Accounting & Taxation
Vol. 7, No. 1, 2015, pp. 103-114
ISSN: 1944-592X (print)
ISSN: 2157-0175 (online)

IBFR
www.theIBFR.org

MEASURING INCOME INEQUALITY: AN APPLICATION OF THE POPULATION DYNAMIC THEIL'S ENTROPY

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ABSTRACT

In this paper we use the index we call Population Dynamic Theil's Entropy to analyze as the income inequality varies on time. The index may consider both the inequality among the classes in which we assign the individuals and the inequality within each class. This inequality measure working in a dynamic way allows to forecast inequality in time. Besides it may capture not only changes in the wealth but also changes in the population composition. The earned results are relevant for adopting a social and economic policy of wealth distribution. We fulfilled the model with statistics from the Organization for Economic Co-operation and Development and we applied it to Mexico, Portugal and Spain. We picked up economic data about population, means and medians of the equivalised net income for the three countries. The data refer to years from 2004 to 2011.

JEL: E64, E27

KEYWORDS: Income Distribution**,** Population Dynamic Theil's Entropy, Markov Chains

INTRODUCTION

A recent approach in economics proposes measuring the income inequality through dynamic indices instead of the classic static indices, like those by Gini, Herfindahl-Hirschman and Theil. Theil (1967) introduced the Theil's entropy, since then most used in scientific papers. It holds the sum of the products of the shares of the total income of each individual (stood for by y_i) multiplied by the logarithm of Ny_i , *being* N the number of the agents in the economic system. The range of values is between 0 and $ln(N)$. The index takes the value 0 when the wealth is equidistributed among the agents and the value $ln(N)$ when one agent holds all the wealth.

This paper belongs to this recent line of research aiming at measuring the income inequality in a dynamic way in the whole population of some countries. We considered countries with comparable socio-cultural life styles and religion but with different rates of change of Gross Domestic Product (GDP), like for example Mexico, Portugal and Spain. For this investigation we adopted the Population Dynamic Theil's Entropy (PDTE) because it may capture not only the changes in wealth but also changes in population composition. Therefore it is possible to justify changes in the index when the population structure varies over time. The results show such analysis to be useful to decision makers to carry out policies of economic integration.

Many papers make use of Markov chain modeling to describe how income changes (Quah, 1993, 1994, 1995, Dardanoni, 1995), also some papers consider Bayesian estimations of persistent income inequality (Nishino, Kakamu and Oga, 2012, Kakamu and Fukushige, 2009). Some applications related to the income inequality indices underline the importance of this research field. They include 1.) Influences of political regimes and financial reforms (Kemp-Benedict, 2011, Baland, Dagnelie and Rey, 2007), 2.) Relevance of geographical reasons (Banerjee, Mookherjee, Munshi and Rey, 2001, Chaudhuri, Ghatak, Guha, Mookherjee and Rey, 2007), 3.) Impact of immigration on the concentration of wealth distribution (D'Amico, Di Biase and Manca, 2011) and 4.) Impact of the fiscal system on wealth redistribution in the population (D'Amico, Di Biase and Manca, 2013). This paper is a follow-up study to the adjustment of the Dynamic Theil's Entropy to forecast the income inequality on a given time horizon in the whole population of some countries. Thanks to decomposing Theil's Entropy into three addenda (D'Amico, Di Biase and Manca, 2014) the paper makes a careful examination of the wealth distribution in Mexico, Portugal and Spain. The rest of the paper is organized as follows. In the next session the paper provides a review of the relevant literature. The following section briefly describes the stochastic model. Next we explain data and method to calculate the PDTE. Next section presents the results of the application to the three countries. The paper closes with some conclusions and further research suggestions.

