniques have brought an increase in the risk-adjusted performance even though there has been, both for Spmres and for Spmresdd, a worsening in terms of weekly average return.

For the Tev model, particularly with regard to Tevres and Tevresdd variants, there are two opposing scenarios from a standpoint of efficiency and risk, for which you can reach the same conclusions with regard to Rap measures. Taking the average weekly return, one can observe a positive differential in relation to the Tevres model (+0.08%) and a negative differential in respect to the Tevresdd model (-0.03%). Conversely, for risk measures an improvement to the Tev model is observed, -0.57% with reference to standard deviation, as a result of the Tevresdd model and a worsening to the Tevresdd model for the Down side risk and Tev (-1.03% and -0.09% respectively). However both Tevres and Tevresdd models improve the Rap indicators of the Tev model. Finally, regarding the application of Resampling techniques to the Markowitz model, the Markres model has a differential output compared to the reference model, negative. The differential for the risk measures, as for that of performance is negative, this evidence is reversed on the Rap indicators leads to a negative differential. The Markresdd model presents characteristics diametrically opposed. It is possible to observe improvements in performance, risk and Rap. In this case the "Resdd" model has not only improved the initial model in all the components of risk and return analyzed, but also has improved version of "res" of that model.

In summary, if we exclude the Markeres models in all other cases the Resampling techniques made an improvement of the Rap indicators compared with strategies derived from the application of reference models. Hitherto Resampling techniques have been considered as an evolution of the reference model, in a way that we can consider the “resdd” models, as evolutions of the “res” models. In fact in the application of “resdd” models, distribution hypotheses can be considered closer to reality than the “res” models, as a matter of fact often the returns can present fat tails or positive and negative asymmetry, not captured by the standard “res” models. In reference to this observation, considering the Rap measures, "resdd" models have a positive differential compared to "res". This means that, except for Sharpe indicators for Tev and Spm models, the application of the technique of "resdd” results in an improvement with respect to the “res” technique. For Tev and Spm models, data shows no valid improvements in terms of Rap. It is noted that the differentials of negative Sharpe are principally a result of the income component, the fact the Dsr, which expresses the negative volatility of the standard deviation, has a negative differential. In conclusion, for the models analyzed, result shows how Resampling techniques identify investment strategies that improve the Rap measures of the portfolios selected compared to standard models. Still, the application of the “resdd" technique led to an improvement in the Rap indicators of the portfolio compared with the “res” technique.

<u>Eux 50 Results</u>

The use of portfolio model strategies on the Eux 50 index did not lead, unlike those used on the SPMib, to a significant improvement from a passive strategy (Table 2). One can try to understand whether the application of Resampling techniques has nevertheless brought a benefit in terms of Rap measures for the models taken into account. Table 2, shows how the Madmres and Madmresdd models have improved not only the Rap indicators of the reference model but also the weekly average return and the various risk indicators. Increasing profitability and reducing risk by the "res" and "resdd" applied to this model reflects, in general, what has been previously discussed with regard to the result shown in Table 1.

Data on average returns shows negative differentials per the related risk measures. Resampling techniques have reduced the risk of the reference model, namely SPM, but not increased profitability. The impact on the Rap measures was opposite to that which occurred in Table 2. For the Spm model, the impact of the condition described above on the Rap measures is positive. In this case there was a worsening of the Rap measures. It can be concluded, that the Res and Resdd techniques created a less risky strategy "sacrificing" the income component of the Rap indicators. The Tevres model empirical findings presents a weekly average return higher than the Tev model but has also led to a worsening of risk components. The Tevresdd model has improved the income component, as Tevres, improving risk measures. Both variants improve the Rap measures of the original model, this means that the Tevresdd model has worked more effectively than Tevres as it has increased profitability by reducing the average risk. Even the Markres and Markresdd models improve in terms of Rap the Rap reference model (Table 2). Summarize comparing applications of the "res" and "resdd" models to the Eux index 50 we can make the following observations: 1) For the Madm model, the Rap differences between “res” and “resdd" models is close to 0. This leads to the conclusion that, given that both techniques improve the basic model, the choice between “resdd” and "res" techniques is, for the Madm model, almost indifferent. 2) For the Spm model, variant “ressdd” provides no benefit over the application of "res"; 3) The variants “resdd” with respect to the variants "res", applied to Tev model, improve Rap measures and risk – return components. 4) The

Markes model and the Markresdd model do not differ in terms of Sortino and Information Ratio Index, whereas variant “resdd" does not make any improvement to the variant "res" in terms of Sharpe Ratio as a result of excessively negative return differential while generating a differential negative standard deviation.

