# **OPTIMAL INVESTMENT FOR INSTITUTIONAL INVESTORS UNDER VALUE-AT-RISK CONSTRAINTS IN CHINESE STOCK MARKETS**

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## **ABSTRACT**

*Value at Risk (VaR) is defined as the worst expected loss under normal market conditions over a specific time interval at a given confidence level. Given the widespread usage of VaR, it becomes increasingly important to study the effects of the portfolio optimization subject to the VaR constraint set by the fund manager. In this paper, we examine the classical portfolio optimization models and the most popular VaR methodologies. We show that the portfolio optimization models under VaR constraint provide the clear insight to the mean-variance decision. We also consider the problem with the extra tracking error constraint. Furthermore, we provide an empirical analysis on the model by using China's market data. VaR estimates are produced via Monte Carlo simulations.* 

**JEL:** G11; G15; G32

**Keywords:** Portfolio optimization, mean-variance, VaR, Monte Carlo

# **INTRODUCTION**

any investment fund managers choose Mean-Variance analysis and Value at Risk (VaR) as their most important supporting tools in their asset allocation and portfolio allocation decision-making. Nowadays, the fund managers turn to focus on the downside possibility of portfolio and the new Many investment fund managers choose M<br>most important supporting tools in their as<br>benchmark for the measure of risk is Value at Risk.

After VaR was introduced by Philippe Jorion (2001), some researchers also discuss the relationship between Mean-Variance analysis and VaR. However, most of their analyses are in terms of absolute return of portfolio, without taking the benchmark into account. In this paper, we try to highlight the similarities and differences between Mean-Variance analysis and Value at Risk and find out how institutional investors, who care about the relative performance of their portfolio to the benchmark, do their risk-return management under Value at Risk constraint using returns relative to the benchmarks.

We will further investigate to solve the institutional investor's utility maximization problem subject to VaR constraint. In other words, we introduce VaR restriction into the problem of Knight (2005) and extend its research. Furthermore, we use the data from Chinese market to examine and support our conclusion. As a young emerging market, China's stock market has experienced extraordinary growth since the inceptions of the Shanghai and Shenzhen Stock Exchange in late 1990s. On its way to go to the matured market, it has a lot of specialties which make our findings more interesting.

The paper is organized as follows. We present the previous studies in Section II. In Section III, we derive the solution for the constrained maximum problem in mathematical framework. In Section IV we further explain the reason why we choose Chinese capital market as our research objectives and illustrate the characteristics of the data. Section V The conclusions are made in Section VI.

# **LITERATURE REVIEW**

VaR was first introduced and popularized in 1994 by J.P. Morgan's famous RiskMetrics software. The subsequent research works, such as Pichler and Selitch (1999), Jorion (2001) and Alexander (2003) provide a complete analysis of VaR methodology and successfully help VaR become a standard concept in risk management. The two most important components of VaR measures are the length of time period over which market risk is to be measured and the confidence or significance level at which market risk is to be measured. In other words, VaR is used as an estimate of the minimum expected loss (alternatively, the maximum loss) over a set time period at a desired level of significance (alternatively, at a desired level of confidence). For example, a 5% VaR of \$1,000 for a 10-day holding period implies that, given the standard deviation and distribution of returns for the portfolio, there is a 5% probability that the portfolio will lose a minimum of (at least) \$1,000 over the next 10 days. Stated differently, there is 95% confidence the loss will be no greater than \$1,000.

However, Rochefellar and Uryasev (2002) imply that as a quantile, VaR has its own serious shortcomings because it has no reason to be convex. Uryasev (1998) provides an alternative risk measure to VaR, called Conditional Value at Risk (CVaR). Pflug (2000) shows that CVaR is a coherent risk measure that it has many attractive properties including convexity. Conditional Value at Risk is also known as mean excess loss, mean shortfall or tail VaR. So, CVaR is the expected loss given that the loss exceeds VaR.

In this paper, we suppose the institutional investors care more about the whole maximum loss of their investment than the potential excess loss. In this respect, we will continue to use VaR, not CVaR, as the risk constraint in this paper in order to simplify the question.

In recent years, there are a lot of research papers that focus on the effects of CAPM and optimal portfolio selection under VaR constraint( Campbell et al (2001), Basak and Shapiro (2001)). The usual discussion is to develop a mean-risk model and a CAPM or utility maximization subject to a VaR constraint and find out some surprising features of VaR usage. Huisman et al (1999) uses mean variance approach to develop an asset allocation model which allocates assets by maximizing expected return subject to the constraint that the expected maximum loss should meet the Value at Risk limits set by the risk manager. Gaivoronski and Pflug (2000) combine the notion of VaR with portfolio optimality and develop a theory similar to Markowitz theory for optimal mean variance portfolios under VaR constraint. Alexander et al (2003) focus on the portfolio selection problem which yields a portfolio of the minimum CVaR with a specified rate of return.

