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SKEWNESS, CRYPTOCURRENCY, AND PEER-TO-PEER LOANS: AN ASSET ALLOCATION EXERCISE FOR A UNIQUE STUDENT-MANAGED FUND

Lynda S. Livingston, University of Puget Sound and Four Horsemen Investments

ABSTRACT

We incorporate skewness and kurtosis into an optimization process for a unique student-managed fund. Unlike the vast majority of such funds, which hold only equity, our fund includes REITs, cryptocurrency, and peer-to-peer loans. Adding these unusual asset classes allows our students to explore portfolio management concepts more generalizable than just picking stocks. While most of our assets cannot be recommended based solely on traditional mean-variance analysis, they nonetheless offer beneficial contributions. Using polynomial goal programming to incorporate higher moments in our optimization, we find that asset classes dominated in mean-variance space can make meaningful contributions to the full risk-return profile of the portfolio. In particular, we find that including cryptocurrency and peer-to-peer loans can increase the skewness and decrease the kurtosis of our portfolio.

JEL: G10, G11

KEYWORDS: Cryptocurrency, Peer-to-Peer Loans, Student Managed Funds, Polynomial Goal Programming

INTRODUCTION

Many universities offer finance students the opportunity to run student-managed funds (SMFs). Most of these funds are large, actively managed equity pools, often funded through the institution's endowment and tied to its structured curriculum. But why do almost all SMFs focus only on equities, if more than 90% of a professionally managed portfolio's return variance is explained by strategic asset allocation across multiple asset classes (Brinson, *et al.*, 1986)? Why do most SMFs stress stock picking, if active equity management is almost certainly doomed to fail? (See Sharpe, 1991, and Sharpe, *et al.*, 2014, for the nuances behind these assertions.) Our unique SMF does not focus on stock picking. Instead, our small, independent fund—run through a not-for-profit corporation established by students—manages assets most others do not: REITs, peer-to-peer loans, and cryptocurrency. Our students explore fundamental asset-class distinctions and portfolio optimization techniques. Our fund demonstrates that any school—no matter how small—can create meaningful, comprehensive experiential learning opportunities for portfolio management students.

In this paper, we show how our members approach the strategic asset allocation for our portfolio. We start with traditional mean-variance criteria, using corner portfolios and Excel's matrix manipulation capabilities (see Arnold, 2002) in a constrained optimization. More interestingly, we then expand the objective function to include skewness and kurtosis, using polynomial goal programming to re-optimize. We find that the inclusion of our non-equity "experience assets" can significantly improve our portfolio's characteristics.

Our approach is not specific to student-managed funds. Practitioners are increasingly confronted with the messy reality of negatively skewed and fat-tailed asset returns. By introducing students to broader optimization techniques like polynomial goal programming, we can help move professional money management beyond the mean-variance efficient frontier.

The paper proceeds as follows. In the next section, we review the relevant literature. We then describe our data, paying particular attention to our cryptocurrency returns. Next, we describe our mean-variance and four-moment asset allocation optimization. We summarize in the final section.

BACKGROUND AND LITERATURE REVIEW

Our research draws on the literature on student managed funds, asset allocation, and optimization with higher moments. We consider each in this section.

Student-Managed Funds and Four Horsemen Investments

Managing financial portfolios is complicated. To help finance students prepare for the challenge, many universities have established student-managed funds, pools of assets run by students and providing them with active, hands-on experience in the financial markets. From our perspective, the problems with most of these funds are in their focus on actively managed equity and in their restrictions on student participation and leadership to a stringent curriculum.

The vast majority of SMFs are equity funds. For example, in Neely and Cooley's (2004) sample of 61 SMFs, only four are dedicated debt funds, while 40 are equity-only. Peng *et al.* (2009) and Morgan (2008) find similar equity bias in their samples. Universities emphasize this bias: websites touting student-managed funds highlight the opportunities for students to research and pick stocks (e.g., University of Chicago), to "pitch" stocks (University of Miami), to employ industry and sector rotation (Villanova), and to practice Benjamin Graham/Warren Buffet-style value investing (University of Connecticut). The opportunity to manage multiple asset classes is noticeably absent. One exception is Fordham University, which proudly states that, "While other universities' students are restricted to investing in domestic stock, Fordham undergraduates have the freedom to experiment with international and domestic securities, bonds, commodities, and foreign exchanges" (Fordham, 2018). However, even Fordham does not allow students to venture into more unusual asset classes, such as peer-to-peer loans or cryptocurrency. (Of course, some may argue that a student-managed fund has no business incorporating these sorts of assets. However, as Yau, *et al.*, 2007, note: "Portfolio managers who understand alternative investments have a substantial advantage over those who do not." Considering this position is one of the goals of this paper.)

In addition to being equity-focused, most SMFs are also funded through the sponsoring university's endowment and supported by its academic curriculum. (For an overview, see Peng, *et al.*, 2009, and Mallett and Lerro, 2001.) For example, Trinity University's SMF was established through a \$500,000 allocation from its endowment. It is managed through a one- or two-semester, senior-level course, requiring three additional prerequisites (Trinity, 2018). The University of South Florida requires its student managers to take a two-semester securities analysis course as part of the requirements to manage its half-million dollar portfolio (USF, 2018). At the University of Puget Sound, our SMF was established through a targeted \$100,000 gift to the endowment, and is supported by two specially created courses, valuation and portfolio management.

As is suggested by the sizes of the three funds just described, student-managed funds are typically hundreds of thousands or millions of dollars in size. In Peng *et al.*'s (2009) sample, only 18% of the funds were below \$100,000; Neely and Cooley (2004) estimate a "modal" SMF size at between \$200,000 and \$400,000. (Give the age of these studies, fund sizes have undoubtedly grown; for example, the Trinity fund mentioned above is now worth more than \$5 million.) The few funds that focus on debt are even larger. Morgan (2008) estimates a \$1 million practical minimum for fixed-income funds, given that round lots in this market are so much larger than those in equities. All of the debt funds in his sample exceed this minimum, with one—Iowa State University's—clocking in at $100 million.

Thus, the typical student-managed fund is a large, equity-only portfolio funded by a university endowment and tied to a structured curriculum. The Four Horsemen Investment fund, in contrast, is a small, diversified (but debt-heavy) fund run through an independent 501(c)(3) charitable corporation. It is also about more than the portfolio. Of course, we offer the experience of managing real money in real time, but we also perform research on alternative investments and fringe lending, and we provide meaningful outreach focused on financial literacy. Unshackled from any university's institutional constraints, we are able to allow our student managers (who run the corporation as well as the fund) to explore asset classes and strategies unavailable through traditional SMFs. Thus, unlike most student fund managers, our members are able to conduct meaningful asset allocation reviews. They also must grapple with the basic definition of an "asset class" and whether our experience assets—especially P2P loans and cryptocurrency—qualify. In the next section, we review relevant literature that has helped inform our understanding of our loan and coin assets and how they can contribute to our overall portfolio.

Peer-to-Peer Loans, Cryptocurrency, and Asset Classes

Four Horsemen Investments was able to create a small, debt-focused fund by investing in peer-to-peer (P2P) loans. These loans are personal loans facilitated by an online platform like Prosper or Lending Club. Potential borrowers post a "listing" on the platform, which assesses their creditworthiness. Lenders on the platform can then offer to fund all or part of the loan, bidding as little as $25 on any given loan. If the loan garners enough offers to be fully funded, the platform will make the loan and handle the distribution of payments among the participating lenders. (See Herzenstein, *et al.*, 2008, Iyer, *et al.*, 2009, and Freedman and Jin, 2008, for descriptions of this market.)

With our founding donation of $1,000, 4HI was able to create a diversified portfolio of these P2P loans. Members were able to do meaningful credit analysis and develop useful default models—not only moving beyond standard active equity management, but moving beyond corporate bonds. This model allowed us to create a unique experience for students even without the support of our university's endowment.

However, the P2P market has changed dramatically since our founding in 2009. Observing the profit potential there, institutional investors have moved into the space, crowding out true "peer" lenders. (See, for example, Shore, 2014, and PwC, 2015.) For example, platforms now make arrangements with hedge funds to screen new loans before they are ever made available to peer lenders, allowing wholesale skimming of the best loans. Banks are also anxious to partner with the P2P companies, to take advantage of the platforms' user-friendly interface, cutting-edge technology, and efficient, low-cost underwriting (Schatt, 2014). Now that we have less access to good loans—and have seen our defaults rise, probably not coincidentally—we have had to reexamine our commitment to P2P loans. This was a major motivator for our first comprehensive asset allocation exercise, which we describe below. First, however, we discuss the other unusual asset in our portfolio: cryptocurrency.

