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INDIRECT LABOR COSTS AND IMPLICATIONS FOR OVERHEAD ALLOCATION

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ABSTRACT

Cost accounting typically allocates indirect labor cost to cost object based on direct labor hours. The allocation process implicitly assumes that indirect labor costs vary proportionally with direct labor hours. The assumption of a linear relationship between indirect and direct labor is particularly suspicious at low production volume levels because there tends to be a fixed component in indirect labor. The linearity assumption is also challenged by recent increasing complexity of indirect labor tasks. As automation technology replaces some work of the of traditional labor, the cost of non-production workers becomes an important element of manufacturing overhead and it may not be related to labor hours in a simple linear manner. A model is derived to show the relationship between indirect labor overhead and direct labor hours under different conditions. The implication for the allocation of indirect labor overhead is also discussed.

JEL: J3, M2

KEYWORDS: Indirect Labor Cost, Labor Cost, Overhead Allocation, Cost Accounting, Indirect Labor Cost Allocation

INTRODUCTION

One of the critical roles of cost accounting is to estimate the cost of product or services. All costing models are trying to find the "true" cost of a particular cost object such as product, service, segment, and department. Traditional costing approach allocates overhead by using volume-driven measure such as unit produced to first estimate a predetermined overhead rate then allocate overhead by applying this average overhead rate to the cost object. Application of such models is valid for facilities producing products with less diversity. However, as product diversity increases, the broad averaging process leads to serious cost distortion (Johnson and Kaplan, 1987, Cooper and Kaplan, 1988).

A more sophisticated overhead allocation method such as Activity Based Costing (ABC) intends to reduce these cost measurement distortions by creating multiple cost pools and allocation bases to allocate overhead to product or service in two stages allocation process (Cooper, 1987a, 1987b, 1988). One issue that relates to the ABC system is that the allocation process assumes a strict proportional relationship between activity and cost. Noreen and Soderstrom (1994) challenge this linear proportional assumption by examining the hospital's time-series behavior of overhead costs and activities. The results show that the proportionality hypothesis can be rejected for most of the overhead accounts. On average across the accounts, the average cost per unit of activity overstates marginal cost by about 40% and in some departments by over 100%. Another study conducted by Noreen and Soderstrom (1997) suggests that costing systems, which assume costs are strictly proportional to activity, grossly overstate the impact of changes in activity on cost. Kim and Hon (2008) comments that when cost behavior shows a nonlinear pattern, conventional ABC may distort product costs. Noreen (1991) develops a mathematical model to demonstrate the conditions under which ABC systems provide relevant costs. One of the conditions that ABC would provide relevant information is that the cost in each cost pool is strictly proportional to its activity.

The assumption of a linear relationship between activity and costs creates a challenge to accurately estimate the product cost. The purpose of this paper is to present a model that shows the relationship between indirect labor overhead and cost drivers such as direct labor hours under different conditions followed by a discussion of the implication for the allocation of indirect labor overhead.

LITERATURE REVIEW

Decisions are often taken assuming linear models for the purpose of simplicity. Cost accounting typically allocates indirect labor costs to products or services (cost object) based on direct labor hours. Traditional cost allocation methods presume a linear relationship between the costs and cost allocation base. This linearity assumption is problematic for allocating overhead when there are various product lines and each of which demands diverse amount of resources (Garrison et al., 2012). The average allocation rate assumes that each unit of product/service consumes the resources at a constant rate. This allocation process will not be able to capture the resource consumption when product/service diversity exists. Balakrishnan et al. (2012) comment that traditional costing, ABC or other costing systems currently do not seem to offer an effective way to estimate product/service cost and suggest a blended model that accommodates nonlinearity. Ramani et al. (2010) presents a case study to emphasize the importance of the use of more accurate models to account for nonlinearity. McNair (2007) suggests that it is necessary to apply a non-linear modeling approach to capture cost dynamics and relationships.