In all listed papers above, the analysis is in terms of the absolute return of the portfolio. None of them take the benchmark into consideration. Today's portfolio managers, especially institutional investors, are usually evaluated by comparing their outperforming performance to that of their peers or to a benchmark published in guidelines made available to investors. There are some research papers that incorporate the benchmarks into the specific utility functions when they deal with the portfolio optimization problem. This type of performance evaluation method obviously motivates the fund managers to pursue the active management return. Markowitz (1987), Roll (1992), Sharpe (1992), Chan, Karceski, and Lakonishok (1999) and some other papers have introduced a related quadratic tracking approach to minimize the variance of the return difference between the managed portfolio and the benchmark. Recently, to better capture the manager's motivation, Morton et al (2003) consider optimal portfolio allocation under four non-standard benchmark-based utility functions. On the other hand, Knight (2005) represents a mathematical solution to the institutional investors' portfolio optimization in terms of the return relative to their benchmark.

Although these previous works consider the fund manager's aim to outperform of the benchmark through the investment process, they fail to formulate risk management requirements in terms of percentiles of loss distribution. The proposal of this paper is to find out a new approach to optimal portfolio allocation method for institutional investors in Value-at-Risk framework. To some extent, this paper is more closely related to Knight (2005), who study efficient portfolios for institutional investors' utility functions with general risk level constraints, than to Campbell et al (2001), who effectively replace mean variance preferences by mean VaR preferences.

Knight (2005) investigates the problem of calculating the exact distribution of optimal investments in a mean variance world under multivariate normality. The main contribution of Knight's paper is that their risk analysis is based on mean-variance analysis using not absolute or unbenchmarked returns, but relative returns to the benchmark. Under the assumption of normal distribution, Knight (2005) considers the institutional investors' expected utility in terms of relative returns and calculates the exact properties of measures. Campbell et al (2001) consider an optimal portfolio selection model which maximizes the expected return of the portfolio subject to Value-at-Risk constraint rather than standard deviation alone. In their paper, Campbell and his co-authors derive an optimal portfolio such that the maximum expected loss would not exceed the VaR for a chosen investment horizon at a given confidence level. The investors' problem described in Campbell et al (2001) is to maximize the expected level of final wealth under downside risk constraint which is measured by VaR. We can easily find out that there is no assumption of normally distributed returns in Campbell et al (2001) model. However, the analysis in Campbell's paper is obviously put on the absolute portfolio.

In this study, what we are going to do is combine the models analyzed in both Campbell et al (2001) and Knight (2005). In other words, we use the VaR as the investors' risk measure and evaluate the performance of portfolio based on the given benchmark.

#### **PORTFOLIO SELECTION MODEL**

The portfolio optimization in the modern portfolio theory is to allocate the assets by maximizing the expected value of a given utility function or minimizing the expected risk level of the portfolio. We assume that the institutional investors we analyze try to optimize their portfolio by maximizing the following utility function of over performed value between the portfolio and benchmark under VaR constraint:

$$
\max \ E(U) = E\big[U(W_{PT} - W_{BT})\big] = E\bigg[U\bigg(W_0 \cdot \sum_{i=1}^n \big(w_i - b_i\big) \cdot R_i\bigg)\bigg] \tag{1}
$$

subject to  $\sum_{i=1}^{n} w_i = \sum_{i=1}^{n} b_i$ *i n i*  $w_i = \sum b$  $i=1$ 

 $w_i > 0, b_i > 0$ 

*i*

1

$$
\Pr\left\{\sum_{i=1}^{n} w_i \cdot R_i < -VaR^*\right\} \le 1 - c
$$

where  $W_{PT}$  is the expected wealth of the Portfolio p at period T;  $W_{BT}$  is the expected wealth of the Benchmark b at period T, if the "portfolio" Benchmark b has the same initial wealth as the Portfolio p at period 0; *wi* , *bi* are the asset i weight of Portfolio p and Benchmark b respectively during the period from time 0 to time T;  $R_i$  is the gross return of asset i during the period T;  $VaR^*$  is the desired level of VaR value set by the institutional investors;  $c$  is the expected level of confidence.

Following Freund (1956), we assume that the institutional investors are "conservative entrepreneurs" and they have the same negative exponential utility function,  $U(r) = 1 - e^{-\lambda r}$  where  $\lambda$  indicates the investors' aversion to risk. That is, the higher the value of  $\lambda$ , the more "conservative" the investors. Under the assumption that the returns follow a normal distribution, Freund (1956) shows that the maximization of expected utility

$$
E(U) = \int_{-\infty}^{+\infty} (1 - e^{-\lambda r}) \cdot e^{-(r - \mu)^2 / 2\sigma^2} dr
$$
 (2)

is easily shown to be accomplished if we maximize the function

$$
E(U^*) = \mu - \frac{\lambda}{2}\sigma^2
$$
 (3)

Using matrix notation we maximize the following:

$$
E(U^*) = \mu^* w - \frac{\lambda}{2} w^* \Omega w \tag{4}
$$

Therefore, similar to Knight (2005), the maximization of expected utility for the institutional investors can be rewritten as follows.

$$
\max E(U) = \mu'(w - b) - \frac{\alpha}{2}(w - b)'\Omega(w - b)
$$
  
subject to  $w'i = b'i = 1$   

$$
\Pr\left\{\sum_{i=1}^{n} w_i \cdot R_i < -VaR^*\right\} \le 1 - c
$$
 (5)

Let  $r_p$  be  $\sum_{i=1}^n w_i$ . *i*  $w_i \cdot R_i$ 1 , and we assume here that the returns are normally distributed.

Then,  $r_p \sim N(E(r_p), \sigma_p^2)$ .