Two years ago, we were very fortunate to receive a donation of two bitcoin from an alumnus. We had to decide whether we would immediately liquidate this windfall or attempt to incorporate the coins into our portfolio. Was our bitcoin just a weird type of cash, or was it a new asset class?

Both art and science are involved in defining an asset class. The definitions we choose directly determine the way we allocate our funds, how we monitor our performance, and how we effect our rebalancings. While there is a tendency to define asset classes the way Justice Potter Stewart defined obscenity ("I know it when I see it"; *Jacobellis v. Ohio*, 1964), there are some commonly accepted guidelines that our members consider. Swensen (2009) stresses that asset classes should be primarily distinguished by function: "broad, sweeping differences in fundamental character." Within a class, assets should share similar statistical properties, and those properties should be different from those of other classes. Asset classes should be mutually exclusive and investable, and the set of classes should span the relevant and available universe (Byrne and Smudde, 2011). Finally, the asset classes should provide the "beta" component of return: since "[s]atisfying investment objectives proves too important to rely on serendipity or the supposed expertise of market players" (Swensen, 2005), asset classes "must raise the expected utility of a portfolio without requiring superior asset selections within the class" (Chauncey, 2002, citing Kritzman, 1999).

While most researchers and practitioners now agree that these criteria admit more than just debt and equity, opinions vary about what other types of assets deserve recognition as a practical class. For example, Goss (2012) evaluates the potential for a popular favorite—gold—to qualify. He finds that since the credit crisis gold's correlation to stocks has become unstable, while its correlation to inflation is "relatively modest." If there is another financial crisis, when any defensiveness of gold would be particularly welcome, Goss expects that the resulting flight to quality would make Treasuries the real hedge, while gold would act more like a risky asset. However, he believes that gold could be useful if "hedging against the implosion of fiat currencies becomes a truly dominant theme." Recognizing gold as an asset class, then, seems to be a consequence of one's assessment of that eventuality.

In other cases, authors have been more willing to assert asset-class status, and to assets more outside the mainstream. For example, Medina-Martinez and Pardo-Tornero (2013) determine that European Union Allowances (EUAs)—entitlements allowing polluters in the European Union to emit one ton of carbon dioxide—are an asset class because their returns exhibit a *set* of characteristics unlike either stocks' or commodity futures'. Like stocks, EUA returns are negatively skewed, but like commodities, they are positively correlated with fixed-income assets and with inflation. Given that the collection of EUAs' returns' statistical properties mirrors neither financial nor commodity assets, the authors declare EUAs a unique asset class.

Similarly, Campbell (2008) identifies art an asset class—and a "financial instrument"—on the strength of its apparent low correlation with equities. However, she also recognizes that the art market is illiquid, opaque, and highly volatile; it provides no periodic cash flows (unless pieces are rented); and it is "whimsical," trendy, and faddish. These latter characteristics make the market subject to bubbles, so she suggests subjectively adjusting any portfolio allocations in art. This is consistent with Swensen's (2009) admonition to portfolio managers to adjust historical optimization inputs to reflect reasonable prospective relationships. It is also the approach that we have taken with our own allocations to cryptocurrency (also called math-based currency, or "MBC"), which shares art's volatility and perhaps its trendiness.

Four Horsemen Investments holds both bitcoin and ethereum (we exchanged some of our initial bitcoin donation for ethereum, to diversify our MBC holdings). These are two of the more than 1,000 types of math-based currencies that exist (39 of which have market capitalizations over $1 billion; Rimkus, 2018). Schatt (2014) describes bitcoin—the oldest and best known of the MBCs—as "a new way of representing and exchanging value using cryptography, a peer-to-peer network, and a public transaction ledger." Bitcoins are long strings of letters and numbers that are generated by a complex, resource-intensive mathematical algorithm. Once generated, a bitcoin's movement—its transaction history—is tracked on an open ledger, visible to everyone (although the entities at either end of a transaction are "obscure"). That transparency is meant to engender trust in the currency, which is overseen by no central authority or government. Burniske and White (2017) view this decentralized, open-source governance scheme as the unique "politico-economic feature" that justifies recognizing MBC as an asset class, asserting—as Goss (2012) does with gold—that "macroeconomic uncertainty underscores its value proposition"; Schatt (2014) agrees.

Nonetheless, there are many bitcoin skeptics. Jamie Dimon called it a "fraud" (Karabell, 2017); Warren Buffet, a "mirage" (Rimkus, 2018). A general manager for the Bank of International Settlements declared that it was "neither a good means of payment, nor a good unit of account, nor… suitable as a store of value"—the three things a currency is supposed to be (Carstens, quoted in Rimkus, 2018).

In contrast, Schatt (2014) sees potential for MBCs to simplify very small payments—pennies—to vendors across the internet (perhaps for gaming or tipping). Nonetheless, in these early days, market participants do not seem focused on cryptocurrency as currency. Instead, most investors appear to hold MBCs because they expect them to appreciate. Glaser, *et al.* (2014) examine volume at bitcoin exchanges (where fiat currency is exchanged for cryptocurrency) and the blockchain (where cryptocurrency is traded for goods and services), and find that uninformed investors, spurred on by volatility, buy MBC as an alternative asset—despite there being no "valid valuation method" or "fundamental pricing methodology available." This enthusiasm by purchasers "limited in their level of professionalism and objectivity" has led many to declare a bitcoin bubble (see, for example, Hankin, 2018).

Bubble or not, given that the vast majority of bitcoin are currently saved, not spent, we see it as purely a speculative asset. Like art, MBC generates no intermediate cash flows, nor does it earn interest, as a currency deposit would. Therefore, we would not judge MBC appropriate for our portfolio in a basic mean-variance sense. However, Four Horsemen Investments has the luxury of being an educational not-for-profit, and, for us, having cryptocurrency is an opportunity to learn. (In fact, offering us this opportunity was one of the main motivations behind the donation.) We have it (and we have a sell discipline), so we are going to use it. Moving beyond the basic "What is it?" question, we are investigating MBC's potential to improve either the skewness or the kurtosis of our portfolio.

Incorporating Skewness and Kurtosis into Asset Allocation

These higher moments are becoming more salient for portfolio managers. The mean-variance approach of Markowitz (1952) defined portfolio optimization for a generation in part because of its relative tractability. However, we suspect that investors' utilities are not quadratic (so that mean and variance are not sufficient parameters to describe their preferences), and we observe that asset returns are not normally distributed (so that mean and variance are not sufficient to describe their choices). Stock returns, for example, are negatively skewed and leptokurtic (Singal, 2011).

In a portfolio, deviations from normality can have significant consequences. For example, positively skewed portfolios have better Sortino ratios and lower semideviations than negatively skewed portfolios (Kim, *et al.*, 2014). On the other hand, Harvey and Siddique (2000) note that assets whose addition would make a portfolio more negatively skewed must compensate investors with higher expected returns. These authors also suggest that conditional coskewness may help explain the equity premium puzzle and the size effect in returns. Davies, *et al.* (2004) assert that higher moments are necessary when assessing the risk profile of hedge funds, since all of the seven strategy groups that they studied (long/short, distressed, merger arbitrage, etc.) were fat-tailed and six were negatively skewed. The high degree of coskewness between funds in the same strategy group means that investors cannot effectively diversify skewness away—they must instead pay for more attractive skewness by accepting higher standard deviation.

These sorts of conjectures have led to an increased focus on skewness and kurtosis among practitioners. For example, these topics are now an integral part of the quantitative methods curriculum of the CFA (see, for example, deFusco, *et al.*, 2015), and are part of the mathematical underpinnings for professional risk managers (see, for example, Parramore and Watsham, 2015). However, it is difficult to incorporate higher moments into optimizations: the algebra is "intractable," giving rise to irrational polynomials that spawn very complicated isovariance curves (de Athayde, *et al.*, 2001). Kim, *et al.* (2014), noting that incorporating higher moments into the traditional objective function makes the problem non-convex, drastically increases the number of parameters, and "makes it practically impossible to obtain reliable estimators," suggest using a quadratically constrained quadratic program (QCQP) to solve a robust version of the mean-variance objective function. Their robust portfolios maximize the worst-case outcome under mean-variance, which favors positive skew and penalizes kurtosis. Nonetheless, this method is computationally and analytically less accessible to undergraduate finance students.