Take the labor and indirect labor costs as an example. Labor is direct when their work and wages can be identified with specific costing units such as departments, products or sales contracts (Horngren et al., 2012). All other employees that cannot be directly traced to the costing units are indirect. From the perspective of manufacturing, wages that directly relate to production are considered as direct labor costs; other work that is performed on the production floor but not on producing the products is considered as indirect labor costs. In accounting, trace direct labor costs is straight forward because there are payroll records to directly connect the direct labor costs to the products. On the other hand, the indirect labor costs requires allocation process, because it cannot be directly traced in an economically way (Horngren et al., 2012). The allocation process will have to first estimate an allocation rate by taking a total indirect labor costs divided by selected allocation basis. This allocation process perceives that labor-related overhead behaves proportionally to direct labor hours. That is, the average indirect labor overhead per direct labor hour is the same as marginal indirect labor overhead per direct labor hour given all other conditions remain the same. This allocation process implicitly assumes that indirect labor hours should vary proportionally with direct man hours. These average allocation rates are useful guides with the relevant range of fluctuation in direct labor, but they cease to be satisfactory when large changes in the direct labor base occur.

The assumption of a linear relationship between indirect and direct labor is particularly suspicious at low production volume levels because there tends to be a fixed component in indirect labor. When a decline in direct labor activity is expected to be short in duration, this fixed component of the indirect work force usually remains intact, because management retains experienced supervisors and others not readily replaceable when needed again. When activity is expected to remain low for an extended period or costs must be reduced to protect the company's financial resources, cuts are made in some organizations at the management's discretion. The number of indirect labor required for the coming budget year also can be determined by analysis of the work to be done. This procedure usually is followed in companies that have little or no direct labor variable with short term production volume. Under this condition, a functional linear relationship between direct and indirect labor is questionable.

The linearity assumption is also criticized by recent increasing complexity of indirect labor tasks. As automation technology replaces some work of traditional labor, the cost of non-production workers becomes an important element of manufacturing overhead and it may not be related to labor hours in a simple linear manner. Many studies discuss the value of non-production workers such as production supervisors, quality control staff, production managers and on-site tooling engineers to a manufacturing plant's productivity (Gunasekaran et al, 1994; Kang & Hong, 2002; Krajewski and Ritzman, 2004). Studies also find a significant effect of non-production labor on a manufacturing plant's productivity (Gray and Jurison, 1995; Wacker et al., 2006). As the indirect labor takes a more essential role in manufacturing plants than it has previously and indirect labor cost may not be linearly related to direct labor hours, the averaging process of allocating overhead would produce misleading cost estimates. As a result, it is essential to conduct a preliminary examination on the nature of indirect costs before allocating them as overhead.

DATA AND METHODOLOGY

Define the Nature of Types of Indirect Labor Overhead

Overhead is ongoing costs of a business which cannot be attributed to any specific cost object. To narrow the analysis focus, this study attempts to build a model that represents one of the overhead items - indirect labor costs that are commonly seen on the production floor or service setting. In general, according to the complexity of the work nature, indirect labor can be broadly categorized into three types of indirect labor according to task complexity and job nature. The first type of indirect labor is the labor force who performs routine tasks such as maintaining, and cleaning the working facility. The second type of indirect labor directly oversees the production lines or service processes and performs higher level of work than the first type of indirect labor. First line supervisors or on-site tooling and equipment engineers are examples of the second type of indirect labor. The third type of indirect labor, for example, a production manager, conducts overall supervision, coordinates all production lines, schedules production runs and communicates with the upper management. In general, considering the overtime paid, fringe benefits, incentive plans and bonus provided to manager or supervisors, these three types of labor costs are not fixed costs even some of them may be on salary basis. The research question that this paper attempts to explore is whether these three types of indirect labor costs change proportionally (linear) or non-proportionally (non-linear) to the total production which is assumed to be directly related to direct labor hours. The following discussion uses production line as an example to develop a model that demonstrates three scenarios of the behavior of indirect labor costs.

Model the Indirect Labor Costs

Based on the nature of indirect labor's tasks, the general three types of indirect labor are defined as follows:

x	front line direct production labor hours
$y = f(x)$	labor related overhead (indirect labor costs) generated from the production floor. y is expressed as a function of x because the indirect labor costs are generated and built upon the direct labor. The more direct labor force, the more indirect labor efforts are involved in the function of supervision, coordination and scheduling.
$\frac{dy}{dx}$	represents front line supervisor overhead at a certain number of direct labors supervised. The cost of supervisors is based on the numbers of direct labor (represents by x) they supervise. The first order derivative captures the dynamics of impact on overall production cost due to more or less supervision. For instance, at a certain point, the more supervision involved in the production

process, the labor obtains more on-site assistance resulting in less errors, material waste and scrap which may decrease overall manufacturing cost.