Therefore, from  $Pr{r_p < -VaR^*} \le 1-c$ , we can get

$$
\Pr\left\{\frac{r_p - E(r_p)}{\sigma_p} < \frac{-VaR^* - E(r_p)}{\sigma_p}\right\} \le 1 - c \tag{6}
$$

Sequentially,

$$
1 - N \left( \frac{VaR^* + E(r_p)}{\sigma_p} \right) \le 1 - c \tag{7}
$$

Obviously, under the assumption of normally distributed returns, the VaR constraint could be simply changed as follows.

$$
VaR^* \ge -(E(r_p) - N^{-1}(c)\sigma_p)
$$
\n<sup>(8)</sup>

where  $N(\cdot)$  is the distribution function of the standard normal distribution.

$$
N(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}
$$
 (9)

It will be a better way to calculate the VaR of portfolio if we use the Monte Carlo simulation to estimate the  $E(r_p)$  land  $\sigma_p$ . To simplify the calculation we will assume normality and we simply use  $E(r_p) = \mu'w$ 

and  $\sigma_p^2 = w^2 \Omega w$  to describe the VaR constraint. If we use Lagrange method, we get the following equation:

$$
L = \mu'(w - b) - \frac{\alpha}{2}(w - b) \Omega(w - b) + \lambda_1(w'i - 1) + \lambda_2(VaR^* + \mu'w - N^{-1}(c)\sqrt{w'\Omega w})
$$
(10)

and find a relative maximum of *L* with respect to *w* and a relative minimum with respect to  $\lambda$ . The Kuhn-Tucker conditions are a complete taxonomy of the first-order necessary conditions for obtaining a saddle point for *L* . These Kuhn-Tucker conditions are given by:

1) *w* is feasible

$$
2) \frac{\partial L}{\partial w} = \mu - \alpha \Omega(w - b) + \lambda_1 i + \lambda_2 \mu - \frac{\lambda_2}{2} N^{-1}(c) \frac{\Omega w}{\sqrt{w' \Omega w}} = 0
$$
\n(11)

 $\lambda_2 \geq 0$  and  $\lambda_1$  unrestricted sign

3) 
$$
VaR^* + \mu^* w - N^{-1}(c)\sqrt{w'}\Omega w \ge 0
$$
 (12)

$$
4) \t w' i - 1 = 0 \t (13)
$$

5) 
$$
\lambda_2 (VaR^* + \mu^* w - N^{-1}(c)\sqrt{w'\Omega w}) = 0
$$
 (14)

and 
$$
\lambda_2 \geq 0
$$

The Lagrange Multipliers  $\lambda_1$  and  $\lambda_2$  represent the sensitivities of the objective function to the first and second constraints, respectively. The key idea of Kuhn-Tucher theorem is that if the inequality constraint of VaR is not precisely satisfied, then the corresponding Lagrange Multiplier  $\lambda_2$  should have to be zero, relaxing a non-binding constraint.

The first possibility is  $\lambda_2 = 0$ . In this case, the VaR constraint will be loosed and the optimization question will be quite similar to the one Knight (2005) faced and solved. Following the method presented in Knight (2005), we re-solve this question here. The first order condition is:

$$
\frac{\partial L}{\partial w} = \mu - \alpha \Omega (w - b) + \lambda_1 i = 0 \tag{15}
$$

We get 
$$
w = b + \frac{1}{\alpha} \Omega^{-1} (\mu + \lambda_1 i)
$$
 (16)

Using  $w' i = b' i = 1$ , that is,  $i' w = i' b = 1$  we can see that,

$$
i'w = i'b + \frac{1}{\alpha}i'\Omega^{-1}(\mu + \lambda_i i) \Rightarrow i'\Omega^{-1}(\mu + \lambda_i i) = 0 \Rightarrow \lambda_1^* = -\frac{i'\Omega^{-1}\mu}{i'\Omega^{-1}i}
$$
\n(17)

So, we finally get the optimized weights  $w^* = b + \frac{1}{\alpha} \Omega^{-1} (\mu - \frac{i^{\prime} \Omega^{-1} \mu}{i^{\prime} \Omega^{-1} i})$ \* = b +  $\frac{1}{2} \Omega^{-1} (\mu - \frac{i^{\prime} \Omega^{-1} \mu}{n \Omega^{-1}} i$  $w^* = b + \frac{1}{\alpha} \Omega^{-1} (\mu - \frac{i^{\prime} \Omega^{-1} \mu}{i^{\prime} \Omega^{-1} i})$ Ω  $= b + \frac{1}{\alpha} \Omega^{-1} (\mu - \frac{i^{\prime} \Omega^{-1} \mu}{i^{\prime} \Omega^{-1} i})$  (18)

However, the difference between Knight (2005) and this paper is that the solved optimized weights should satisfy the VaR constraint. In the case  $\lambda_2 = 0$ , if  $VaR^* + \mu^*w^* - N^{-1}(c)\sqrt{w^*}'\Omega w^* \ge 0$ , then we can say we find a best weights of portfolio, which make the objective function have its maximum value:

$$
E(U_1^*) = \mu^*(w_1^* - b) - \frac{\alpha}{2}(w_1^* - b)^{\prime} \Omega(w_1^* - b)
$$
\n(19)

here  $(w_1^* = w^*)$ .