We use instead the much more user-friendly approach of polynomial goal programming, which determines optimal portfolio weights given specific investor preferences over mean, variance, skewness, and kurtosis (see, for example, Lai, *et al.*, 2006, and Kemalbay, *et al.*, 2011). We will apply this approach to our portfolio, to gauge the portfolio impact of our experience assets (P2P loans, MBCs, and REITs) when we incorporate higher moments into our assessment. Before describing this process, however, we will describe our data.

ASSETS AND RETURN DATA

Assets

As noted above, we currently own equity (45% weight), peer-to-peer loans (8%), math-based currency (32%), and REITs (15%). In this section, we describe the use of these assets in our portfolio.

Our equity allocation is a return driver—as it is for almost all portfolios; nonetheless, unlike most SMFs, we approach it as an opportunity to teach members about the efficacy of indexing (rather than the probable futility of stock picking).

We break the equity class into domestic, foreign, and small-cap value, for which we use the Vanguard Index Funds S&P500 ETF (VOO; 13% of total portfolio), the Vanguard International Equity Index (VEU; 5%), and the Vanguard CRSP U.S. Small-cap Value Index ETF (VBR; 27%), respectively. Incorporating international stocks allows us to get some additional diversification (see Swensen, 2009); using a Vanguard index fund allows us to get it cheaply. On the domestic side, we clearly have not broken the space into mutually exclusive groups, so our approach is not based on a well-defined, macro-consistent asset class scheme (see Idzorek and Kowara, 2013). Nonetheless, we are unrepentant. We are very strong believers in the efficacy of the size and value factors, and we embrace any tilts in our domestic exposure with eyes wide open.

While equity is our return driver, math-based currency, the P2P loans, and REITs are our "experience assets" —assets we want our members to learn about. (We will consider the most novel of these, the MBC, in detail in the next section.) Of these three experience assets, we have the longest track record with the P2P loans. Our portfolio was created to hold these loans, and for many years they were the only asset that we held. To inform our expectations for our P2P investments going forward, we draw on a comprehensive review of the performance of our loans from our portfolio's inception in 2009, based on a census of our 131 Prosper loans and a matched sample of 343 Lending Club loans (described in Livingston and Crosby, 2017). As we note below, we will adjust our historic performance for the deteriorating state of the P2P market for small lenders like us.

Our third experience asset is our REIT, which is the newest addition to our portfolio. REITs are common in institutional portfolios (and individual ones, too, as they are one of the five asset classes that Swensen recommends for any personal portfolio; see Swensen, 2009). Nonetheless, they are not at all common in the equity-heavy world of student-managed funds. We added this allocation to help us diversify, to protect against inflation, and to generate cash for member reinvestment. Their versatility offers our members numerous options and research opportunities, since REITs can invest in properties from hotels and commercial real estate to farm and timberland. We have decided to invest in Weyerhaeuser Co. (WY), a timber REIT that gives us a natural resources play reflecting our home in the U.S. northwest. (See Paolone, *et al.*, 2015, for an overview of REITs' investment characteristics; see Morawski, *et al.*, 2008, for a discussion justifying viewing REITs as a unique asset class.) Having described our assets, we now consider the return data that will drive our optimization.

Daily Math-based Currency Data

Later, we will be using monthly data to perform our asset allocation optimization. Nonetheless, understanding the daily behavior of our math-based currencies—experience assets—will help our members better appreciate our portfolio's inputs and potential.

We retrieved daily pricing data for both bitcoin and ethereum from Coinbase (coinbase.com). Coinbase has bitcoin data from January of 2013 and ethereum data from August, 2015, but we choose to look only at the last year (9/30/16 through 9/29/17). Before late 2016, both series showed relatively little variability (with the possible exception of bitcoin around December, 2013) and were not as familiar to most investors as they are now. We do not believe that the statistical properties of the return series pre-2016 will inform our estimates of their future behavior.

Table 1 provides the summary statistics for the two MBC ("coin") return series, along with those for the Dow Jones/Wilshire 5000, a broad-based market indicator. The "math-based currency portfolio" is simply an equally weighted portfolio of the two MBCs, which we will use in our asset allocation below. Both the means and the variances of the coins' returns are significantly larger than those for the market: the *p*-values for the equality of means tests for bitcoin and ethereum are 0.032 and 0.052, respectively, while those for chi square tests for equality of variances are both << 0.0001. (Our variance figure for bitcoin is roughly consistent with that shown in Burniske and White, 2017; see their Figure 23.) All of the series are positively skewed (which is not the expected situation for the market), although the coins are much more so. Similarly, while all of the series are highly leptokurtic, the coins' returns have about twice the excess kurtosis of the market.

Table 1: Summary Statistics for Daily Math-based Currency Returns

		DJW5000	**Bitcoin**	**Rthereum**	**Math-Based Currency Portfolio**
Full year:	mean	0.00035	0.00685	0.00837	0.00851
	variance	0.00002	0.00238	0.00477	0.00225
	coefficient of variation	13.3	7.1	8.3	5.6
	skewness	0.03	0.09	1.19	0.41
	excess kurtosis	2.5	5.6	4.6	4.4
Matched data:	beta	1	-0.12	1.58	0.73
	alpha	0	0.0069	0.0078	0.0074
	coskewness (gamma)	1	-4.8	-6.0	-5.4

This table provides the summary statistics for a year-long, daily series of returns for our two math-based currencies, for an equally weighted portfolio of those MBCs, and for a market indicator. The coin series are more positively skewed and fat-tailed than the market, and they show strong negative coskewness.

The bottom panel of Table 1 reports on some of the coins' returns relationships with the market. To find these values, we used a subsample of 199 of the daily returns. We used this subset because Coinbase gives coin values for all seven days of the week, while market data are reported only for business days. Since we are unwilling to make assumptions about the relative size of coin returns for market trading and non-trading days, we omit returns for both the market and the coins for weekends and holidays, leaving us with 199 return-days. (Thus, for example, we omit Monday returns, leaving us with four returns per week for weeks without holidays.) Using this sample, we find strong comovement between ethereum and the market, but slight oppositional movement for bitcoin. We do not have enough confidence in our data to view these values are much more than suggestive, however.

The magnitude of the coskewness numbers makes them interesting in spite of the data deficiencies. Both coin series have large, negative gammas. (Gamma is calculated as the asset's coskewness with the market, standardized by the market's skew: $\sum_{t=1}^{n}(r_{it}-\mu_i)*(r_{Mi}-\mu_M)^2/\sum_{t=1}^{n}(r_{Mt}-\mu_M)^3$, where M denotes the Wilshire 5000; see Smith, 2007.) We would therefore expect them to make meaningful contributions to a portfolio when the market is negatively skewed (as it usually is). In Smith's (2007) estimates, assets with positive gamma would need to offer a premium of 1.81% when the market is negatively skewed, to compensate for their exacerbation of unattractive skewness; our negative-gamma coins should therefore be attractive in such a market. (This is the same sort of argument that supports commodity investing; see, for example, Medina-Martinez and Pardo-Tornero, 2013.) We will evaluate the coins' contributions to portfolio skewness further later in the paper.

To complete our discussion of the daily cryptocurrency data, we consider the possibility of autocorrelation. Using the 364-day sample, we find that the monthly variance is 39 times larger than the daily variance for bitcoin, and 137 times larger for ethereum; both are significantly higher than 30.33 times the daily variance ($p << .001$). We therefore could suspect some positive serial correlation. However, runs tests on both coins cannot reject randomness. Regressions of errors on lagged errors (using predicted values based on regressions of the coins' returns against the DJW5000, using the 199-day sample) also show no relationship. Thus, we have mixed signals about possible daily serial correlation. However, we will be using monthly data for our actual asset allocation work, which should mitigate any problems (via "aggregational gaussianity"; see Medina-Martinez and Pardo-Tornero, 2013). We turn to that monthly data now.