LITERATURE REVIEW

Many authors successfully applied econometric indices to measuring the income inequality in a given economic system (Tirole, 1988, Sen, 1993, Dagum 1990, Davies and Hoy, 1995, Bajo and Salas, 2002, Campano and Salvatotre, 2006, Shahrestani and Bidabad, 2010). Scientific community recognized Theil's Entropy thanks to its properties. They include the scale independence, the invariance to replication of population, the Pigou-Dalton principle of transfers (Dalton, 1929, Athanasopoulos and Vahid, 2003), the strong principle of transfers and the additive decomposability (Cowell, 1995, Kakamu and Fukushige, 2009). In the nineties Quah (1993, 1994, 1995) and Dardanoni (1995) used Markov chains to modeling income dynamics. Nevertheless these random processes may not consider properly the randomness in the waiting times in the states. Indeed, in the authors opinion, the lapsed time in a class of income, influences the probability distributions of the income. If an agent is in the rich class for a long time, has a likelihood to remain rich that is different from that of an individual who is rich by few time.

Bickenbach and Bode (2003) outlined the inadequacy of the Markov chain model. In a recent paper D’Amico and Di Biase (2010) proposed the use of a semi-Markov process to calculate inequality indices in a dynamic way. So they surmounted the inadequacies previously highlighted. The generalization of the indices considered a population that changes overtime according to a semi-Markov process and by considering the production of each economic agent as a reward process. This approach can capture not only changes in the wealth but also changes in the population structure and justifies changes in the inequality when the population composition varies overtime.

Nishino, Kakamu and Oga (2012) proposed a different approach based on Bayesian estimation of persistent income inequality by Lognormal stochastic volatility model. A paper by D’Amico, Di Biase and Manca (2011) performed a model simulation and calculated some dynamic indices for different economic scenarios. In particular the paper considered the Herfindahl-Hirschman index (Hirschman, 1964), the Gini index (Gini, 1912) and the Theil's entropy (Theil, 1967). Also D’Amico, Di Biase and Manca (2011) showed how the model could be useful for analyzing the immigration effects about the inequality of wealth distribution in the economy. They paid a particular attention to the effects caused by a population simulation increase of 10%. Since the direct application of the model to real case studies needs microdata about income evolution of agents which are often unavailable, D’Amico, Di Biase and Manca (2012) proposed a method to use the model by knowing only averages and medians of the incomes. The method considered Markov chains to model income evolution.

D’Amico, Di Biase and Manca (2013) proved the force of suitable fiscal policies as fundamental tool of macroeconomic planning to spread out wealth and decrease income inequality. The paper recovered the gross income distributions by using the individual income tax rates in some European countries. The paper by D’Amico, Di Biase and Manca (2014b) proposed a decomposition of the Population Dynamic Theil's Entropy (PDTE). This decomposition into three addenda let us evaluating the inequality on the whole considered population and not only among the classes of agents as done in D'Amico and Di Biase (2010). The PDTE relax the hypothesis of homogeneity among the agents belonging to the same income class. The more correct hypothesis of diversity fulfill more correct applications.

The Model

We can represent the classic Theil's Entropy (TE) as follows:

$$TE = \frac{1}{N}\sum_{i=1}^{N}\frac{y_i}{\bar{y}}\left(log\frac{y_i}{\bar{y}}\right), \tag{1}$$

where $\bar{y}$ is the average income in the population and y_i is the income of the i-th agent.
If we allocate all agents in K classes $E = \{C_1, C_2, \dots, C_K\}$, we can represent the TE by using the decomposability property, see D'Amico, Di Biase and Manca (2014b)

$$TE = \sum_{g=1}^{K} a_{C_g}\, TE\left(y_{C_g}; n_{C_g}\right) + \sum_{g=1}^{K} a_{C_g} \log K a_{C_g} + \sum_{g=1}^{K} a_{C_g} \log\frac{N}{Kn_{C_g}} \tag{2}$$

where

$$a_{C_j} = \frac{n_{C_j} y_{C_j}}{\sum_{g=1}^{K} n_{C_g} y_{C_g}}, \tag{3}$$

n_{C_g} is the number of agents of the class C_g, y_{C_g} is the average per capita income of class C_g and $TE(y_{C_g};\ n_{C_g})$ is the Theil's Entropy of class C_g.