If we find  $VaR^* + \mu^*w^* - N^{-1}(c)\sqrt{w^*Qw^*} < 0$ , which means that the optimized weights w<sup>\*</sup> violates the VaR constraint, then we can't solve for a potential maximum point by using the Lagrangian and the Lagrange Multiplier conditions for the optimal point. In other words, the VaR constraint may be such a strict one that it is impossible for us to find out any optimal point for the problem with this inequality constraint.

The only other possibility to solve this maximum utility problem is when  $\lambda_2 > 0$ . From the Kuhn-Tucker

constraint 5, we can see that the VaR constraint is changed to  $VaR^* + \mu'w - N^{-1}(c)\sqrt{w'\Omega w} = 0$  in this

case. Let 
$$
\beta
$$
 be  $N^{-1}(c)$  and substitute  $w' \Omega w = \left(\frac{VaR^* + \mu'w}{\beta}\right)^2 = \frac{VaR^{*2} + 2VaR^* \mu'w + (\mu'w)^2}{\beta^2}$ 

into the institutional investors' utility function. The objective optimization problem is changed to:

$$
\max E(U) = \mu'(w-b) - \frac{\alpha}{2}(w-b)'\Omega(w-b)
$$
\n(20)

$$
\Rightarrow E(U) = \mu'(w - b) - \frac{\alpha}{2}(w'\Omega w - 2w'\Omega b + b'\Omega b)
$$
\n(21)

$$
\Rightarrow E(U) = \mu'(w - b) - \frac{\alpha}{2} \left( \frac{VaR^{*2} + 2VaR^* \mu'w + (\mu'w)^2}{\beta^2} - 2w'\Omega b + b'\Omega b \right)
$$
(22)

subject to  $w'i = b'i = 1$ 

Let us use the Lagrange Multiplier method to solve it. The Lagrange function is:

$$
L = \mu'(w - b) - \frac{\alpha}{2} \left( \frac{VaR^{*2} + 2VaR^* \mu'w + (\mu'w)^2}{\beta^2} - 2w'\Omega b + b'\Omega b \right) + \lambda_1(w'i - 1)
$$
\n(23)

The First Order Conditions are:

$$
\frac{\partial L}{\partial w} = \mu - \frac{\alpha}{\beta^2} V a R^* \mu - \frac{\alpha}{\beta^2} \mu \mu^* w + \alpha \Omega b + \lambda_1 i = 0
$$
\n(24)

$$
\frac{\partial L}{\partial \lambda_1} = w^i i - 1 = 0 \tag{25}
$$

Let  $(N \times 1)$  matrix Ψ be  $\mu - \frac{\alpha}{\rho^2} VaR^* \mu + \alpha \Omega b$ β  $\mu - \frac{\alpha}{\beta^2} VaR^* \mu + \alpha \Omega b$ , then from equation (24) we can get:

$$
w = \frac{\beta^2}{\alpha} (\mu \mu^{\prime})^{-1} (\Psi + \lambda_1 i). \tag{26}
$$

Combining it with the equation (25), we have  $i' \frac{\beta^2}{\alpha} (\mu \mu')^{-1} (\Psi + \lambda_1 i) = i' w = 1$ . Finally, we get

$$
\lambda_1^* = \frac{\frac{\alpha}{\beta^2} - i'(\mu\mu')^{-1}\Psi}{i'i}.
$$
\n(27)

We know that  $i' i = N$ , so  $\lambda_1^* = \frac{\alpha}{N \cdot \beta^2} - \frac{i'(\mu \mu')}{N}$  $=\frac{\alpha}{N\cdot\beta^2}-\frac{i'(\mu\mu')^{-1}\Psi}{N}$ 2 \* 1  $^{\prime}(\mu\mu')$ β  $\lambda_1^* = \frac{\alpha}{N} - \frac{\mu(\mu\mu)Y}{N}$  land  $w^* = \frac{\beta}{\alpha}(\mu\mu')^{-1}(Y + \lambda_1^*i)$  $w^* = \frac{\beta^2}{\alpha} (\mu \mu^{\prime})^{-1} (\Psi + \lambda_1^* i)$ . Similarly, we

can calculate the maximum utility function based on the optimized  $w^*$  as follows.  $(U_2^*) = \mu'(w_2^* - b) - \frac{a}{2}(w_2^* - b)' \Omega(w_2^* - b)$ \* 2 \* 2  $E(U_2^*) = \mu'(w_2^* - b) - \frac{\alpha}{2}(w_2^* - b)' \Omega(w_2^* - b)$  (28) where  $(w_1^* = w^*)$ .