Table 1: Spmib40 Model Results

	Madm	Madmres	Madmddres	Madmres vs	Madmddres vs Madm
Total Return	265.05%	301.34%	377.88%	36.29%	112.83%
Average Return	0.34%	0.38%	0.44%	0.04%	0.10%
Standard Deviation	3.01%	2.97%	2.71%	-0.04%	-0.29%
Downside risk	2.98%	2.80%	3.25%	-0.18%	0.27%
Tem	0.53%	0.62%	0.74%	0.09%	0.20%
Tev	2.93%	2.66%	3.03%	-0.27%	0.10%
Information Ratio	9.91%	11.91%	12.14%	2.00%	2.23%
Sharpe	8.35%	9.74%	12.91%	1.39%	4.56%
Sortino	10.08%	12.53%	13.03%	2.45%	2.95%
Beta	53.58%	62.10%	39.17%	8.52%	-14.41%
Treynor	0.55%	0.54%	1.01%	-0.01%	0.46%
	Spm	**Spmres**	**Spmddres**	**Spmres vs Spm**	**Spmddres vs Spm**
Total Return	379.37%	342.75%	335.90%	-36.62%	-43.46%
Average Return	0.49%	0.43%	0.42%	-0.07%	-0.07%
Standard Deviation	4.24%	3.30%	3.24%	-0.94%	-0.99%
Downside risk	4.36%	3.29%	3.01%	-1.07%	-1.34%
Tem	0.85%	0.70%	0.68%	-0.15%	-0.17%
Tev	3.99%	3.14%	2.86%	-0.85%	-1.13%
Information Ratio	10.28%	11.63%	12.43%	1.35%	2.15%
Sharpe	9.53%	10.25%	10.19%	0.72%	0.66%
Sortino	11.23%	12.20%	13.09%	0.97%	1.87%
Beta	64.72%	57.94%	66.25%	-6.78%	1.53%
Treynor	0.69%	0.66%	0.57%	-0.03%	-0.13%
	Tev	**Tevres**	**Tevddres**	**Tevres vs Tev**	**Tevddres vs Tev**
Total Return	252.94%	325.55%	239.04%	72.61%	-13.91%
Average Return	0.32%	0.41%	0.29%	0.08%	-0.03%
Standard Deviation	2.90%	3.15%	2.33%	0.25%	-0.57%
Downside risk	2.67%	2.96%	1.64%	0.29%	-1.03%
Tem	0.51%	0.67%	0.42%	0.16%	-0.09%
Tev	2.58%	2.79%	1.63%	0.21%	-0.95%
Information Ratio	10.41%	12.22%	15.03%	1.81%	4.62%
Sharpe	8.05%	10.09%	8.64%	2.04%	0.60%
Sortino	10.77%	12.96%	15.05%	2.19%	4.28%
Beta	62.17%	64.86%	68.65%	2.69%	6.48%
Treynor	0.45%	0.56%	0.36%	0.11%	-0.09%
	Markowitz	**Markres**	**Markresdd**	**Markres vs Mark**	**Markresdd vs Mark**
Total Return	393.45%	310.86%	407.37%	-82.59%	13.92%
Average Return	0.45%	0.38%	0.45%	-0.07%	0.00%
Standard Deviation	2.63%	2.66%	2.12%	0.03%	-0.50%
Downside risk	3.04%	3.09%	2.79%	0.05%	-0.25%
Tem	0.72%	0.63%	0.74%	-0.09%	0.02%
Tev	2.90%	2.88%	2.72%	-0.01%	-0.17%
Information Ratio	13.32%	10.81%	14.45%	-2.51%	1.13%
Sharpe	13.72%	10.89%	16.92%	-2.83%	3.20%
Sortino	13.97%	11.57%	14.80%	-2.40%	0.83%
Beta	41.33%	42.56%	31.94%	1.23%	-9.39%
Treynor	0.98%	0.78%	1.26%	-0.19%	0.28%

This table shows the Spmib40 Model results.