$x\frac{dy}{dx}$ stands for total costs of front line supervisors. $x\frac{dy}{dx}$ represents the "true" total cost of front line supervisors that incorporates the increasing or decreasing rate of efficiency (rate of returns) of hiring a supervisor for a certain number of direct labor force. The x is actual numbers of front line production labor supervised.

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$ is the next level of supervision which can be represented by production manager. The production manager's cost is the second order differential relationship to the x. The second order of derivatives intends to capture the overall impact on the total manufacturing costs by adding this level of supervision and management.

$x * x\frac{d^2y}{dx^2}$ represents the total "true" cost of production manager who coordinates all production lines, maintains scheduling, manages material movements, monitors the flow of manufacturing process, and communicates with upper management. Therefore, $x * x$ is the weight for the more complex level of supervision.

If there are n layers of supervision (supervisors or managers) in the production plant factory, the total indirect labor overhead costs can be captured in the following equation:

$$a_n x^n \frac{d^n y}{dx^n} + a_{n-1} x_{n-1} \frac{d^{n-1}y}{dx^{n-1}} + \ldots + a_1 x \frac{dy}{dx} + a_0 y = g(x)$$

This paper will narrow the analysis focus specifically on three types of indirect labor as explained in the previous section. The total indirect labor costs from production floor can be represented by a Cauchy-Euler Equation (Rabenstein, 1975):

$a_2 x^2 \frac{d^2y}{dx^2} + a_1 x \frac{dy}{dx} + a_0 y = g(x)$, the total indirect labor cost is expressed in a brief form of the Cauchy-Euler Equation

(let $a = a_2, b = a_1$ and $c = a_0$)

For the purpose of discussion, we confine our analysis to solving the homogeneous second-order equation

$$ax^2 \frac{d^2y}{dx^2} + bx\frac{dy}{dx} + cy = 0 \qquad (1)$$

For $y = f(x)$, indirect labor cost is the function of direct labor hours, we can assume that $y = x^m$ is a general solution of (1), where m is to be determined. The first and second derivatives are, respectively,

$\frac{dy}{dx} = mx^{m-1}$ and $\frac{d^2y}{dx^2} = m(m-1)x^{m-2}$

Consequently the differential equation (1) becomes

$$ax^2\frac{d^2y}{dx^2} + bx\frac{dy}{dx} + cy = ax^2 * \ m(m-1)x^{m-2} + bx * mx^{m-1} + cx^m$$

$$= \ am(m-1)x^m + bmx^m + cx^m$$

$$= x^m(am(m-1) + \ bm + c)$$

Thus $y = x^m$ is a solution of the differential equation whenever m is a solution of the trivial equation (2).

$am(m-1) + bm + c = 0$ or $am^2 + (b-a)m + c = 0$ (2)

There are three different cases to be considered, depending on the whether the roots of this quadratic equation are real and distinct, real and equal, or complex conjugate.
[Basic solutions of quadratic equation]:

$$Am^2 + Bm + C = 0$$

$$m = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (3)$$

Scenario 1:

$B^2 - 4AC > 0$: Two distinct real roots

Let $m_1$and m_2 denote the real roots of (2) and $m_1 \neq m_2$. Then

$y_1 = x^{m_1}$ and $y_2 = x^{m_2}$

Hence the general solution is

$$y = c_1x^{m_1} + \ c_2x^{m_2} \quad (4)$$

Scenario 2:

$B^2 - 4AC = 0$: Repeated real roots

If the roots of (2) are repeated, that is, $m_1 = m_2$ then we obtain only one solution, namely, $y = x^{m_1}$. From the quadratic formula, the root must be $m_1 = -(b-a)/2a$.

To solve for a second solution y_2, divide the brief form of the Cauchy-Euler equation (1) by ax^2 to obtain the following form

$\frac{d^2y}{dx^2} + \frac{b}{ax}\frac{dy}{dx} + \frac{c}{ax^2}y = 0$ Thus (see appendix for supplemental derivation),

$$y_2 = x^{m_1}\int\frac{e^{-\int(b/ax)dx}}{(x^{m_1})^2}dx$$

$$= x^{m_1}\int\frac{e^{-\int(b/a)lnx}}{x^{2m_1}}\text{dx}$$

$= x^{m_1} \int x^{-b/a} * x^{(b-a)/a} dx$

$= x^{m_1} \int \frac{dx}{x} = x^{m_1} lnx$

The general solution is then

$$y = c_1 x^{m_1} + c_2 x^{m_1} lnx \qquad (5)$$

Scenario 3:

$B^2 - 4AC < 0$: Conjugate complex roots

If the roots of (2) are the conjugate pair
$m_1 = \propto + i\beta, m_2 = \propto - i\beta$ (i: imaginary number)
where $\propto$ and β are real numbers, then a solution is

$$y = c_1 x^{\propto + i\beta} + c_2 x^{\propto - i\beta}$$

As in the case of the equation with constant coefficients, when the roots of equation (2) are complex, the solution will be expressed in terms of real numbers only. Note the identity [$e^{i\theta} = \cos\theta + i\sin\theta$]

$$x^{i\beta} = (e^{lnx})^{i\beta} = e^{i\beta lnx}$$

$$x^{i\beta} = \cos(\beta lnx) + i\sin(\beta lnx)$$

Similarly,

$$x^{-i\beta} = \cos(\beta lnx) - i\sin(\beta lnx)$$

Adding and subtracting the last two results yield, respectively,

$x^{i\beta} + x^{-i\beta} = 2\cos(\beta lnx)$ and

$x^{i\beta} - x^{-i\beta} = 2i\sin(\beta lnx)$

From the fact that $y = c_1 x^{\propto + i\beta} + c_2 x^{\propto - i\beta}$ is a solution of $ax^2 y'' + bxy' + cy = 0$

for any values of the constants c_1 and c_2 we see that

$y_1 = x^{\propto}(x^{i\beta} + x^{-i\beta}), (c_1 = c_2 = 1)$

$y_2 = x^{\propto}(x^{i\beta} - x^{-i\beta}), (c_1 = 1, c_2 = -1)$ or

$y_1 = 2x^{\propto} (\cos(\beta lnx))$

$y_2 = 2i\, x^{\propto}(\sin(\beta lnx))$

Hence, the general solution is

$$y = x^{\propto}[c_1 \cos(\beta lnx) + c_2 \sin(\beta lnx)] \quad (6)$$

where $\propto = \frac{m_1 + m_2}{2}$, $\beta = \frac{m_1 - m_2}{2i}$

Example:

The model can also be applied to a service context. Following is an example of airline carriers. This example uses American Airlines and United Airlines because of their similar operating scale evidenced by the closeness of the costs of Aircraft and Traffic Servicing Labor ($106,418 ≈ $104,150). Cost of labor inputs of these two airline carriers are used to illustrate the second scenario repeated real roots where $B^2 - 4AC = 0$.

Table 1: Inputs Labor Costs by Category (Means in Thousands)

Carrier	Aircraft & Traffic Servicing Labor	Promotions & Sales Labor	Flying Operations Labor	Passenger Service Labor	General Overhead
American Airlines	106,418	112,297	98,098	52,685	135,352
United Airlines	104,150	122,467	141,899	80,965	143,744

Data source: The labor inputs costs are partially adopted from the data used in Banker and Johnson (1993).

Using this model, we can insert the relative value of labor costs of two airline carriers into equation 1, Cauchy-Euler homogeneous second-order equation:

$$ax^2 \frac{d^2y}{dx^2} + bx\frac{dy}{dx} + cy = 0 \quad (1)$$

The Promotion & Sales labor category gives us the 1.09 relative value of United Airlines and American Airlines (the division of 122,467 by 112,297). Applying this relative value to equation (1) yields equation (1.1):

$$a * 1.09^2 y'' + b * 1.09y' + cy = 0 \quad (1.1)$$

To simplify equation (1.1) for trivial solutions, let $a = {}^1/_{1.09^2} = 0.842$ and $b = {}^1/_{1.09} = 0.917$

As previously defined, A= a = 0.842 and B = (b-a) = (0.917- 0.842) = 0.075

Solve the roots by following $m = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Under Scenario 2, m = -B/2A= -0.045