Monthly Data for Optimization

We base our asset allocation model on data from October, 2016 through September, 2017—twelve months' worth of returns. This is obviously a very small sample. However, as we noted above, we do not have confidence that earlier math-based currency data will be especially useful for projections. The same is true for our P2P data: our early portfolio data includes a significant number of Prosper loans, loans from lower grades, and loans chosen based on data available under now-defunct loan listing rules. As noted earlier, the P2P market has changed drastically over the past several years, as institutions have recognized the potential in the space, created funds devoted to P2P investing, and been given priority access to the best loans. Retail investing in P2P is much more treacherous than it was when we began our student-managed fund, reducing the relevance of our early experience to our current portfolio optimization. Table 2 summarizes the data from the monthly sample. The top half of the table is the correlation matrix; the lower half contains summary statistics, including skewness and excess kurtosis.

The math-based currencies are negatively correlated with the broader market indicators, to the REIT, and to the small-cap value ETF, mitigating their extremely high relative volatility. (Burniske and White, 2017, show a similar -0.39 correlation between bitcoin and the MSCI REIT index, using five one-year returns; their correlation between coins and the S&P500 was a higher 0.35.) The coins are also positively skewed and less leptokurtic than all of the other assets except for the international equity ETF and the REIT.

The P2P's potential is less obvious, despite the negative correlations. First, its mean return is negative. We have seen accelerating defaults as our portfolio has aged, and, as noted above, we do not expect dependable access to good loans going forward (see also Gillum, 2018). The small variance is not much consolation, since we expect that it is more an artifact of the measurement problems created by the relative opacity and illiquidity of the market than a reflection of return stability. P2P is also negatively skewed and fat-tailed. Nonetheless, given that our portfolio is a teaching portfolio, these undesirable P2P characteristics must be seen as challenges, not as disqualifiers.

Table 2: Correlations and Summary Statistics for Monthly Sample

	BIT	*ETH*	*P2P*	*VEU*	*VOO*	*VBR*	*WY*	*W5000*
BIT	1							
ETH	0.34	1						
P2P	0.19	0.42	1					
VEU: International	0.02	0.40	0.15	1				
VOO: S&P500	**-0.09**	**-0.24**	**-0.38**	0.11	1			
VBR: SCV	**-0.45**	**-0.52**	**-0.48**	**-0.36**	0.71	1		
WY: REIT	**-0.50**	**-0.07**	**-0.22**	0.12	0.65	1	1	
W5000	**-0.19**	**-0.28**	**-0.44**	0.07	0.98	0.81	0.67	1
Mean	0.196	0.469	-0.004	0.015	0.014	0.014	0.009	0.013
Standard Deviation	0.25	0.82	0.01	0.02	0.02	0.04	0.04	0.02
Maximum	0.70	2.16	0.01	0.04	0.04	0.10	0.08	0.04
Minimum	-0.11	-0.31	-0.03	-0.02	-0.02	-0.03	-0.05	-0.02
Skewness	0.98	1.24	-1.18	-0.75	-0.31	**1.31**	0.17	-0.36
Excess Kurtosis	0.48	0.66	0.93	-0.04	0.70	2.45	**-0.63**	0.98

This table summarizes the inputs to the portfolio optimization. Our "experience" assets are BIT and ETH (the math-based currencies); P2P (the peer-to-peer loans); and WY (the timber REIT). VEU, VOO, and VBR are our equity ETFs: international, S&P500 index, and small-cap value (SCV), respectively. The Wilshire 5000 is our market benchmark. Cells in black highlight potentially useful relationships: negative correlations, positive skewness, and negative excess kurtosis. Note that the coins and the P2P loans offer most of these promising relationships.

The historical data from Table 2 provide our optimization starting point. However, as Swensen (2009) stresses, "Some of the most egregious errors committed with mean-variance analysis involve inappropriate use of historical data," because "past returns provide perverse signals to backward-looking investors." We have therefore made some subjective adjustments to the realized metrics for our coin and P2P assets to reflect the performance we expect going forward, paying particular attention to expected returns, which are more influential on optimization outcomes than variances and correlations by at least an order of magnitude (Sharpe, *et al.*, 2014).

First, we raised our P2P return expectation from its small negative mean to 0.2%/month. Historically, our Lending Club portfolio has returned -.05% and 0.45% per month for higher- and lower-A ratings, respectively, including charge-offs (see Livingston, 2017). Since we intend to concentrate our future lending in these grades, we will use the average, 0.20%, as our expected return for the P2P asset class.

For the coin portfolio, we adjusted both the mean and the standard deviation. Coins have been incredibly lucrative since we put them into our portfolio in early 2017, but we certainly do not expect 20%/month return in the future. We have lowered our expected return to 2%/month—still high, but not excessive in our view, given the 1.5%/month historical return for the international ETF. As for the coins' standard deviation, we reduced the stunning 47%/month to 8%, based on the 28% annual standard deviation projected for private equity in Wilshire Consulting's June, 2018 asset class assumptions.

Finally, we evaluated our estimates for the timber REIT using NCREIF's timberland index (5.2% average return over 3- and 10-year periods; 7.1% over five years); the timberland/farmland/infrastructure assumptions in Milliman's 2017 public pension funding study (5.5% geometric mean; 14.5% annual standard deviation; Sielman, 2017); and the historical returns for the FTSE NAREIT All REITs index as quoted in Blanchett (2014) (12.21% mean; 18.48% standard deviation) and Blanchett and Straehl (2015) (12.36% mean; 19.76% standard deviation). These REIT values were comparable to our historical results, so we made no changes for this asset class.

(We recognize that using a property indicator like the NCREIF to estimate a REIT return involves an inherent mismatch. In the short run, REITs correlate much more closely with stock indexes than they do with returns on real property indicators, since measuring returns on real estate is hampered by frictions long turnover times, the stickiness of appraisals, the heterogeneity of properties, and high transactions costs. In addition, leverage tends to magnify the risk and return of REITs relative to direct real estate portfolios—as is reflected by our estimates above. Nonetheless, over long holding periods, the correlation between

REITs and real estate increases, so that "the return characteristics of REIT holdings of several years are very likely to resemble those of real estate markets where the companies are active"; see Morawski, *et al.*, 2008). Armed with these inputs, we now describe how we approach mean-variance optimization, which we will use later to compare with our four-moment portfolios.

METHODOLOGY AND RESULTS

To assess the potential for our experience assets to benefit our portfolio, we look first at traditional mean-variance optimization, then expand our portfolio measurement criteria using polynomial goal programming.

Mean-Variance Portfolios Using MMULT

Using our forward-looking inputs, we used Excel's MMULT function to determine the efficient set. While many pedagogical expositions of portfolio management recommend using Excel's Solver estimator (e.g., Sharpe, *et al.*, 2014; Carter, *et al.*, 2002), undergraduate students can easily handle the precise and flexible matrix multiplication approach explained in Arnold (2002). (Solver from versions of Excel before 2016 may also give incorrect weights; see Livingston, 2013, and Winston, 2016.)

First, we find the global minimum-variance portfolio (GMVP) using MMULT with no return constraint. The GMVP has short positions in coins (-1%) and REITS (-6%), and—given the loans' relatively small variance—puts over half into P2P (54%). International stocks, the S&P500, and small-cap value make up 15%, 29%, and 9%, respectively. The expected return for this portfolio is 0.8%/month, with a standard deviation of 0.71%/month.

With the GMVP as our starting point, we now add a return constraint to trace out the efficient frontier. MMULT makes it easy to find these efficient portfolios: once the matrix system is set up, all a student has to do is change the expected portfolio return to generate the efficient weighting scheme. Using return targets between 0.8% (the GMVP) and 2% (the mean of our highest-yielding asset, coins), we find a set of efficient portfolios whose standard deviations range from 0.71% to 1.65%/month. REITS are always sold short, and the P2P portfolio—so heavily emphasized at the low-return levels—gets negative weights in the highest-return portfolios. The allocation to coins rises steadily with return, but never tops 8%; the allocation to the three ETFs also rises, with the S&P500 ETF being assigned weights greater than 50% in almost half of the portfolios. (Not surprisingly, this ETF has the highest Sharpe ratio of all of our assets.) The weights in the highest-return portfolio (the 2% portfolio) are -28% P2P, -27% REIT, 8% coins, 56% international stocks, 66% S&P500, and 24% small-cap value.