Table 2: Ex50 Model Results

	Madm	**Madmres**	**Madmddres**	**Madmres vs Madm**	**Madmddres vs Madm**
Total Return	61.56%	77.98%	74.28%	16.42%	12.72%
Average Return	-0.09%	-0.03%	-0.05%	0.06%	0.04%
Standard Deviation	3.33%	2.96%	2.91%	-0.37%	-0.42%
Downside risk	2.69%	2.45%	2.77%	-0.24%	0.08%
Tem	-0.23%	-0.11%	-0.15%	0.12%	0.08%
Tev	2.76%	2.40%	2.69%	-0.35%	-0.07%
Information Ratio	-4.26%	-2.23%	-2.56%	2.03%	1.70%
Sharpe	-5.46%	-4.11%	-4.74%	1.34%	0.72%
Sortino	-4.16%	-2.27%	-2.64%	1.88%	1.52%
Beta	70.76%	67.75%	57.45%	-3.00%	-13.30%
Treynor	-0.16%	-0.08%	-0.12%	0.08%	0.04%
	Spm	**Spmres**	**Spmddres**	**Spmres vs Spm**	**Spmddres vs Spm**
Total Return	103.50%	88.69%	80.30%	-14.81%	-23.20%
Average Return	0.08%	0.01%	-0.02%	-0.06%	-0.09%
Standard Deviation	3.61%	3.16%	3.11%	-0.45%	-0.50%
Downside risk	3.33%	2.75%	2.65%	-0.58%	-0.69%
Tem	0.11%	-0.02%	-0.09%	-0.13%	-0.20%
Tev	3.18%	2.56%	2.53%	-0.62%	-0.65%
Information Ratio	1.57%	-0.36%	-1.55%	-1.92%	-3.12%
Sharpe	-0.41%	-2.43%	-3.48%	-2.02%	-3.07%
Sortino	1.64%	-0.38%	-1.63%	-2.02%	-3.27%
Beta	67.33%	70.45%	69.48%	3.12%	2.15%
Treynor	0.08%	-0.01%	-0.06%	-0.09%	-0.14%
	Tev	**Tevres**	**Tevddres**	**Tevres vs Tev**	**Tevddres vs Tev**
Total Return	58.39%	73.15%	115.04%	14.76%	56.66%
Average Return	-0.12%	-0.05%	0.08%	0.07%	0.19%
Standard Deviation	3.06%	3.10%	2.67%	0.04%	-0.39%
Downside risk	2.28%	2.62%	0.83%	0.35%	-1.44%
Tem	-0.29%	-0.15%	0.10%	0.13%	0.39%
Tev	2.33%	2.50%	0.79%	0.17%	-1.54%
Information Ratio	-6.11%	-2.66%	6.63%	3.45%	12.74%
Sharpe	-6.74%	-4.42%	-0.44%	2.32%	6.30%
Sortino	-5.97%	-2.79%	6.97%	3.18%	12.94%
Beta	73.49%	69.92%	88.74%	-3.57%	15.24%
Treynor	-0.19%	-0.10%	0.06%	0.09%	0.25%
	Markowitz	**Markres**	**Markresdd**	**Markres vs Mark**	**Markresdd vs Mark**
Total Return	68.37%	134.37%	114.77%	66.00%	46.40%
Average Return	-0.07%	0.21%	0.07%	0.28%	0.14%
Standard Deviation	2.89%	4.16%	2.37%	1.27%	-0.52%
Downside risk	3.72%	3.73%	1.14%	0.01%	-2.58%
Tem	0.51%	0.52%	0.09%	0.00%	-0.42%
Tev	3.65%	3.61%	1.16%	-0.04%	-2.49%
Information Ratio	7.54%	7.37%	4.10%	-0.17%	-3.44%
Sharpe	2.87%	2.80%	-0.85%	-0.07%	-3.72%
Sortino	7.69%	7.62%	4.04%	-0.07%	-3.64%
Beta	85.40%	81.66%	75.34%	-3.75%	-10.07%
Treynor	0.33%	0.34%	0.06%	0.01%	-0.27%

This table shows the Ex50 Model results.