According to the general solution for the repeated real roots, $y = c_1 x^{m_1} + c_2 x^{m_1} lnx$ (equation 5), the equation becomes

$$y = c_1 x^{-0.045} + c_2 x^{-0.045} lnx$$

The general relative solutions for Flying Operation Labor and Passenger Service Labor can be solved in the same calculation process:

$$y = c_1 x^{-0.215} + c_2 x^{-0.215} lnx$$

$y = c_1x^{-0.27} + c_2x^{-0.27}lnx$

The Passenger Service Labor is considered as direct labor. The general solution for the Passenger Service Labor is $y = c_1x^{-0.27} + c_2x^{-0.27}lnx$ which indicates that the relationship between indirect labor costs and direct labor activity is not linear. For example,

when x = 1,000

$y = c_1x^{-0.27} + c_2x^{-0.27}lnx = c_1 1{,}000^{-0.27} + c_2 1{,}000^{-0.27}\ln(1{,}000)$
$= 0.155c_1 + 1.074c_2$

If x increases 10 times to 10,000, y increases in different magnitude as follows:

$y = c_1x^{-0.27} + c_2x^{-0.27}lnx = c_1 10{,}000^{-0.27} + c_2 10{,}000^{-0.27}\ln(10{,}000)$
$= 0.083c_1 + 0.764c_2$

It is noted that when the relative direct labor value of the two airlines increases 10 times, the relative indirect labor costs do not increase proportionally. In fact, in this specific example, when the direct labor activities increase 10 times, the resulting relative indirect labor weights decrease from 0.155 to 0.083 and from 1.074 to 0.764.

RESULTS

As shown in the cost model, the diversity of indirect labor overhead was captured by incorporating weights to each type of labor in the model. When we limit the types of indirect labor to three general types, the equation that captures total "true cost" of indirect labor is simplified to the second order of equation: $ax^2\frac{d^2y}{dx^2} + bx\frac{dy}{dx} + cy = 0$. When the indirect labor costs were incurred in a small, simple business environment, the coefficients of a and b become zero and the second order equation becomes a linear function ($cy = 0$). Note that y is a function of x and can be represented by $y = c_1x + c_0$. The relationship between y (indirect labor cost) and x (direct labor hours) is linear and is identified by the coefficients c_1 and c_0. This happens when there is only one line of variable indirect labor whose working hours are driven by direct labor hours (i.e., quality control operator) or when there is no indirect labor at all, the labor overhead is simply the wages paid to the direct labor for working overtime.

In another case that cost of a first line supervisor or manager does not add additional value or contribution such as improving productivity or saving production costs (when the production floor is indifferent to the additional supervision), the term $\frac{dy}{dx}$ also becomes 0. When $\frac{dy}{dx}$ is equal to zero, the equation is a linear function ($cy = 0$). Therefore, in a simple context with less diversity of indirect labor or where the function of supervision does not add value to the production, the cost function approaches linear where indirect labor cost is proportionally related to direct labor hours. This represents a perfect context to allocate overhead by calculating the average predetermined overhead rate based on direct labor hour.

Depending on the supervision layers, management style and diversity of supervising tasks, there are three possible solutions to the equations where total indirect labor overhead can be $y = c_1x^{m_1} + c_2x^{m_2}$ (see equation 4), or $y = c_1x^{m_1} + c_2x^{m_1}lnx$ (equation 5), or $y = x^{\propto}[c_1\cos(\beta lnx) + c_2\sin(\beta lnx)]$ (equation 6). These solutions demonstrate that the effect of indirect labor on overhead is so dynamic that the indirect labor costs and direct labor hours are not linearly related and can be specified by three different non-linear forms as demonstrated above. The effect of change in direct labor hours on indirect labor costs cannot be captured by a simple linear coefficient but rather can be expressed by several non-linear forms. There tends to be a fixed portion of indirect labor when increase in direct labor activity is expected to be short in duration; this fixed portion of the indirect work force usually remains the same without hiring additional staff. This is accomplished by upgrading certain positions. A higher graded indirect labor (for example, supervisors or production managers) should be able to handle an additional workload, thereby absorbing any additional work to accommodate the extra demand. The total production increases even though the total indirect labor cost remains the same. The change in the mix of the authorized positions can be one of the reasons that the cost function is not purely linear to direct labor hours. Another reason that contributes to the non-linear relationship is the variability of indirect labor which reflects management policy to control or not to control the amount of indirect labor. For example, a maintenance crew of a given size may be kept in readiness at all times, or the number of maintenance employees may be varied with current work load by various means such as hiring outside contractors. Given those facts, direct labor hours do not proportionally drive the labor-related overhead. The model can also apply to the service context. Airline carriers are used as an example to demonstrate the second scenario of repeated real roots where B^2-4AC = 0.). This example also shows that the changes in indirect labor costs are not proportional to the changes in the direct labor costs.