Corner Portfolios: Sign-constrained Optimization

Once students have found the efficient frontier, they can better appreciate the efficiency losses that attend sign-constrained optimization. Since our portfolio is long-only, we are restricted to nonnegative weights in our asset classes. Thus, we will adjust the efficient frontier, starting by finding the corner portfolios. Corner portfolios define the no-shorting efficient frontier: as we move from one corner to another, assets either enter the portfolio from a zero weight, or they exit entirely. Once the corners are identified, portfolios whose returns fall between the returns of adjacent corners can be found through simple linear interpolation.

(Corner portfolios are not covered in traditional undergraduate finance texts. However, Level III of the CFA curriculum highlights the use of corner portfolios, and gives numerous examples of their application to both individual and institutional asset allocation: see Sharpe, *et al.*, 2014. For more background, Chen and Plemmons, 2007, provide a description of the basic approach to actively solving nonnegative least squares problems. Finally, Markowitz, 1952, Figure 3, offers a visual representation of the process for a three-asset portfolio; I always go over this figure with my undergraduate investments students.)

To find the corner portfolios, we start with the unconstrained global minimum-variance portfolio (GMVP). Since our GMVP assigns negative weights to coins and REITs, those asset classes are then constrained to have zero weight, and the portfolio is optimized over the rest of the asset classes. The result is the constrained minimum-variance portfolio, which will be one of the corners—the corner with the lowest expected return and variance. Since the corner with the highest expected return and variance will be simply 100% invested in the highest-mean asset class, we now have the two outside corners. Any others must lie within the return/risk range defined by these extremes.

Students can find many other corners using the unconstrained efficient portfolios as a guide. Since a corner portfolio is defined by the expected return level at which an asset's weight changes from zero to positive (or vice versa), we focus on those assets assigned both positive and negative weights along the unconstrained efficient frontier. First, students should plot the weights from the unconstrained frontier, noting at what approximate expected return level an asset's weight crosses over from positive to negative. (They should omit assets whose unconstrained weights are always negative.) Next, handling each potential corner-asset one at a time, they can set up an optimization in MMULT—not constraining the weight in the target asset—then use Goal Seek to find the portfolio expected return level at which the optimized weight in that asset is zero. Assuming that this portfolio return is near the crossover return indicated by the unconstrained efficient frontier graph, the portfolio identified by Goal Seek is likely to be a corner.

(Not all corners can be found this way. For example, sometimes an asset will enter the constrained efficient set for a very short return interval, or assets leave the constrained set but not the unconstrained. For example, in Sharpe, *et al.*'s [2014] first optimization, international bonds enter in corner portfolio 4 and exit in corner portfolio 5, while they do not exit again in the unconstrained optimization. Following the procedure described above changes the portfolio expected returns and standard deviations in this example in the affected interval by only a few basis points. Finally, it is also possible to use the improved Solver in Excel 2016 to find corners, since it allows weights to be constrained to be nonnegative; see Winston, 2016. We nonetheless prefer to use the MMULT method with students, since it is less opaque.)

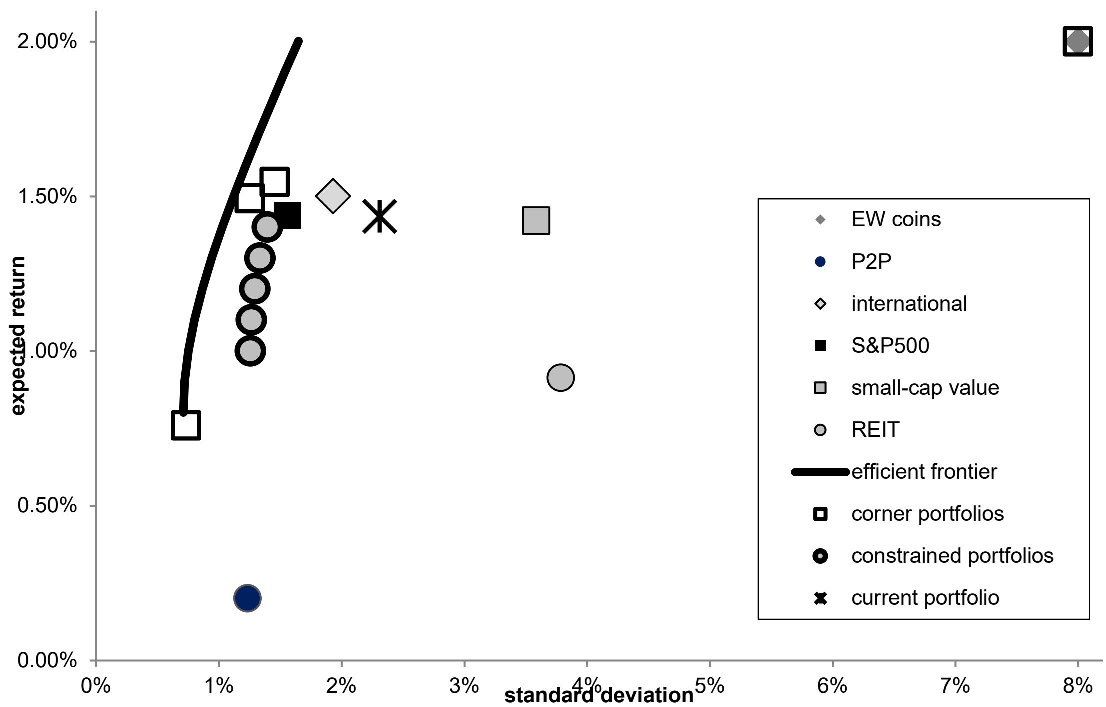
The corner portfolios for our monthly data are plotted in Figure 1. International and small-cap value stocks are in all but the highest-return corner, which is 100% coins (the highest expected-return asset). Coins are not in the constrained minimum-variance portfolio or the next corner; peer-to-peer loans are only in the minimum-variance portfolio. REITs are omitted entirely. (Blanchett and Straehl's 2015 optimizations also allocate 0% to REITs in every case. Their portfolios are designed to incorporate industry-specific human capital, which the authors assert to have "relatively high" correlations with REITs.)

<u>Weight-constrained Optimization</u>

Given the optimizer's relative dislike for "experience" assets—math-based currency, P2P loans, and REITs—and its love for the S&P500, we reran the optimization constraining coins to be 15% (a recognition of reality, given the size of our coin endowment), REITs to be 5% (a minimum for reasonable experience), and the S&P500 ETF to be 10% (the maximum we are willing to commit). This resulted in positive weights for all portfolio assets for expected returns between 1.0% and 1.4%/month. For this weight-constrained set, as return rises, the P2P allocation falls from 40% to 9%; international equity rises strongly from 5% to 37%; and small-cap value rises slightly from 25% to 32%. These portfolios are also plotted in Figure 1.

Portfolios including math-based currency and REITs are clearly dominated in a mean-variance framework. However, they have definite benefits when we broaden the portfolio evaluation criteria to include skewness and kurtosis. For each corner and weight-constrained portfolio, we used the optimized weighting scheme to create a portfolio that we tracked using our 12 months' of data (October, 2016 through September, 2017). (Of course, we would have preferred to test our portfolios using out-of-sample data, but such data is not available. Remember, though, that the optimized weighting schemes were created using <u>adjusted</u> data based on this historical period—not on the pure historical data itself.) Figure 2 plots the weights in the corner portfolios (bars in left-hand section of figure) and in the weight-constrained portfolios (right-hand side), as well as the skewness and excess kurtosis of each set (circles and triangles, respectively, plotted using the right-hand axis). All of the corner portfolios have negative skewness and are strongly leptokurtic. On the other hand, the portfolios that include math-based currency and REITs are positively skewed and essentially mesokurtic—dominating the corners along these dimensions. (This includes our current portfolio, whose skewness is 1.05 and whose excess kurtosis is 0.15.)

Figure 1: Portfolio Assets, Efficient Portfolios, Corners, and Asset-Constrained Portfolios



This figure illustrates the relationships among the various portfolio sets in mean-variance space. All assets and constrained portfolios are obviously dominated by the efficient frontier. Our weight-constrained portfolios—which fix the weights of coins, REITS, and the S&P, and which are shown as circles with heavy borders—are also dominated by the corner portfolios (the optimal long-only portfolios, shown as white squares). However, our weight-constrained portfolios offer a trade-off with their superior skewness and kurtosis, as will be shown below; our optimum portfolio—considering all four moments—may well lie below the pictured frontier (see Davies, et al., 2004).