Now if we assume the shares of income a_{C_g} to be random, then (2) becomes the Dynamic Theil's Entropy on the whole Population (PDTE):

$$\begin{aligned} PDTE(t;N) = {} & \sum_{g=1}^{K} a_{C_g}\left(\underline{n}(t)\right) TE\left(y_{C_g}; n_{C_g}(t)\right) \\ & + \sum_{g=1}^{K} a_{C_g}\left(\underline{n}(t)\right) \log K a_{C_g}\left(\underline{n}(t)\right) + \sum_{g=1}^{K} a_{C_g}\left(\underline{n}(t)\right) \log\frac{N}{Kn_{C_g}(t)}. \end{aligned} \tag{4}$$

In calculate formula (4) we supposed the agents can leave the early class and enter a new income class according to a discrete time Markov chain with transition probability matrix **P** whose element p_{ij} denotes the likelihood that an agent, now assigned in C_i, will enter C_j. Notice the second addendum of (4) coincides with the Dynamic Theil's Entropy (DTE), as defined in D'Amico and Di Biase (2010). As remarked before it measures the income inequality among the classes after standardizing the population. We may summarize the entire process by computing the first order moment addendum by addendum as proved in D'Amico, Di Biase and Manca (2014b). It is worth noticing the variability of each income y_{C_g} can be more properly considered when we treat the vector $\underline{y}(t) = \left(y_{C_1}, y_{C_2}, \dots, y_{C_K}\right)$ as random. We could address this major complexity by using Markov (or semi-Markov) reward processes as done for computing the Dynamic Herfindahl-Hirschman index in D'Amico, Di Biase and Manca (2014a).

DATA AND METHODOLOGY

We used the Organization for Economic Co-operation and Development (OECD) Income Distribution Database (http://stats.oecd.org) and we picked up data about population, means and medians of the equivalised net income for Mexico, Portugal and Spain. Data refer to years from 2004 to 2011. We motivate the choice of the countries since they represent countries with comparable socio-cultural life styles and religion but with different rates of change of GDP. We dictate the choice of the time interval by practicable considerations on data availability. We report the input data in Table 1.

Table 1: Population and Net Income Evolution for Spain, Portugal and Mexico

	SPAIN			PORTUGAL			MEXICO		
Years	**Population**	**Mean**	**Median**	**Population**	**Mean**	**Median**	**Population**	**Mean**	**Median**
2004	42,873,973	13,292	11,530	10,529,255	10,593	8,154	102,988,791	45,286	30,147
2005	43,586,854	14,188	12,600	10,569,592	10,684	8,306	**104,661,644**	**48,298**	**32,293**
2006	44,339,161	15,243	13,442	10,599,095	11,109	8,573	**106,334,496**	**51,309**	**34,439**
2007	45,109,464	16,434	14,647	10,617,575	11,508	9,219	**108,007,349**	**54,321**	**36,585**
2008	45,607,945	16,793	14,979	10,627,250	11,609	9,232	109,680,201	57,333	38,731
2009	45,757,233	16,592	14,685	10,637,713	11,728	9,595	**111,140,392**	**56,769**	**38,866**
2010	45,900,276	15,993	14,000	10,636,979	11,610	9,357	112,600,583	56,205	39,001
2011	46,354,779	15,503	13,356	10,541,840	11,190	9,232	**114,942,506**	**58,780**	**40,215**

This Table shows Population, Means and Medians of the equivalised net income for the considered countries picked up from web site http://stats.oecd.org. We expressed the incomes of Spain and Portugal in Euros whereas that of Mexico in Pesetas. We approximated all data to the units. The numbers in bold characters represent the results of a linear interpolation for missing data.

First we recovered the income distributions by assuming that they follow a Lognormal distribution function, theory well supported by the Offices for National Statistics of the considered countries. Known means and medians we recovered the parameters reported in Table 2.

Table 2: Parameters of the Lognormal Distributions

COUNTRIES	μ	Σ
Spain	9.35271	0.53331
Portugal	9.00626	0.723443
Mexico	10.3138	0.902123

This Table shows the values of the parameters of the Lognormal distributions for the considered countries: the mean μ and the standard deviation σ.