CONCLUDING COMMENTS

The purpose of this paper is to present a model that shows the relationship between indirect labor overhead and cost drivers such as direct labor hours under different conditions. This paper explores the nature of overhead, specifically examining the linearity of indirect labor costs.

According to the model, indirect labor costs relate to direct labor hours in different ways which may not be fully captured by using predetermined average rates to allocate the overhead. It is in a special condition that indirect labor overhead will demonstrate linear relationship to the labor hours. Under that condition, the traditional process or the ABC approach will provide appropriate overhead estimates. However, indirect labor variety and variability lead to indirect labor costs not purely proportional to direct labor hours. If indirect labor cost functions appear to be non-linear after preliminary analysis, caution should be exercised when determining how to allocate the overhead.

It is suggested that costs be further classified into variable and fixed costs. The variable portion can be allocated based on direct labor hours or other appropriate measures to the product. However, the fixed portion should not be allocated down to the product level. Instead, fixed portion can be included in the overall profitability analysis and considered as a share of total cost that should be covered by the revenue. The example shows two airlines' relative costs for demonstration and if labor costs for four or more consecutive years are available, an exact solution of second order differential equation (1) can be solved by using Cauchy-Euler and Runge-Kutta methods. Therefore, future costs and profits can be predicted more accurately.

This study is subject to a few limitations. First, three types of indirect labor costs were used to derive the model and second, the Cauchy-Euler homogeneous second-order equation may not fully capture the magnitude of nonlinear relationship if there were more indirect costs elements incorporated into the model.

Two possible lines of research could be conducted to further explore the nature of overhead. First, models can be established to closely investigate other types of overhead; second, the model in this paper attempts to generally describe the behavior of overhead for a short-run time frame. The model could be further refined by incorporating other conditions or macro factors such as labor contracts and size of segment to extend the model that might assist in a company's long term planning and controlling.

APPENDIX

To derive from $\frac{d^2y}{dx^2} + \frac{b}{ax}\frac{dy}{dx} + \frac{c}{ax^2} = 0$ to $y2 = x^{m_1} \int \frac{e^{-\int (b/ax)dx}}{(x^{m_1})^2} dx$

$$y'' + P(x)y' + Q(x)y = 0 \qquad \text{(a)}$$

where $P(x) = \frac{b}{ax}$ and $Q(x) = \frac{c}{ax^2}$

Assume that $y_1(x)$ is a solution of (a) and that $y_1(x) \neq 0$. If we define y = u(x) $y_1(x)$, it follows that

$y' = uy'_1 + y_1u'$

$y'' = uy''_1 + 2y'_1u' + y_1u''$

$y'' + Py' + Qy = y_1u'' + (2y'_1 + Py_1)u' + [y''_1 + Py'_1 + Qy_1]u = 0$

From equation (a), $[y''_1 + Py'_1 + Qy_1]u = 0$, we have

$y_1u'' + (2y'_1 + Py_1)u' = 0$ or

$y_1w' + (2y'_1 + Py_1)w = 0$

Where we have let $w = u'$, we obtain

$$\frac{dw}{w} + 2\frac{y'_1}{y_1}dx + Pdx = 0$$

$$\ln|w| = 2\ln|y_1| = -\int Pdx + c$$

$$\ln|wy_1{}^2| = -\int Pdx + c$$

$$wy_1{}^2 = c_1e^{-\int pdx}$$

$$w = u' = c_1\frac{e^{-\int pdx}}{y_1{}^2}$$

Integrating again gives $u = c_1 \int \frac{e^{-\int p(x)dx}}{y_1{}^2(x)} dx + c_2$ and therefore

$$y = u(x)y_1(x) = c_1y_1(x)\int \frac{e^{-\int p(x)dx}}{y_1{}^2(x)} dx + c_2y_1(x)$$

By choosing $c_2 = 0$ and $c_1 = 1$, we find that a second solution of equation (1) is

$$y_2 = y_1(x) \int \frac{e^{-\int p dx}}{{y_1}^2(x)} dx \qquad \text{(b)}$$

Substitute P by $b/_{ax}$, and knowing that $y_1 = x^{m_1}$, (b) becomes

$$y_2 = x^{m_1} \int \frac{e^{-\int (b/ax) dx}}{(x^{m_1})^2} dx$$

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BIOGRAPHY

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