We choose the maximum one between  $E(U_1^*)$  based on  $w_1^*$  and  $E(U_2^*)$  calculated by  $w_2^*$  as the final potential maximum value of the institutional investors' utility function and its best portfolio selection. In summary, the mathematical solution of this class of optimization utility function is shown as follows. One of the possible optimal asset allocation solutions is

$$
w_1^* = b + \frac{1}{\alpha} \Omega^{-1} (\mu - \frac{i^{\prime} \Omega^{-1} \mu}{i^{\prime} \Omega^{-1} i} i) \text{ if } VaR^* + \mu^{\prime} w_1^* - N^{-1}(c) \sqrt{w_1^* \Omega w_1^*} \ge 0
$$
 (29)

The other possibility is

$$
w_2^* = \frac{\beta^2}{\alpha} (\mu \mu^*)^{-1} (\Psi + \lambda_1^* i)
$$
 (30)

where 
$$
\lambda_1^* = \frac{\alpha}{N \cdot \beta^2} - \frac{i'(\mu\mu^*)^{-1}\Psi}{N}
$$
,  $\Psi = \mu - \frac{\alpha}{\beta^2}VaR^*\mu + \alpha\Omega b$  and  $\beta = N^{-1}(c)$ 

The optimized asset weight  $w^*$  which could make the utility function  $E(U^*)$  bigger is the solution we are looking for. Next section gives information about the stock exchanges in China and explains why we decided to apply our model to Chinese market.

## **CHINA'S CAPITAL MARKET AND DATA**

In order to put more pressure on the State Own Enterprises (SOE) to increase their accountability, reduce the SOEs' debt, liquidate the government's state assets, and enable the non-SOEs have access to capital, two stock exchanges were set up in Shenzhen and Shanghai in 1990 respectively. Over the past sixteen years, China stock market has facilitated the development of China's economic growth and market oriented reform, but it is still a young and immature market. At the end of 2005, there were 1378 listed companies in Chinese stock exchanges, which included 834 listed companies in Shanghai Stock Exchange and 544 listed companies in Shenzhen Stock Exchange.

A share market is for Chinese investors while the international investors can only invest in B share market because Chinese currency RMB is a domestic currency and the Chinese people can't exchange foreign currency freely. In both stock exchanges, there are A shares, B shares, T-notes, and some corporate bonds available for investors to trade. Although the stock markets in China have developed rapidly, China's stock markets remain relatively small in proportion to GDP, only 23.76% shown in the table above.

On its road to join the international market and finally become a well-developed market, Chinese stock market has been opening more and more to overseas institutional investors. Due to the promise made to World Trade Organization (WTO), China now allows the establishment of Sino-foreign joint venture securities firms and fund management companies and the Qualified Foreign Institutional Investors (QFII) have begun to participate in securities investment in Chinese stock market. We have reasons to believe that the gradual opening up of China's stock market will provide foreign institutional investors with excellent opportunities to invest in China. The potential opportunity of investing in Chinese stock market is the main reason why we choose it as the object of our empirical research.

 On the other hand, Chinese stock market is an emerging market as mentioned above. Because the economic and political circumstances are different from those of the developed capitalist countries, emerging markets, including Chinese market, are usually considered to be much more risky. The severity of the Asian financial crisis in the late 1990's has stressed the importance of identifying the market risk and credit risk, especially in the emerging economies. Value at Risk which is a mathematical measurement of market risk is primarily concerned with the maximum loss in portfolio value over a given holding period to be experienced under a specific probability level. The VaR approach encourages the institutional investors to think of the portfolio as a set of assets exposed to, in theory, all sources of market risk. Therefore, adding VaR constraint will efficiently help our institutional investors to manage the high risk of investing in emerging markets and to pursue a higher risk-adjusted return of their portfolio to some extent.

<b>Stock</b> Exchange	Listed Companies	A Stock <b>Shares</b>	<b>B</b> Stock <b>Shares</b>	Market Cap. $(C\$ , bn)	Market Cap. to GDP	Investor Accounts (mn)	Turnover in Value $(C\$ , bn)	Average P/E
Shanghai	834	827	54	329.94	16.92%	38.56	711.08	16.33
<b>Shenzhen</b>	544	531	55	133.35	6.84%	35.37	189.65	16.36
Total	378	1358	109	463.29	23.76%	73.93	900.73	16.35

Table 1: Summary Statistics of China Stock Market

*This table shows basic information about two main stock exchanges in China.*

#### Data

In order to show how the model works, we use the data obtained from DataStream database which is one of familiar international financial information providers. We try to find out the optimal portfolio from 25 listed companies that have the biggest market values from Shanghai Stock Exchange and 20 listed companies that have the biggest market values from Shenzhen Stock Exchange such that a VaR constraint over various time horizons is met. The reason why we use the market value as the stock selection criteria is that most institutional investors in Chinese market prefer to invest their funds in the big market value stocks. We employ daily data from these stocks from January 1996 or the date when the stocks were listed in the boards until June 2006.

We first calculate the number of observation, average return, standard deviation, median return, minimum return, maximum return, skewness, kurtosis and ratio of skewness to kurtosis for each stock we analyzed. Then we summarize their average values in the Table 2. A normal distribution has a skewness equal to zero and a kurtosis of 3. The negative or positive skewness implies that the distribution has a higher probability of a large loss or gain than the normal one. A kurtosis greater than 3 indicates that the distribution has longer tails than the normal distribution. One less than 3, on the other hand, means that the values of the distribution are bunched up near the mean. The further the skewness/kurtosis ratio from zero, the more likely it is that the returns are not normally distributed. If we take a look at the skewness and kurtosis of the monthly returns and daily returns, we can find that the stock returns do not conform well to a normal distribution. However, the skewness/kurtosis ratio shows us that the distribution of the daily returns is much closer to the normal distribution due to its larger observations.