After that we built the states of the Markov Chain model by assigning each agent in one of the states according to the following rules as suggested by Quah (1996) 1.) If an economic agent has less than a quarter of the country's average per capita income, then we assign it in the poorest class C_1, 2.) If the agent has an income between a quarter and one half of the country's average per capita income, then we assign it in the class C_2, 3.) If the agent has an income between one half and the country's average per capita income, then we assign it in the class C_3, 4.) If the agent has an income between the country's average per capita income and its double, then we assign it in the class C_4 and 5.) If the agent has an income more than the double of the country's average per capita income, then we assign it in the richest class C_5. Next we calculated the income of classes $C_k, k \in \{1, 2, 3, 4, 5\}$ by the complete income distribution of its agents as the conditional expectation of the income distributions got just before, given the income is in class C_k.

RESULTS AND DISCUSSION

In Table 3 we reported the income of the classes got for each country.

Table 3: The Income of the Classes

Classes	Spain	Portugal	Mexico
C_1	2,910	2,092	7,931
C_2	5,456	4,062	16,740
C_3	9,769	7,560	31,885
C_4	17,197	14,046	60,752
C_5	31,351	26,362	116,197

This Table shows the income of the classes got for each country. We expressed the incomes of classes of the Spain and Portugal in Euros whereas that of Mexico in Pesetas. We estimated all incomes to the units.

Next by iterating the procedure from the year 2004 to the year 2011 we got the evolution of the population outline in time. We report these results in Table 4, Table 5 and Table 6 for Spain, Portugal and Mexico respectively.

Table 4: Evolution of Population in Spain

CLASSES	2004	2005	2006	2007	2008	2009	2010	2011
C_1	421,469	198,914	256,094	172,937	167,399	225,478	324,572	502,848
C_2	6,043,493	4,911,702	5,271,368	4,721,828	4,678,613	5,087,413	5,628,350	6,336,233
C_3	19,479,480	20,452,423	20,153,642	20,606,729	20,641,534	20,307,553	19,847,939	19,210,153
C_4	14,415,746	15,260,098	14,996,083	15,398,417	15,429,819	15,131,490	14,730,527	14,188,847
C_5	2,513,785	2,050,836	2,196,785	1,974,062	1,956,608	2,122,040	2,342,585	2,635,893

This Table shows the population changing in Spain from 2004 to 2011 subdivided for each of the income classes, that is for the five states of the Markov chain model.

Table 5: Evolution of Population in Portugal

CLASSES	2004	2005	2006	2007	2008	2009	2010	2011
C_1	632,068	578,382	618,188	423,126	459,897	322,896	393,524	285,497
C_2	2,268,253	2,232,602	2,259,457	2,099,101	2,135,620	1,978,153	2,066,947	1,922,551
C_3	3,851,247	3,913,325	3,866,978	4,115,755	4,064,133	4,272,429	4,159,371	4,338,662
C_4	2,793,797	2,841,580	2,805,887	2,998,841	2,958,513	3,122,303	3,033,045	3,175,027
C_5	983,890	963,365	978,745	892,432	911,091	833,475	876,368	807,517

This Table shows the population changing in Portugal from 2004 to 2011 subdivided for each of the income classes, that is for the five states of the Markov chain model.

Table 6: Evolution of Population in Mexico

CLASSES	2004	2005	2006	2007	2008	2009	2010	2011
C_1	14,296,777	14,052,813	13,837,974	13,647,352	13,477,081	12,726,504	11,968,078	12,765,171
C_2	24,376,718	24,363,089	24,349,336	24,335,724	24,322,415	24,250,171	24,152,897	24,254,455
C_3	30,743,761	30,911,007	31,059,697	31,192,760	31,312,539	31,851,401	32,415,221	31,823,192
C_4	22,104,372	22,228,556	22,339,029	22,437,944	22,527,029	22,928,326	23,349,158	22,907,297
C_5	11,467,162	11,433,325	11,402,755	11,375,011	11,349,727	11,232,388	11,103,438	11,238,675

This Table shows the population changing in Mexico from 2004 to 2011 subdivided for each of the income classes, that is for the five states of the Markov chain model.

Finally we estimated the probability matrices by minimizing a χ-squared type expression, as in D'Amico, Di Biase and Manca (2012). We reported the results got in Table 7, Table 8 and Table 9 for Spain, Portugal and Mexico respectively.