Sp 100 Results

For both reference models, "res" and "resdd", resampling techniques improved not only Rap measures, but also measures of performance and risk. The Tevres model shows an increase in performance compared to a general increase in all measures of risk. This increase did not negatively affect Rap measures. For Rap, positive differentials are reported. Tevresdd worsened the performance of the reference model but reduced the risk producing a net positive effect (excluding the Sharpe Ratio) on the Rap measures. Unlike with Tevresdd, Markresdd and Markres models while presenting the same characteristics in the income and risk components (respectively worsening and improvement over the reference model), has a worsening in Rap measures.

Table 3: Sp100 Model Results

	Madm	**Madmres**	**Madmddres**	**Madmres vs Madm**	**Madmddres vs Madm**
Total Return	86.52%	153.43%	134.81%	66.91%	48.29%
Average Return	0.07%	0.22%	0.18%	0.15%	0.10%
Standard Deviation	4.90%	4.37%	4.23%	-0.53%	-0.67%
Downside risk	4.34%	3.74%	3.97%	-0.60%	-0.37%
Tem	0.27%	0.52%	0.46%	0.25%	0.19%
Tev	4.25%	3.68%	3.82%	-0.57%	-0.43%
Information Ratio	3.31%	7.84%	6.23%	4.54%	2.92%
Sharpe	-0.31%	3.08%	2.10%	3.39%	2.41%
Sortino	3.38%	7.97%	6.48%	4.59%	3.10%
Beta	93.16%	91.06%	74.20%	-2.10%	-18.96%
Treynor	0.15%	0.32%	0.33%	0.17%	0.18%
	Spm	**Spmres**	**Spmddres**	**Spmres vs Spm**	**Spmddres vs Spm**
Total Return	118.15%	266.12%	232.12%	147.97%	113.97%
Average Return	0.21%	0.40%	0.36%	0.18%	0.15%
Standard Deviation	5.77%	4.44%	4.68%	-1.34%	-1.09%
Downside risk	5.41%	3.66%	4.09%	-1.75%	-1.32%
Tem	0.53%	0.78%	0.75%	0.25%	0.21%
Tev	5.07%	3.71%	3.97%	-1.36%	-1.10%
Information Ratio	5.22%	12.67%	10.58%	7.45%	5.36%
Sharpe	2.15%	6.88%	5.84%	4.73%	3.70%
Sortino	5.57%	12.49%	10.88%	6.92%	5.31%
Beta	105.82%	94.29%	96.01%	-11.53%	-9.81%
Treynor	0.27%	0.49%	0.45%	0.22%	0.18%
	Tev	**Tevres**	**Tevddres**	**Tevres vs Tev**	**Tevddres vs Tev**
Total Return	163.82%	199.67%	110.55%	35.86%	-53.27%
Average Return	0.24%	0.31%	0.08%	0.07%	-0.16%
Standard Deviation	4.26%	4.51%	2.98%	0.25%	-1.29%
Downside risk	3.54%	3.86%	1.32%	0.32%	-2.21%
Tem	0.55%	0.68%	0.23%	0.13%	-0.32%
Tev	3.49%	3.77%	1.59%	0.28%	-1.91%
Information Ratio	8.72%	9.83%	10.85%	1.10%	2.13%
Sharpe	3.52%	4.89%	-0.50%	1.37%	-4.02%
Sortino	8.83%	10.06%	9.05%	1.22%	0.21%
Beta	93.99%	95.53%	96.18%	1.54%	2.19%
Treynor	0.33%	0.40%	0.15%	0.07%	-0.18%
	Markowitz	**Markres**	**Markresdd**	**Markres vs Mark**	**Markresdd vs Mark**
Total Return	149.57%	148.67%	130.62%	-0.90%	-18.95%
Average Return	0.21%	0.21%	0.11%	-0.01%	-0.10%
Standard Deviation	4.26%	4.16%	2.58%	-0.10%	-1.68%
Downside risk	3.72%	3.73%	2.22%	0.01%	-1.50%
Tem	0.51%	0.52%	0.33%	0.00%	-0.18%
Tev	3.65%	3.61%	2.18%	-0.04%	-1.47%
Information Ratio	7.54%	7.37%	8.20%	-0.17%	0.67%
Sharpe	2.87%	2.80%	0.93%	-0.07%	-1.94%
Sortino	7.69%	7.62%	8.37%	-0.07%	0.69%
Beta	85.40%	81.66%	64.03%	-3.75%	-21.38%
Treynor	0.33%	0.34%	0.29%	0.01%	-0.04%

This table shows the Sp100 Model results.