Table 2: Summary Statistics of the Analyzed Data (Statistics Period: 01/01/1996 – 30/06/2006)

*This table shows the risk and return information of 45 stocks we pick. The skewness and kurtosis of the monthly returns and daily returns indicate that the time series data has longer tails than the normal distribution while the distribution of the daily returns is closer to the normal distribution.*

Since Fama (1965), it has been well known and accepted by academic researchers and real investors that the asset returns do not always follow a normal distribution. In spite of this fact, the normality assumption is still working as a popular assumption in mainstream finance, as we do it in this paper. The only reason, why we use the assumption of normal distribution even when we know it is not true in the real world, is that it helps us to simplify the question and to clean the technical impediments in our research way. Also, as the number of observations increase, distribution approach would be normal.

## Benchmark

The seemingly simple construction and rebalancing rules for price weighted index cause it to be the most popular index in the markets all over the world. As a basic benchmark, we use a price weighted index that includes all the stocks we chose from Chinese stock market. It means that each stock in the benchmark is weighted by its stock price as a proportion of the total price of all stocks in the index. Apparently, the price weighted index is a passive benchmark and it represents the buy and hold strategy. If at inception each stock in the index is weighted by its share of the index's total price and no new stock is introduced into the index after then, no adjustments to the index are necessary for it to keep its construction strategy. In other words, the performance of the price weighted index is the most easily replicate with a very low degree of tracking error.

We know that the index requires keeping rebalanced as soon as any price of the stock in the index changes after the index is constructed. That means that the equally weighted index is a good choice to be a benchmark because of its easy calculation, but its performance is nearly impossible to replicate with a low degree of tracking error. Because of its easy calculation and few rebalancing, we decided to construct a price weighted index which includes all analyzed stocks and used it as our benchmark. The rebalancing of this benchmark portfolio only takes place at the first trading day for each year in order to keep consistency between the benchmark and the real invested portfolio. On the other hand, we also choose the average monthly and daily return of the Shanghai A share stock index and Shenzhen A share stock index as another benchmark. Comparing the performance of our optimized portfolio with the performance of the whole markets will help us find out the extent to which the portfolio get the extra return compared to the average return of the market.

In this paper, what we investigate whether or not the optimized method is able to lead to outperformed portfolios for institutional investors. The statistics of Table 3 show that the average and the standard deviation of the monthly return of the equally weighted benchmark are 1.1478% and 8.0826%, respectively. It also tells us that the average daily return of the benchmark is much lower, 0.0571%, while its standard deviation is also lower, 1.7753%. Both the monthly and daily average returns of the whole market are less than those of benchmark, while their standard deviations are also much lower. The data shows us that the performances of the stocks whose market values are the greatest in the market are more outstanding. On the other hand, the average returns and standard deviations of the Shanghai A share stock index and the Shenzhen A share stock index present the similar risk and return tradeoff as shown in the following table. Monthly return and daily return of average stock index are 0.9789% and 0.0487% respectively, while the standard deviations are 7.9938% and 1.7136%. Again, the skewness, kurtosis and the ratio of skewness to kurtosis in Table 3 show us that neither the monthly return of the benchmark nor daily return is normally distributed. The average returns and standard deviations of both monthly and daily data from the 45 sample stocks are shown at Table 4.



Table 3: Summary Statistics of Equally Weighted Benchmark and Stock Index (Statistics Period:  $01/01/1996 - 30/06/2006$ 

*This table shows statistical information about equally weighted benchmarks and main stock indices. The data in the table tells that the higher the risk, the better the returns. It also indicates that neither the monthly return series nor the daily return series is normally distributed.*



Table 4: Summary Statistics of Sample Stocks (Statistics Period: 01/01/1996 – 30/06/2006)



*This tab le shows the return and risk information of all stock we select from two Chinese stock exchanges.*

Apparently, there is a positive relationship between the expected reward measured by average return and the risk level estimated by the standard deviation. In order to examine the empirical support for this risk-return tradeoff, which is the most important assumption in Markowitz's mean variance model and accordingly our model, we use the following regression equation to identify the relationship between the average return and the standard deviation.

$$
\mu_d = \alpha + \beta \sigma_d \tag{31}
$$

where  $\mu_d$  = Expected (Average) return of the daily return of the stock  $\sigma_d$  = Standard deviation of the daily return of the stock. The Ordinary Least Squares (OLS) estimates were obtained. The results are presented in Table 5.

Table 5: Regression between Each Year's Average Returns and Standard Deviations

Data	Variable	<b>Coefficients</b>	<b>Standard Error</b>	t Stat
Daily Data	Intercept $\alpha$	$-0.000018$	0.000449	$-0.040493$
	Slope $\beta$	0.021801	0.016530	1.318863
Monthly Data	Intercept $\alpha$	0.001695	0.004421	0.383436
	Slope $\beta$	0.080032	0.035491	2.255012

This table shows the regression estimates of the equation:  $\mu = \alpha + \beta \sigma$ . The first two rows show the estimated results using daily data while other two<br>rows show the results for monthly data. The regression coefficients a *the time series data and the figure in each cell under last column is the t-statistic at the 5 percent level.*

The estimated regression line based on daily data is:

$$
\mu_d = -0.000018 + 0.021801\sigma_d + \varepsilon \tag{32}
$$

where  $\mu_d$  = Expected (Average) return of the daily return of the stock  $\sigma_d$  = Standard deviation of the daily return of the stock. The estimated regression line based on monthly data is:

$$
\mu_m = 0.001695 + 0.080032\sigma_m + \varepsilon \tag{33}
$$

where  $\mu_m$  = Expected (Average) return of the monthly return of the stock,  $\sigma_m$  = Standard deviation of the

monthly return of the stock.