Figure 2 demonstrates that our experience assets are valuable when we examine portfolio features beyond mean and variance. In the next section, we explore this potential by running a different type of optimization meant to incorporate higher moments: polynomial goal programming.

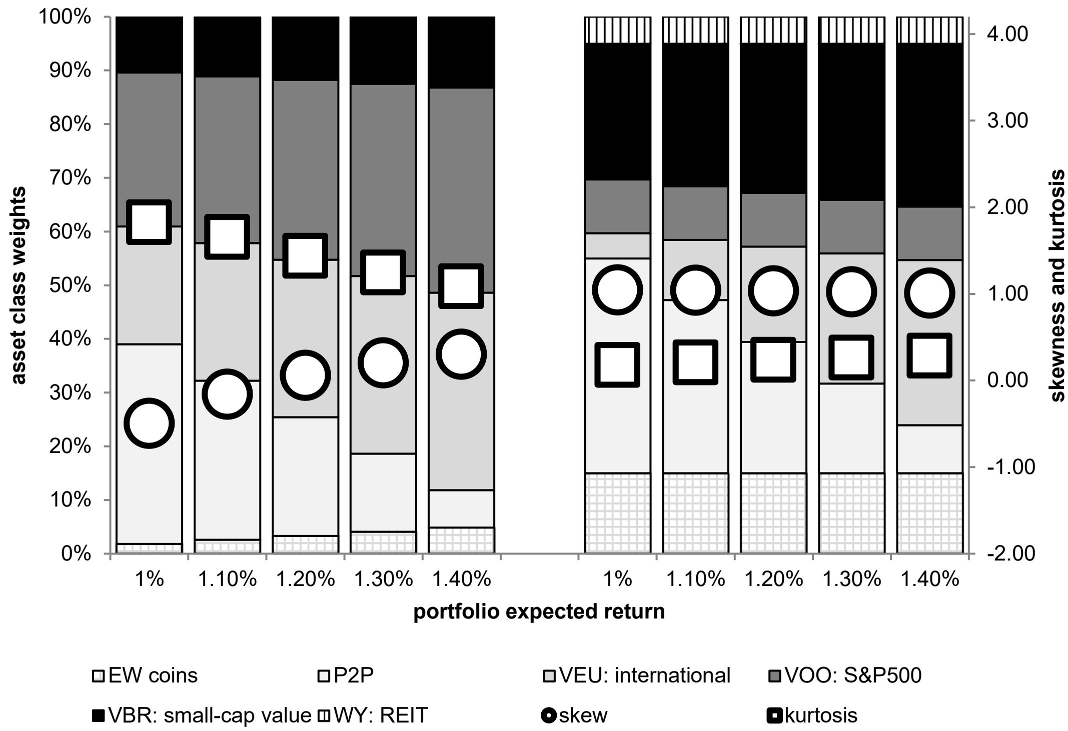
<u>Mean-Variance-Skewness-Kurtosis Portfolios Using Polynomial Goal Programming</u>

Incorporating skewness and kurtosis into an asset allocation means confronting "a nonconvex and nonsmooth multiobjective optimization problem" (Kemalbay, *et al.*, 2011). One way to handle this problem, while incorporating an investor's preferences over the first four moments, is to use polynomial goal programming (PGP). To implement this approach, we minimize the following objective function:

$$Z = \left|\frac{d_1}{R*}\right|^{\lambda_1} + \left|\frac{d_2}{V*}\right|^{\lambda_2} + \left|\frac{d_3}{S*}\right|^{\lambda_3} + \left|\frac{d_4}{K*}\right|^{\lambda_4}.$$

Here, R*, V*, S*, and K* are the "aspired" levels for mean, variance, skewness, and kurtosis, respectively, given the structure of the capital market expectations. Thus, V* is the lowest level of variance possible (from the global minimum-variance portfolio); S* is the maximum possible skewness; K* is the minimum possible kurtosis. R* is the maximum possible mean return; restricting the analysis to nonnegative weights makes this an operational concept (and implies that the max-mean portfolio will plunge into the highest-mean asset, unless weights are restricted). The first step in the PGP process is to find these aspired values. (See Lai, *et al.*, 2006, for an overview of this procedure, and Kleniati, 2004, for an overview of its development; see Massett and Henderson, 2010, for an application of PGP to the wine market and Davies, *et al.*, 2004, for an application to hedge funds.)

Figure 2. Skewness and Kurtosis of Corner and Weight-Constrained Portfolios



This figure plots the weights (bars), skewness (circles), and kurtosis (triangles) of the corner and weight-constrained portfolios. Corners are on the left-hand side of the figure; weight-constrained portfolios on the right. The corners, which do not include REITs or math-based currency, are all negatively skewed and leptokurtic. The weight-constrained portfolios, on the other hand, are positively skewed and essentially mesokurtic, which investors prefer.

Once the market opportunities are characterized, we add representations of the investor's preferences. These are the λ values: λ_1 reflects the investor's preference for the first moment, mean; λ_2 reflects variance; λ_3, skew; and λ_4, kurtosis. Preferences guiding portfolio creation are therefore described as (λ_1, λ_2, λ_3, λ_4); for example, (1100) is the traditional mean-variance portfolio, while (1110) adds skewness as a criterion and (1111) adds both skewness and kurtosis. An investor with a stronger preference for a particular moment will tend to see more attractive values for that moment in her optimized portfolio (Davies *et al.*, 2004).

The PGP optimization proceeds by choosing the *d* terms to minimize the objective function. These *d* terms measure the distance between the portfolio's moment and the optimal level of that moment. Thus, d_1 represents the amount by which the portfolio's mean falls below M*: d_1 = (M* - portfolio mean), and d_3 measures the difference in skew: d_3 = (S* - portfolio skew). The d_2 and d_4 values measure portfolio distances from V* and K*, respectively, defined by subtracting the aspired values (which will be smaller) from the portfolio values. (Thus, *d* values are defined to be positive.) The absolute value functions correct for possible negative values of M* and S*. (For our optimizations, M* and S* are positive, so we can ignore the absolute values; we therefore optimize using Excel Solver's GRG Nonlinear/Multistart engine rather than its Evolutionary engine. See Winston, 2016.)

The results of the initial PGP tests are shown in Table 3. In the top third of the table, the weights were unrestricted (except for the max-mean portfolio, which did not converge; it was restricted to positive weights); in the middle, only nonnegative weights were allowed; at the bottom, coins were set at 15% and REITS at 5%. For each of these three schemes, we give summary statistics, the value of the objective function (Z), and the optimal weights for each of the twelve λ_i preference sets used by Kemalbay, *et al.* (2011). We have roughly grouped these preference sets by the moment they highlight. We will focus primarily on the first three sets: the traditional mean-variance portfolio (whose λ_i values are 1100), the portfolio that adds skew as an equally important criterion (1110), and the portfolio that adds both skewness and kurtosis (1111).

The light grey cells in the table highlight the weights that are less than 5% in absolute value; most of these are zero. (Not surprisingly, the greatest variation occurs when the weights are unrestricted.) No portfolio, under any of the three allowed weighting schemes, uses all six assets.

We can get a sense of what the optimizer "likes" by looking at the unrestricted weight sets at the top of Table 3. Coins are essentially ignored throughout. REITS are sold short—by up to 15%—in all but two portfolios (low-variance portfolios, in which they are simply ignored). REIT weights are lowest in portfolios emphasizing skewness and kurtosis. Small-cap value is similarly deemphasized, being sold short up to 15%; its highest weight—7%—is in the traditional mean-variance portfolio. P2P loans are allocated about half the assets across the board, a reflection of its variance-reduction contributions (from its own low variance and its plethora of negative correlations). The S&P makes up the majority of what is left, along with a 10-20% allocation to international stocks. We consider these results merely indicative, however; most PGP portfolio applications do not permit negative weights (see, for example, Lai, *et al.*, 2006 and Kleniati, 2004). Thus, the sign-constrained portfolios in the middle of Table 3 are a more meaningful baseline.