Table 7: One Step Transition Probability Matrix for Spain

CLASSES	C_1	C_2	C_3	C_4	C_5
C_1	0.1500	0.8500	0.0000	0.0000	0.0000
C_2	0.0475	0.0500	**0.9025**	0.0000	0.0000
C_3	0.0000	0.2400	0.6000	0.1600	0.0000
C_4	0.0000	0.0000	0.2100	0.7000	0.0900
C_5	0.0000	0.0000	0.0000	0.6500	0.3500

This Table shows the transition probability matrix of the Markov chain model for Spain. Each number gives the probability that an agent being in this year in class i (represented on the rows) next year will get to the class j (represented on the columns). For example the value 0.9025 *stands for the probability that a Spanish individual, now assigned in class C_2 will enter the next allocation in class C_3.*

Table 8: One Step Transition Probability Matrix for Portugal

CLASSES	C_1	C_2	C_3	C_4	C_5
C_1	0.0500	0.9500	0.0000	0.0000	0.0000
C_2	0.2100	0.3000	0.4900	0.0000	0.0000
C_3	0.0000	0.2400	0.6000	0.1600	0.0000
C_4	0.0000	0.0000	0.2100	0.7000	0.0900
C_5	0.0000	0.0000	0.0000	0.3000	0.7000

This Table shows the transition probability matrix of the Markov Chain model for Portugal. Each number gives the probability that an agent being in this year in class i (represented on the rows) next year will get to the class j (represented on the columns).

Table 9: One Step Transition Probability Matrix for Mexico

CLASSES	C_1	C_2	C_3	C_4	C_5
C_1	0.5500	0.4500	0.0000	0.0000	0.0000
C_2	0.2475	0.4500	0.3025	0.0000	0.0000
C_3	0.0000	0.2275	0.6500	0.1225	0.0000
C_4	0.0000	0.0000	0.1600	0.8000	0.0400
C_5	0.0000	0.0000	0.0000	0.1000	0.9000

This Table shows the transition probability matrix of the Markov Chain model for Mexico. Each number gives the probability that an agent being in this year in class i (represented on the rows) next year will get to the class j (represented on the columns).

To estimate all transition probabilities needed in our model allows gaining valuable information about to evolve the population shares in each class and thus the mobility in the distribution. Indeed, assuming as initial population distribution that of the year 2004 and using the probability matrices reported in Table 7, Table 8 and Table 9, we calculated the evolution in time of the expected values of the population of Spain, Portugal and Mexico. We reported it in Table 10.

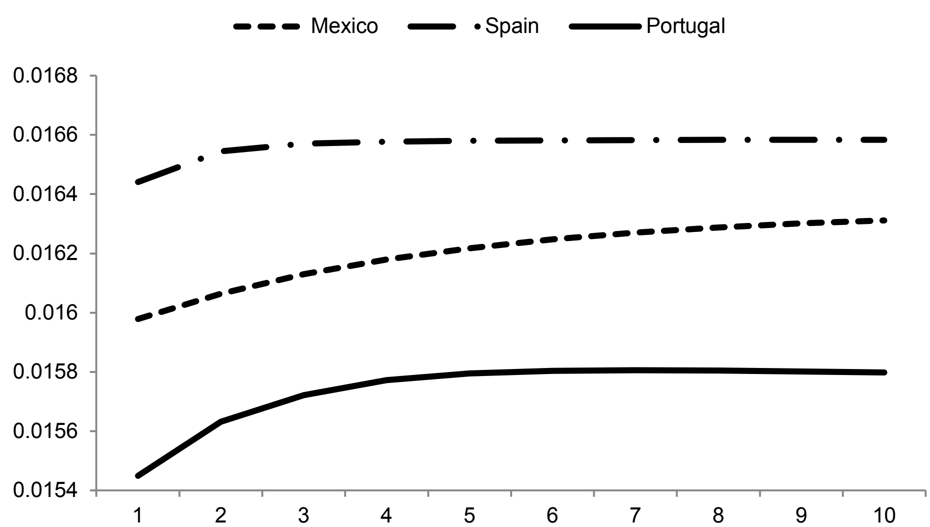
Now we can show the forecasts of the inequality measure through Theil's entropy considering the complete population, that is the PDTE. In Figures 1, 2 and 3 we report respectively to evolve the first, the second and the third addenda of PDTE for Spain, Portugal and Mexico. The graph on Figure 1 displays the expectation, on a time horizon of ten years, of the inequality within each income class for the three countries. The values are of two orders of size lower than those of the two graphs shown in Figures 2 and 3. This applies for all the considered countries. We expected the inequalities within the classes to increase in the three countries. The highest inequality within each class occurs for Spain, then for Mexico and the lowest occurs for Portugal.