T-stat test and p value in Table 5 show us that the estimated coefficient of standard deviation in monthly regression model is statistically significant at the 5% level, while the one in daily regression model is statistically significant at the 20% level. In other words, all these stocks have the identical characteristic: the greater the risk, the higher the return investors demand as compensation on them. That makes them relatively suitable to our institutional investors' utility model.

#### **EMPIRICAL RESULTS**

The investment period is from January 1, 1997 to June 30, 2006. The computation steps of the invested weights of the portfolio are presented as follows: We choose the first day of each year as the rebalancing date for the portfolio. At each rebalancing date, we calculate the optimized weights of the portfolio by using the model mentioned above and the data of previous year. For example, at the first trading day in 1997, we analyze the data between the first trading day and the last trade day in 1996 and use the utility maximum function subject to VaR constraint to generate the optimized weights for each stock in the invested portfolio. We allocate the actual weights of the portfolio by using the optimized weights calculated previously and keep the proportion in the whole year. Therefore, during the whole year in 1997, we adopt the hold strategy in the portfolio investment after the asset allocation at the beginning of 1997. At next rebalancing date, we repeat the process. In other words, at the first trading day of 1998, we use the optimized weights calculated by using the data in 1997 to allocate the assets for1998.

According to Chinese stock market policy, the investors are prohibited from short selling stocks in both Shanghai and Shenzhen Stock Exchange by the China Securities Regulatory Commission. Therefore, in the following scenario analysis, we also add non-short selling restrictions in the model, which is different from the academic one. Another important assumption in the calculation process is how to estimate the annual return and standard deviation by using the daily and monthly returns in order to calculate the VaR. We estimate the annual return by simply timing the daily or monthly average returns by the number of trading days or months in a year, while we calculate the annual standard deviation by multiplying the daily or monthly standard deviation by the square root of the number of trading days or months in a year. In this paper, we use the formulas as follows.

$$
E(r_{annual}) = E(r_{daily}) * 250; \quad (34) \quad \sigma_{annual} = \sigma_{daily} \cdot \sqrt{250} \tag{35}
$$

$$
E(r_{annual}) = E(r_{monthly}) * 12; \quad (36) \quad \sigma_{annual} = \sigma_{monthly} \cdot \sqrt{12} \quad (37)
$$

where 250 is the number of the trading days in a year and 12 is the number of the trading months in one year. Let us assume the investor's risk aversion score  $\alpha = 1$  and the utility function will be  $E(U) = \mu'(w-b) - \frac{1}{2}(w-b)' \Omega(w-b)$  (38). We also suppose that  $VaR^* = 2\%$  and the confidence

level c = 95%. That is, the institutional portfolio managers are 95% confident the loss will be no greater than 2% of the initial investment of the initial or rebalancing date during the same year. Based on these assumptions, we solve this optimization problem with the initial \$1,000 thousand investment.

Finally, we get the result as shown in Table 6. From it, we can find that both monthly and daily returns are higher than the return of the equally weighted benchmark and the average return of the whole market. Both monthly data and daily data show us that the benchmark has the risk and return similar to those of the market. Once again, the skewness, kurtosis and the ratio of the skewness to kurtosis show that the return distribution of our investment portfolio is a little far away from the assumption of the normal distributed return.

Table 6: Summary Statistics of Optimized Portfolio, Equally Weighted Benchmark and Stock Index (Statistics Period: 01/01/1997 – 30/06/2006)



*This table indicates that both monthly and daily returns of our optimized portfolio are higher than those of the equally weighted benchmark and the market's returns while the portfolio has slightly higher risk than benchmarks and market main indices. The skewness and kurtosis show that the return distribution of our optimized portfolio unlikely satisfies the assumption of the normal distributed return.*

When we use the daily data to do our optimization, the returns of our optimized portfolio, the equally weighted benchmark and the stock index are shown in Graph I. It tells us that the total return, average return and standard deviation of the optimized portfolio are the highest. The risk and return properties of the benchmark and the stock index are similar and both of them follow the rule of risk-return tradeoff. In our case, the equally weighted benchmark has a similar risk-return behavior to the whole market.

Using the daily data gives us the similar result in the relationship among the total returns of our portfolio, equally weighted benchmark and stock index. On this angle, we can make a conclusion that the performance of our optimized portfolio is much better than the normal performance of the whole market, and exceeds the equally weighted benchmark as well.

These results from both monthly data and daily data show us that our portfolio outperforms the equally weighted benchmark and has a better performance than the market. However, the relative return of our portfolio to the equally weighted benchmark and the stock index is much higher than the result shown by the daily data. In other words, the weights optimized from daily data can improve the performance of our portfolio more than those calculated by using monthly data. On the other hand, the return distribution of daily data is much closer to a normal one comparative to that of monthly data. We know that the normal distribution is one of the most important assumptions in our model. Therefore, we can say that the result is somewhat close to our expectation of the paper.