CONCLUSIONS

Optimal asset allocation has generated considerable interest in finance but important caveats remain. Once caveat is estimation error in the parameters. Empirical evidence indicates that the behaviour of stock market returns does not agree with the frequently assumed normal distribution. In the presence of higher order moments, optimizing with respect to mean and variance only can lead to highly undesirable effects. According to Markowitz and Usmen (2003) Resampled Efficiency optimized portfolios exhibited superior performance on average. RE technology also avoid the often ineffective and costly rebalancings making resampled portfolios more stable and have the added benefits of simplifying portfolio management. It is worth noting that Michaud's approach does not consider tail dependences and extreme (negative) returns (tail risk).

Other research focuses on the mean-variance approaches. To cover this gap in literature we perform high order resampling, taking into account Skewness and Kurtosis, with regard to several model portfolios. Specifically, we examine Mean-Variance, Tracking Error Minimization (TEM), Mean Absolute Deviation Minimization (MADM) and Shortfall Probability Minimization Models (SPM). We apply the method to a set of blue chip equities from 3 stock indexes (Sp100, Smib40, Ex50). Our result show that the Resampling techniques improved all three markets considered, the Rap measures of the portfolio in 70% of cases analyzed. Considering individual markets: 80% for the Ex 50 index, 63% for the SP100 index and 80% for the Spmib index. High order moments resampling techniques have improved the Rap measures of the respective standard resampling technique in 48% of cases. The results suggest procedures for improving the investment value of estimates are always worthwhile.

REFERENCES

C.J. Adcock (2002), "Asset Pricing and Portfolio Selection Based on the Multivariate Skew-Student Distribution", paper presented at the *Non-linear Asset Pricing Workshop*, April, Paris

A. Ang, G. Bekaert (2001), "International Asset Allocation with Regime Shifts", *SSRN Working Paper Series*

G.M. Athayde, R.G. Flôres (2004), "Finding a Maximum Skewness Portfolio – A General Solution to the Three-Moments Portfolio Choice", *Journal of Economic Dynamics and Control*, n.28, 2004

J.F. Bacmann, U. Bosshard (2004), "Optimising portfolios with asymmetric return distributions: An application to hedge fund styles", *SSRN Working Paper Series*

N. Barberis (2000), ""Investing for the Long Run When Returns Are Predictable"", *Journal of Finance*, No. 55, p. 225-264

D.S. Bates (1996), "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options", *Review of Financial Studies*, No. 9, p. 69-107

T. Bollerslev, R. Chou, K. Kroner (1992), "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence", *Journal of Econometrics*, No. 52, p. 5-59

S. Ceria, R. Stubbs (2005), "Incorporating Estimation Error into Portfolio Selection: Robust Efficient Frontiers", *Axioma Working Paper*

P. Chunhachinda, K. Dandapani, S. Hamid, A.J. Prakash (1997), "Portfolio selection and skewness: evidence from international stock markets", *Journal of Banking and Finance*, No. 21, p. 143–67

CRSP, Center for Research in Security Prices (2005). Graduate School of Business, The University of Chicago.

V. DeMiguel, F. Nogales (2006), "Portfolio Selection with Robust Estimates of Risk", *Working Paper, London Business School*

P. Embrechts, A.J. McNeil, D. Straumann D. (2002), "Correlation and Dependency in Risk Management: Properties and Pitfalls", *in Risk Management: Value-at-Risk and Beyond (ed. M. Dempster), Cambridge University Press*

R. Gallant, G. Tauchen (1989), "Seminonparametric Estimation of Conditionally Constrained Heterogeneous Processes: Asset Pricing Applications", *Econometrica*, No. 57, p. 1091-1120

C. Harvey, R. Campbell, A. Siddique (1999), "Autoregressive conditional skewness", *Journal of Financial and Quantitative Analysis*, No.34, p. 465–487

C. Harvey, A. Siddique (2000), "Conditional Skewness in Asset Pricing Tests", *Journal of Finance*, No. 55, p.1263-1295

C. Harvey, R. Campbell, J. Liechty, C., Liechty, W. Merril, P. M¨uller, (2003), "Portfolio Selection with Higher Moments", *Working Paper, Duke University*