When we restrict weights to be nonnegative, the biggest changes occur—not surprisingly—in the asset classes which were most often sold short. However, small-cap value (SCV) and REITs change differently: while SCV is now generally omitted where once it was sold short, REITs are now given meaningful positive weights—about 11%—in half of its previously negative-weight portfolios, including our benchmark (1110) and (1111) portfolios. Thus, considering higher moments results in higher weights for REITs, relative to the mean-variance portfolio. Our other experience assets, P2P and coins, on the other hand, have changed little: coins are still ignored, and P2P is still emphasized (albeit even more so: its weights now range from 55% to 71%, with the minimum weight in the mean-variance portfolio).

Things change dramatically when we fix the weights of coins and REITs. Now, the S&P disappears, replaced with a heavy weight in small-cap value. This SCV weight is lowest when the portfolio makes low variance a priority, but is still a minimum of 33%; it is highest in the mean-variance portfolio, at fully 80%. International, on the other hand, is most heavily weighted when variance is highlighted. P2P disappears in half of the cases, but still contributes in the mean-focused portfolios.

Figure 3 summarizes the weighting schemes just discussed, focusing on the mean-variance, 1110, and 1111 portfolios. These three portfolios are shown for the no restrictions case (left-hand side of Figure 3), nonnegative weights case (middle of figure), and restricted-weights case (right-hand side).

Table 3: Polynomial Goal Programming Portfolio Weights

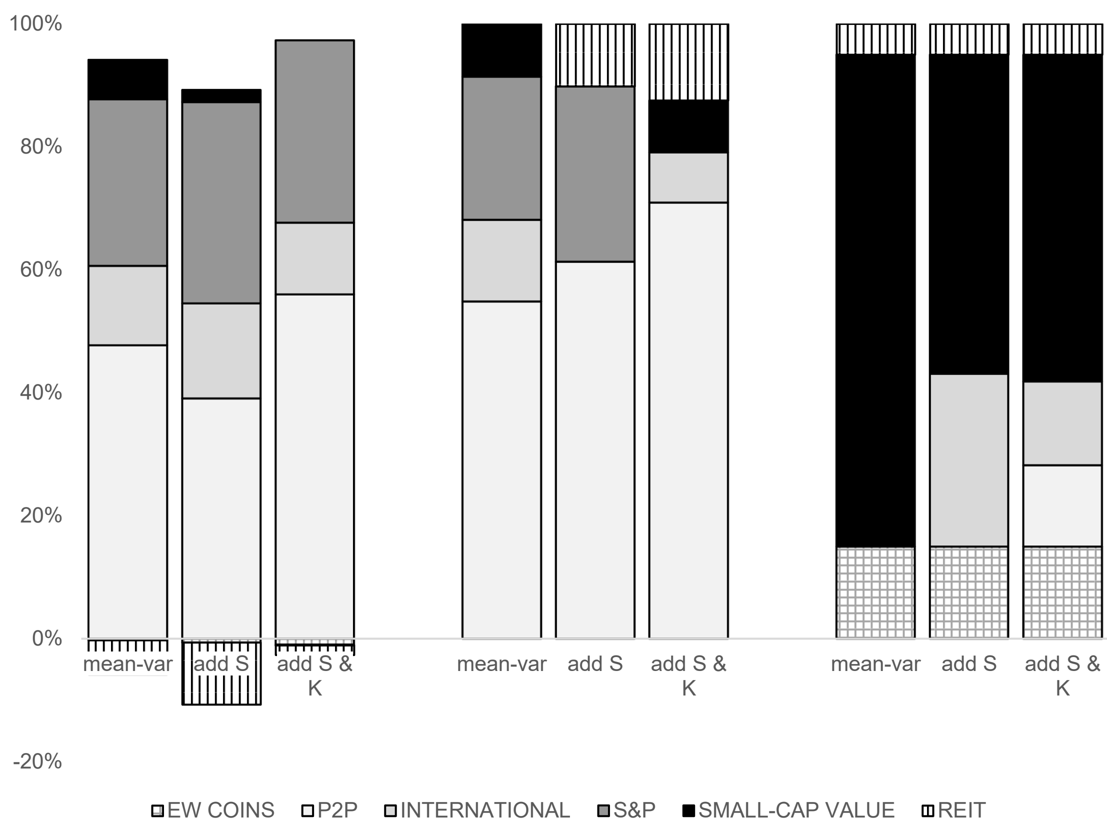
				MEAN OF HIGHEST IMPORTANCE				VARIANCE OF HIGHEST IMPORTANCE				
	Mean-Var	**Add Skew**	**All Equal**	**Add Skew**		**Max Kurt**	**Max Skew**		**Max Kurt**	**Max Skew**	**Max Kurt**	**Max Skew**
KOF (2011):	J	K	L	A	B	H	C	D	F	I	E	G
λ_1	1	1	1	3	3	3	3	1	1	2	1	1
λ_2	1	1	1	1	1	1	1	3	3	3	1	2
λ_3	0	1	1	1	2	2	3	1	1	3	1	3
λ_4	0	0	1	0	1	3	1	1	3	1	3	2
UN-CONSTRAINED												
mean (%)	0.39	0.34	0.05	0.36	0.06	0.38	0.44	-0.10	0.27	-0.07	0.31	0.43
variance (E-05)	4.93	5.74	5.67	5.76	5.86	6.01	6.51	6.56	6.59	6.78	5.50	6.61
skew	-0.74	0.33	-0.09	0.34	0.09	0.43	0.37	0.07	0.51	0.26	0.15	0.53
kurtosis	3.18	3.42	2.36	3.44	2.49	3.33	2.62	2.20	3.07	2.45	3.05	3.08
objective	2.99	3.05	2.11	3.03	2.11	1.96	2.00	1.89	1.87	1.79	2.07	1.72
EW COINS	**0.00**	**-0.01**	**-0.01**	**-0.01**	**-0.01**	**-0.01**	**-0.01**	**-0.01**	**-0.01**	**-0.01**	**-0.01**	**-0.01**
P2P	0.54	0.50	0.59	0.49	0.57	0.48	0.47	0.62	0.49	0.59	0.51	0.46
INT'L	0.15	0.20	0.12	0.20	0.10	0.17	0.07	0.11	**0.04**	0.08	0.16	0.13
S&P	0.31	0.42	0.31	0.42	0.41	0.52	0.70	0.29	0.71	0.43	0.45	0.64
SCV	0.07	**0.03**	**0.00**	**0.03**	**-0.04**	**-0.02**	-0.11	**-0.01**	-0.15	-0.08	**-0.01**	-0.07
REIT	-0.06	-0.13	**-0.02**	-0.13	**-0.03**	-0.14	-0.12	**0.01**	-0.07	**0.00**	-0.10	-0.15
SIGN-CONSTRAINED												
mean (%)	0.44	0.26	0.08	0.27	0.71	0.76	0.80	0.08	0.15	0.70	0.16	0.73
variance (E-05)	5.41	7.59	8.52	7.55	10.5	10.7	12.3	8.54	8.33	9.87	8.01	10.3
skew	-1.02	0.04	-0.04	0.03	-0.12	-0.09	0.03	-0.03	0.13	-0.18	0.08	-0.13
kurtosis	3.90	4.13	2.70	4.18	3.25	3.39	3.35	2.70	3.45	3.30	3.48	3.35
objective	2.99	3.36	2.67	3.34	3.40	3.12	3.43	2.29	2.08	3.39	2.46	3.30
EW COINS	**0.00**	**0.00**	**0.00**	**0.00**	**0.01**	**0.01**	**0.01**	**0.00**	**0.00**	**0.01**	**0.00**	**0.01**
P2P	0.55	0.61	0.71	0.61	0.59	0.58	0.59	0.71	0.67	0.59	0.66	0.58
INT'L	0.13	**0.00**	0.08	**0.00**	**0.04**	**0.03**	**0.03**	0.08	**0.00**	**0.04**	**0.00**	**0.03**
S&P	0.23	0.28	**0.00**	0.29	0.35	0.38	0.37	**0.00**	0.21	0.37	0.22	0.37
SCV	0.09	**0.00**	0.08	**0.00**	**0.00**	**0.00**	**0.00**	0.08	**0.00**	**0.00**	**0.00**	**0.00**
REIT	**0.00**	0.10	0.12	0.10	**0.00**	**0.00**	**0.00**	0.13	0.12	**0.00**	0.11	**0.00**
CONSTRAINED WEIGHTS												
mean (%)	6.18	6.20	5.95	5.60	5.91	5.88	6.06	6.21	6.21	5.80	6.20	6.19
variance (E-03)	3.50	4.05	3.97	4.06	3.71	3.74	3.58	4.59	4.53	3.82	4.05	3.85
skew	0.59	0.91	0.92	0.96	0.80	0.82	0.68	1.00	0.99	0.86	0.91	0.83
kurtosis	4.11	4.36	4.29	4.21	4.18	4.19	4.15	4.33	4.34	4.20	4.36	4.33
objective	2.01	1.30	0.37	1.25	0.15	0.12	0.09	0.16	0.09	0.04	0.30	0.03
EW COINS	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**	
P2P	**0.00**	**0.00**	0.13	0.32	0.15	0.16	0.06	**0.00**	**0.00**	0.21	**0.00**	**0.00**
INT'L	**0.00**	0.28	0.14	**0.00**	**0.00**	**0.00**	**0.00**	0.47	0.45	**0.00**	0.28	0.20
S&P	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
SCV	0.80	0.52	0.53	0.48	0.65	0.64	0.74	0.33	0.35	0.59	0.52	0.60
REIT	**0.05**	**0.05**	**0.05**	**0.05**	**0.05**	**0.05**	**0.05**	**0.05**	**0.05**	**0.05**	**0.05**	**0.05**