As shown in Table 7, stocks listed on Shenzhen exchange in our optimized portfolios are riskier than those listed on Shanghai exchange, but they have much more return contributions to our optimized portfolio.

Table 8 gives us another chance to take a closer look at stock return contributions of our optimized portfolios. Over years, the sector Construction & Materials has the biggest return contribution to our portfolio with the highest standard deviation. The sectors Autombiles & Parts, Beverages, and Technology Hardware & Equip. have also played very important roles in the nearly 10 year return of our optimized portfolio.

Table 7: Contribution Statistics of Optimized Portfolio by Stock Exchanges (Statistics Period: 01/01/1997 – 30/06/2006)



*Besides Table 6, this table gives us a closer look at the performance of our optimized portfolio by stock exchanges. The data tells a similar story as Table 6 does.*

Table 8: Contribution Statistics of Optimized Portfolio by Sectors (Statistics Period: 01/01/1997 – 30/06/2006)



*This table provides us another view of the return and risk of all sectors in our optimized portfolio.* 

However, before we decide to use this optimization model in our real investment world, we should pay attention to the followings in advance: 1.The greater the number of stocks in the portfolio, the more complicated the calculation of the model. 2. One of the most important requirements for the model is that the stocks in the portfolio should have the similar risk-return characteristics. The stock which has a high risk is supposed to have a high expected return. Although most stocks in our case seems to have this type of the risk-return tradeoff, other stocks in the market may not satisfy this requirement. 3. The deterministic optimization approach typically uses historical data to forecast the weights of portfolio in the coming year and expect the trends of stocks similar to those in the past year. The premise of this type of forecasting is that all current market information has always been reflected in the price movement of the stocks.

In summary, there are some potential problems which are needed to be researched further, although we can use this mathematical model in the real world to provide the suggestion on portfolio allocation to decision-makers.

Figure 1: The Daily Returns of Our Portfolio, Equally Weighted Benchmark and Stock Index



*This figure uses the daily time series data to compare the historical performance among our optimized portfolio, equally weighted benchmark and market stock index.*

## **CONCLUSIONS**

One of the main contributions of our paper is to provide a theoretical solution to our institutional investors' portfolio optimization problem. Solving this optimal investment in the mathematical way helps us to develop a framework for portfolio allocation under Value-at-Risk constraint. The measure for the risk of our portfolio depends not only on the variance or standard deviation, as normal mean variance analysis theories do, but also on the portfolio's potential loss, which is measured by Value at Risk in the paper. Introducing VaR into the institutional investors' optimal utility function has the benefit of allowing the risk-return tradeoff analyzed through focusing on the control of our portfolio's maximum potential loss. The mathematical solution provides a practical way for our institutional investors to carry out the investment decisions in the well-know mean-variance allocation framework, which also satisfy the common Value at Risk restrictions imposed by the internal and/or external regulators.

On the other hand, in order to study the feasibility of our solution, we collect the data from Chinese stock markets to do the empirical analysis and examine how our model works. China has been becoming a hotter and hotter investment zone with the more and more advanced opening of her financial market coming along with WTO. The analysis based on about 10-year monthly and daily data from Chinese markets shows us that our optimized portfolio can be expected to do slightly better than the market return and outperforms than the equally weighted benchmark. Consistent with the risk-return tradeoff, the result also shows us that the higher the return among our portfolio, benchmark index, and market index, the greater the risk.

During the period we analyze in this paper, Chinese stock markets were still relatively small and mainly domestic because they didn't allow any foreign players. Because of this, some global financial events such as global financial crisis have very limit effect on this emerging market. However, it will be interesting and meaningful to see how our model will perform when exposed to global financial events. This will be one of areas in our future research.





*This figure uses the monthly time series data to compare the historical performance among our optimized portfolio, equally weighted benchmark and market stock index.*

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# **BIOGRAPHY**

ZhengXiong Chen, CFA, is senior programmer analyst of Hillsdale Investment Management Inc., an independent investment firm that provides a full range of traditional equity and alternative investment strategies. Mr. Chen is responsible for providing research support and investment strategies, designing and developing investment platforms, and building and improving quantitative and qualitative models to the portfolio management. Prior to his current position, he worked as a financial budget analyst at City of Toronto, a risk analyst at TD Canada Trust and an analyst in Treasury Credit department at TD Securities. Mr. Chen has been an active regular member of the Toronto CFA Society since 2006.

Before moving to Canada with his family in 2005, Mr. Chen had worked in Chinese financial market for 5 years. As a programmer at Industrial and Commercial Bank of China (ICBC), he was one of core members who built the first data warehouse based on SAS in Chinese banking industry. He also had worked as a senior financial engineer for three years at DaCheng Fund Management Co., one of the biggest investment management companies in Mainland China.

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Her research concentrates on Investments and on International Finance. She wrote 3 books, one book chapter, and more than thirty articles . She has presented at various national and international conferences. She has papers on Canadian Merger and Acquisitions, on Chinese Stock Markets, on short and long term relationships among the stock exchanges and on emerging markets. Last year, she got a \$128,000 SSHRC grant to examine foreign direct investments of multinational corporations. Currently she is working on that project, also on international investments of Chinese venture capital fund companies with a Chinese professor.

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