C. Harvey, J. Liechty, M. Liechty, P. M¨uller (2004), "Portfolio Selection with Higher Moments", *mimeo, Duke University*

S. Hwang, S.E. Satchell (1999), "Modelling Emerging Market Risk Premia Using Higher Moments," *International Journal of Finance and Economics*, No. 4, p. 271-296

R.G. Ibbotson (1975), "Price performance of common stock new issues", *Journal of Financial Economics*, Vol. 2, No. 3, p. 235–272

E. Jondeau, M. Rockinger (2004), "Optimal Portfolio Allocation Under Higher Moments", *EFMA 2004 Basel Meetings Paper*

E. Jondeau, M. Rockinger (2005), "Conditional Asset Allocation under Non-Normality: How Costly Is the Mean-Variance Criterion?", *Working Paper, Institute of Banking and Finance, HEC Lausanne*

E. Jondeau, M. Rockinger (2006), "Optimal Portfolio Allocation under Higher Moments", *European Financial Management*

P. Jorion (1988), "On Jump Processes in the Foreign Exchange and Stock Markets," *Review of Financial Studies*, No. 1, p. 27-445

G.G. Kautt (2001), "Stochastic Modelling: The New Way To Predict Your Financial Future", *Virginia: Monitor Publishing Company*

H. Konno, H. Shirakawa, H. Yamazaki, (1993), "A mean-absolute deviation-skewness portfolio optimization model". Ann. Operat. Res., No.45, p. 205–220

T.Y. Lai (1991) "Portfolio selection with skewness: a multiple-objective approach", *Review of Quantitative Finance and Accounting*, No. 1, p. 293–305

A. Lynch, (2001), "Portfolio Choice and Equity Characteristics: Characterizing the Hedging Demands Induced by Return Predictability", *Journal of Financial Economics*, No. 62, p. 67-130

H. Markowitz, N. Usmen (2003), "Resampled Frontiers VersusDiffuse Bayes: An Experiment", *Journal of Investment Management*, Vol. 1, No. 4, p. 9–25

R. Merton (1969), "Lifetime Portfolio Selection: the Continuous-Time Case", *Review of Economics and Statistics*, No. 51, p. 247-257

R. Michaud (1998), "Efficient Asset Management", *Cambridge, MA, Harvard Business School Press*
New Frontier Advisors, LLC (2001), "The Resampled Efficient Frontier – An Introduction: Background, Theory, Tests and Application", *New Frontier Advisors, LLC,* August 3, 2001

Niu, Jijun, Cui, Momo (2002), "Distributional Characteristics of Emergent Market Equity Returns and Contagion Studies of the 2002 Argentinian Peso Crisis", *Working Paper. University of Lausanne*
A.J. Prakash, M. de Boyrie, R. Moncarz (2001), "Applicability of CAPM in Latin American capital markets", paper presented at the *Annual International Trade and Finance Conference*, Montpelier, France

A.J. Prakash, C. Chang, T.E. Pactwa (2003), "Selecting a portfolio with skewness: recent evidence from US, European, and Latin American equity markets", *Journal of Banking and Finance*, No. 27, p. 1375–90

P. Samuelson (1969), "Lifetime Portfolio Selection by Dynamic Stochastic Programming", *Review of Economics and Statistics*, No. 51, p. 239-246

B. Scherer, D. Martin (2005), "Modern Portfolio Optimization with Nuopt for S-Plus", *Springer*, New York

Sun, Qian, Yan, Yuxing (2003), "Skewness persistence with optimal portfolio selection", *Journal of Banking and Finance*, No. 27, p. 1111–1121

G. Tayi, P. Leonard (1988), "Bank balance sheet management: an alternative multi-objective model", *Journal of Operational Research Society*, No. 39, p. 401–410

R.H. Tuntucu, M. Konig, (2004), 'Robust asset allocation', *Annals of Operations Research*, No. 132, p. 132–157

BIOGRAPHY

Giuseppe Galloppo works at Dipartimento CEIS, Università di Roma Tor Vergata**.** He can be contacted at Dipartimento CEIS, Università di Roma Tor Vergata giuseppe.galloppo@uniroma2.it.