This table presents the results of the polynomial goal programming optimization. Each column represents a different set of investor preferences, described by the λ values at the top of the column. λ_i refers to the investor's preferences over the i^{th} moment: e.g., λ_3 represents her preference for portfolio skewness. Larger λ values imply more importance. (The lettered labels for the λ sets refer to Kemalbay, et. al.'s [2011] sets, for comparison. These are comparable to the sets used in Lai, et al., 2006. In contrast, Davies, et al., 2004, set a unit variance, then restrict the kurtosis parameter to fall between 0 and 1.) In the top third of the table, the weights are unconstrained; in the middle third, they are constrained to be nonnegative; in the bottom third, the weight of coins was set at 15% and the weight of REITs at 5% (as highlighted by the dark cells). The light grey cells throughout the table highlight weights close to zero—assets essentially ignored in specific portfolios. Note that the kurtosis values are not excess kurtosis.

Having considered the portfolio weights, we now review their expected performance statistics. To begin, we note that the unrestricted weighting scheme has the most favorable target moments (highest M* and S*, and lowest V* and K*); the restricted-weight scheme has the least favorable. For both the unconstrained and nonnegative-weights schemes, the mean-variance portfolio (1100) has the highest mean and lowest variance; (1110), which adds skewness, has the highest skew, while (1111) has the lowest kurtosis. (For the constrained-weight scheme, mean is highest for (1110), skewness is highest for (1111), and kurtosis is lowest for the mean-variance portfolio; these deviations from the expected outcomes are not material, however.) For all weighting schemes, the mean-variance portfolio has the lowest skew, justifying the point quoted in Davies, *et al.* (2004) that "mean-variance optimisers may be nothing more than skewness minimizers."

What is meaningfully different about the constrained-weights case is the magnitude of the portfolios' means and variances. Requiring a 15% allocation to coins—an asset ignored in almost all other portfolios—makes both means and variances magnitudes higher for these portfolios. We also observe that the variation in parameter values is much smaller when weights are constrained, which is consistent with Davies, *et al.*'s (2004) findings. Perhaps more interestingly, the skewness values for the restricted portfolios are all positive, and are all many times larger than their non-negative counterparts'. We see clearly the trade-off required when incorporating our experience assets into our portfolio: we incur greater risk (much higher variance and slightly higher kurtosis) in exchange for higher expected return and much higher skew.

Figure 3: Weighting Sets for PGP Optimization



The figure shows the weights derived from polynomial goal programming, given no weight restrictions (left-hand side), nonnegative weights (middle), and constrained weights (right-hand side). For each weighting scheme, we show weights for three preference sets (λ_i choices): mean-variance (first bar), mean/variance/skew (second bar), and mean/variance/skew/kurtosis (third bar). Results are mixed for our three "experience" assets. Coins are ignored in the unconstrained and nonnegative-weight cases, while P2P loans are emphasized. REITS are sold short in the unconstrained cases, but are added in the nonnegative weight cases that include skewness and kurtosis.

Bootstrapping

For additional insight into our results, we repeated the PGP sign-constrained and weight-restricted optimizations using bootstrapped data. (These results are not tabulated, but are available from the authors upon request.) We first compressed and shifted the coin returns to achieve a 2% monthly mean and 8% monthly standard deviation, consistent with our forward-looking assumptions, then generated 1,000 draws. These draws became the inputs for the PGP routine.

In the sign-constrained optimization, (1110) and (1111) contain only the S&P, international equity, and P2P loans, with the latter having a supermajority weight in both cases. The mean-variance portfolio adds a sliver of coins and a 6% allocation to small-cap value. In contrast, the restricted weight mean-variance, (1110), and (1111) portfolios all include all six assets, with P2P being much more important in the latter two (weights of 41% for the skewness portfolio and 35% in the skew/kurtosis, compared to 3% in the mean-variance portfolio). Of perhaps more interest are the comparisons among the bootstrapped portfolios and those in Figure 3. Focusing on the restricted-weight cases, we see much less small-cap value in the bootstrapped portfolios, and less international and more P2P in the (1110) and (1111) portfolios. Overall, we interpret these results as mitigating the highly concentrated allocations from the initial PGP results from Figure 3, and suggesting a meaningful portfolio roles for P2P, given our commitment to our other "experience" assets.

CONCLUSIONS

Most universities sponsor student-managed funds, and most of these funds are long-only equity. However, there is a lot more to portfolio management than picking stocks, and there are many who argue (including us) that equity funds of just a few hundred thousand dollars are best managed when allocated completely to mutual funds. Nonetheless, students (and many other investors) can be seduced by the behavioral biases (e.g., overconfidence, illusion of control) that attend picking stocks.

Our fund is different. It is very small, and it is independent of our university. We incorporated in in 2009, as the peer-to-peer loan market was beginning; using microloans in the P2P market allowed us to form a debt-focused fund (highly unusual in itself) using unique assets. Adding additional asset classes—like REITs and math-based currencies—has given our students the opportunity to explore concepts of diversification, portfolio development, and asset allocation that cannot be studied in a traditional long-only equity fund.

In this paper, we describe our asset allocation process. We begin with traditional mean-variance analysis. Not surprisingly, our current portfolio is not on the efficient frontier. We identify various "corner portfolios"—whose weights are constrained to be nonnegative—and find that our "experience assets" are not well represented: coins are only in the higher-return corners; P2P loans are only in the lowest; REITs are ignored completely. Nonetheless, we find that these assets do find more prominent places when we broaden the criteria by considering skewness and kurtosis. The math-based currency is more skewed than the market, and is strongly negatively coskewed with it (a useful feature, since market returns are usually negatively skewed). The MBCs and the P2P loans are also generally negatively correlated with the stock ETFs. These features manifest themselves in higher skewness and lower kurtosis on our weight-constrained optimizations. Thus, asset classes that look unattractive using traditional two-moment optimization reveal useful characteristics when the scope is broadened.

We are not exclusively, or even primarily, concerned with portfolio performance, however; as a teaching portfolio, we have the luxury of also being able to consider the educational opportunities offered by our asset classes. Incorporating unusual assets like math-based currencies and peer-to-peer loans allows our students to explore Excel's MMULT, Solver, and GoalSeek capabilities in the context of mean-variance optimization. Exploring those assets' potential also allows us to apply more novel techniques like polynomial goal programming to evaluate the higher portfolio moments of skewness and kurtosis.

Introducing students to these techniques will better prepare them for the increasingly quantitative nature of portfolio management. The assets we incorporate and the methods we employ are not relevant only to student-managed funds; our couching of our process within the SMF literature simply reflects our own experience. The process, however, can be applied to real-world funds as well.

Our SMF is unique in assets, form, and mission. By moving beyond stock picking, we prepare our students to make immediate, meaningful contributions to a broader array of financial organizations. Any university can duplicate our approach: a tiny amount of money, coupled with motivated students, is all that is required.

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BIOGRAPHY

Lynda S. Livingston is the Nat S. and Marian W. Rogers Professor of Finance at the University of Puget Sound. She is a founding member of Four Horsemen Investments. She can be reached at University of Puget Sound,1500 N Warner St, Tacoma, WA. 98416.