Table 10: Population Expected Evolution for Spain, Portugal and Mexico

	SPAIN					PORTUGAL					MEXICO				
Years	n_{C_1}	n_{C_2}	n_{C_3}	n_{C_4}	n_{C_5}	n_{C_1}	n_{C_2}	n_{C_3}	n_{C_4}	n_{C_5}	n_{C_1}	n_{C_2}	n_{C_3}	n_{C_4}	n_{C_5}
2004	0.0421	0.6043	1.9479	1.4416	0.2514	0.0632	0.2268	0.3851	0.2794	0.0984	1.4297	2.4377	3.0744	2.2104	1.1467
2005	0.0350	0.5335	2.0169	1.4842	0.2177	0.0508	0.2205	0.4009	0.2867	0.0916	1.3896	2.4397	3.0894	2.2596	1.1205
2006	0.0306	0.5405	2.0034	1.5031	0.2098	0.0489	0.2106	0.4088	0.2930	0.0905	1.3681	2.4261	3.1077	2.2982	1.0988
2007	0.0303	0.5338	2.0055	1.5091	0.2087	0.0467	0.2077	0.4100	0.2980	0.0902	1.3529	2.4144	3.1216	2.3291	1.0808
2008	0.0299	0.5337	2.0020	1.5129	0.2089	0.0460	0.2051	0.4104	0.3014	0.0903	1.3417	2.4055	3.1320	2.3538	1.0659
2009	0.0298	0.5326	2.0006	1.5151	0.2093	0.0454	0.2037	0.4100	0.3037	0.0905	1.3333	2.3987	3.1401	2.3733	1.0535
2010	0.0298	0.5321	1.9992	1.5167	0.2096	0.0450	0.2026	0.4096	0.3052	0.0908	1.3270	2.3938	3.1464	2.3886	1.0431
2011	0.0297	0.5317	1.9983	1.5178	0.2099	0.0448	0.2019	0.4091	0.3064	0.0916	1.3223	2.3901	3.1515	2.4007	1.0343

This Table shows the evolution in time of the expected values of the population structure for Spain, Portugal and Mexico respectively. We multiplied all numbers for 10^7*. We approximated all data to the fourth decimal digit.*

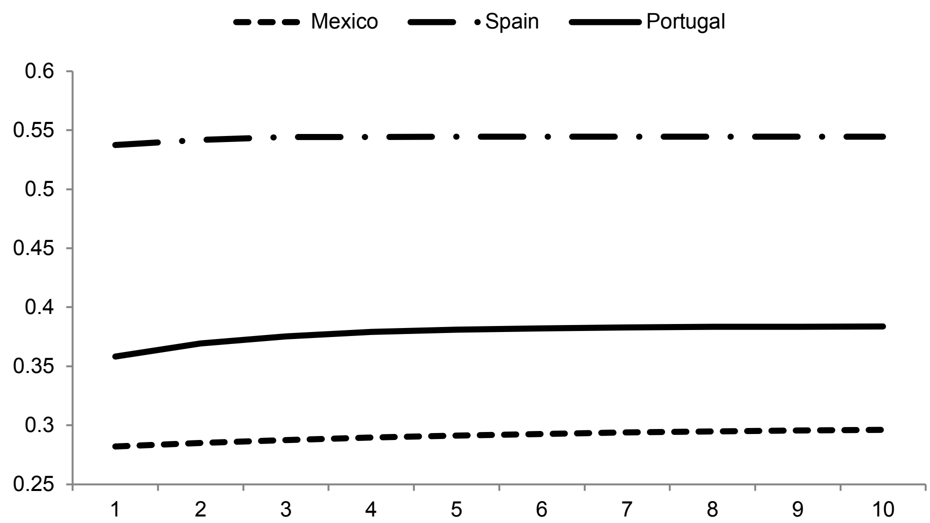
Figure 1: First Addendum of PDTE



This figure shows the forecast on a time horizon of 10 years (x-axis) of the values (y-axis) of the first addendum of decomposition of the Population Dynamic Theil's Entropy. It expresses a measure of the weighted sum of the inequalities within each class.

The graph on Figure 2 shows the values of Dynamic Theil's Entropy (DTE). It give a measure of the inequality among the classes $C_k, k \in \{1,2,3,4,5\}$ after the standardizing the population. As shown before it coincides with the second addendum of the decomposition of PDTE. As we can see in Figure 2 we expect the inequalities to increase slightly in time for Mexico and Portugal whereas in Spain we expect the inequality among the classes to be constant in time. The highest inequality among the classes occurs for Spain, then for Portugal whereas the lowest occurs for Mexico.

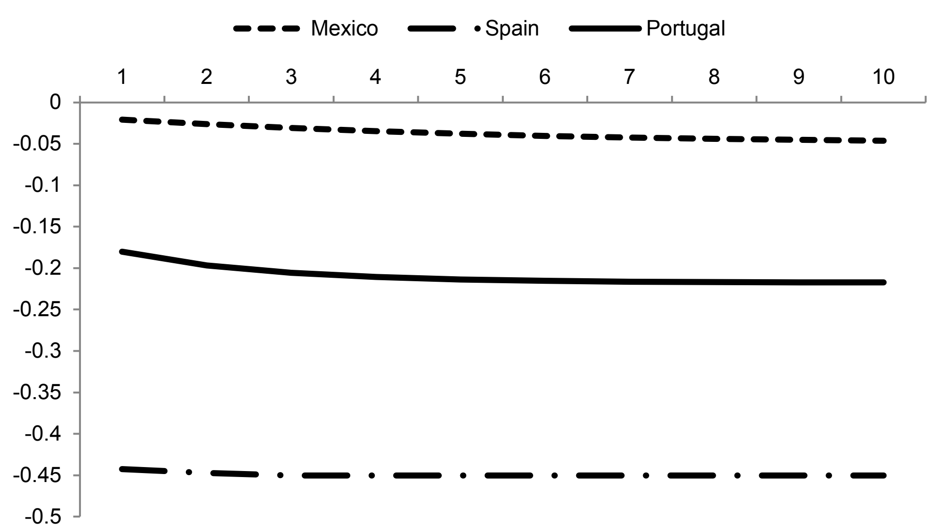
Figure 2: Second Addendum of PDTE



This figure shows the forecast on a time horizon of 10 years (x-axis) of the values (y-axis) of the Dynamic Theil's Entropy that coincides with the second addendum of the decomposition of the Population Dynamic Theil's Entropy. It expresses a measure of the inequality among the classes.

The graph on Figure 3 shows the opposite of the mean logarithmic deviation of the real population structure on the uniform distribution $(\overline{n}, \overline{n}, \dots, \overline{n})$. It represents a correction term, always negative as proved in D'Amico, Di Biase and Manca (2014b), that we added to the DTE when computing the inequality among the classes. This addendum may compensate the increase in the inequality caused by standardizing the population.

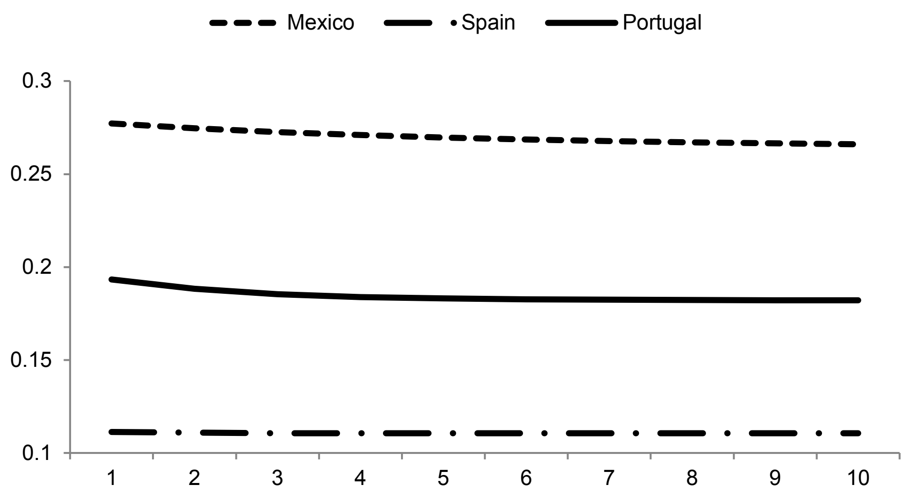
Figure 3: Third Addendum of PDTE



This figure shows the forecast on a time horizon of 10 years (x-axis) of the values (y-axis) of the third addendum of the decomposition of the Population Dynamic Theil's Entropy. It expresses a correction term.

Finally in Figure 4 we got the forecast of the PDTE, on a time horizon of ten years, by summing the curves of Figure 1, Figure 2 and Figure 3 for each of the three considered countries. The three curves incorporate also the effect because of the diversity of the agents within the same income class. As we can see, it exists a net ranking of the inequalities in the whole population: Mexico is the country where the inequality is higher; it comes before Portugal and Spain, where we expect the income to be more equally spread. We expect the inequality to decrease in time in Mexico and Portugal although the decreasing rates are small.

Figure 4: Forecast of PDTE for Mexico, Portugal and Spain



This Figure shows the forecast on a time horizon of 10 years (x-axis) of the values (y-axis) of the Population Dynamic Theil's Entropy (PDTE) for Mexico, Portugal and Spain. The PDTE evaluates the income inequality in the whole population and not only among the classes in which we classified the economic agents.

CONCLUSIONS

In this paper we used the Population Dynamic Theil's Entropy to forecast the income inequality in the years for Mexico, Portugal and Spain. The index, considering both the inequality among the classes in which we assigned the individuals and the inequality within each class, measures the inequality in the whole population. The data used refer to population, means and medians of the individual income for the three countries. They refer to the years from 2004 to 2011. First we recovered the net income distributions and we built the states of the Markov chain model, that is the classes of richness in which we assigned the individuals. Then we computed the net income of the classes and the population configuration evolution for the three countries. After that we estimated the transition probability matrices by minimizing a χ-squared type expression. Next we evaluated the evolution in time of the expected values of the population structure for Mexico, Portugal and Spain. Finally for these three countries we got the forecasting, on a time horizon of ten years, of the values of PDTE. It evaluate the income inequality in the whole population and not only among the classes in which we classified the economic agents.

The results of the application highlight different values of the index. We can remark that it exists a net ranking of inequalities: Mexico is the country where the inequality is the highest; it comes before than Portugal and Spain, where the income is expected to be more equally distributed. In Mexico and Portugal we expect the inequality to decrease in time although the decreasing rates are small. We think the knowledge of the time evolution of the inequality indices plays a fundamental role in programming better economic policies. Therefore the model could be of great help to decision makers. Indeed, although in the considered countries the population mobility among income classes (see Tables 7, 8 and 9) is higher than the mobility registered in others countries (e.g. France, Germany, Greece, Italy, as estimated in D'Amico, Di Biase and Manca, 2012), the values of the index in Spain are constant in time whereas in Mexico and Portugal decrease slightly. This fact leads to the conclusion the changes in the inequalities in these countries are possible only if a consistent change of the income in each class occurs. Proper economical policies could address such income changes. We could measure the income inequalities in a more accurate way by recovering series of microdata on income evolution. In this way we could calculate the transition matrices by its maximum likelihood estimators. Therefore possible avenues for development of our model could be 1.) A real data application involving microdata, 2.) To set up a geographical model and 3.) The research of numerical bounds aimed to explain the differences among the indices values.

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