FINANCIAL DECISIONS AND TAX SHIELDS

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ABSTRACT

This paper addresses two questions related to tax shields. Firstly, what is the real moment of tax shield realization? Secondly, what influence shall the real tax shield realization moment have on tax shield present value? The real moment of tax shield realization is defined at the moment when the tax shield is reflected in the taxpayer's cash flow, in the form of income tax reconciliation and paid tax deposits. Subsequently, authors have investigated the dependence of tax shield present value on the discount rate, and on the size of a time span between the moment of tax-shield claim origination and tax-period's end. The largest difference between basic value of the tax shield and its present value was found at the maximum discount rate considered in the model, and at the maximum size of the time span between the moment of tax-shield claim origination and tax-period's end. The dependence was not always linear. In conclusion modifications of financial models that employ tax shields are suggested. These modifications are based on including coefficients reflecting the real moment of tax shield realization.

INTRODUCTION

Taxes are not merely a financial managers's "nightmare", but they mainly represent an important factor influencing most financial decisions, primarily through tax wedges and shields. While tax wedges diminish investor returns (Holeckova, 2002), tax shields mitigate the impact of certain decisions on financial performance (Marek, Radova, 2002). Tax shield can be regarded as a sum of money the tax liability is reduced by, due to the occurrence of a transaction that decreases the company's tax base, or entitles to tax relief. The transaction can be understood as posting of tax deductible costs and expenses in taxpayer's accounts, or occurrence of a different matter resulting in tax liability alteration, e.g. acquisition of fixed assets.

Mainstream finance has recently turned away from highly abstract theories in favour of detailed models of actual functioning of real world. In this world, tax shields play an irreplaceable role – they alter capital budgeting decisions, influence capital structure, etc. All classic models of finance (see e.g. Copeland & Weston, 1988) assume that the real moment of tax shield realization is identical to the moment the tax-deductible expense is incurred. However, this does not correspond to the real world. In our contribution, we aim to elaborate the tax shield issue in the suggested direction, under conditions of the Czech Republic tax law, and subsequently apply obtained findings through adjustments of financial models. In our discussion, we shall seek the answers to the following questions:

1. What is the real moment of tax shield realization?
2. What influence shall the real tax shield realization moment have on tax shield present value?

In various countries more or less different tax and deposit settlement systems are applied. For the purpose of the paper we shall consider only one settlement system, the present system adopted in the Czech Republic. We shall abstract from the differences in other countries tax settlement systems that may influence the real moment of tax shield realization and may lead to different financial decisions.

WHAT IS THE REAL MOMENT OF TAX SHIELD REALIZATION?

Cash flow based financial models must logically reflect the real tax shield realization in company's cash flows at the time of real execution of a relevant tax payment into state budget. For illustration, a tax-deductible payment incurred anytime in 2007 is not be reflected in tax payments into state budget until the 2007 income tax is settled, i.e. by 31st March of the next consecutive year (2008 in our example), or by 30th June 2008 in case of taxpayers who are liable for annual financial statements audits, or whose tax assessments are elaborated and presented by tax advisors. Decreased tax bases subsequently affect the amount of income tax deposits paid in the next consecutive tax deposit period. Consequently, the difference between tax deposits paid and real income-tax liability for the relevant period shall equal the amount the reduction in tax deposits payments.

For the sake of simplicity, we assume the tax shield claim originates at the same time as realization of this payment, or execution of this payment shall sooner or later lead to origination of the claim, for example, based on incurrence of tax-deductible costs and expenses.

Illustrative example:

01. 9.2007 … tax-deductible payment at amount of 1,000 (payment A);
30. 6.2008 … tax balance reconciliation date – difference paid for 2007 is decreased by the tax shield, i.e. 1,000 × 0.24 = 240 (payment B);
15. 9.2008 … income tax deposit payment decreased by ¼ of tax shield originated in 2007, i.e. 240 / 4 = 70 (payment C);
15.12.2008 … income tax deposit payment decreased by ¼ of tax shield originated in 2007, i.e. 240 / 4 = 70 (payment D);
15. 3.2009 … income tax deposit payment decreased by ¼ of tax shield originated in 2007, i.e. 240 / 4 = 70 (payment E);
15. 6.2009 … income tax deposit payment decreased by ¼ of tax shield originated in 2007, i.e. 240 / 4 = 70 (payment F);
30. 6.2009 … tax balance reconciliation date – difference paid for 2008 is increased by the equal amount the tax deposits paid in 2006 were reduced by, i.e. 60 × 2 = 140 (payment G);
30. 6.2010 … tax balance reconciliation date – difference paid for 2009 is increased by the equal amount the tax deposits paid in 2007 were reduced by, i.e. 60 × 2 = 140 (payment H).

WHAT INFLUENCE SHALL THE REAL TAX SHIELD REALIZATION MOMENT HAVE ON PRESENT VALUE OF TAX SHIELD?

This paper focused primarily on the present value of tax shield calculation methodology. The calculation procedure can be viewed in the following illustration depicted in Table 1.

This illustrative example assumes that a) the investor uses the services of a tax advisor and exploits the opportunity of tax assessment submission by end of June of the next consecutive year, b) the tax period is calendar year, c) the income tax rate is equal 24%, and d) the last known tax liability of the taxpayer exceeded 150.000 CZK. Here we shall add two additional assumptions: the interest is compounded on monthly basis and the fixed monthly discount rate is 1% (i.e. annual discount rate of 12%).

Table 1: Present Value of Tax Shield – Illustrative Example

Date	Payment	Number of Months	Original Payment	Tax Shield	Present Value of Tax Shield	
1. 9.2007	A	0	–1 000			
30. 6.2008	B	10		+240	$+240/(1+0.01)^{10}=$	**+217.27**
15. 9.2008	C	12		+60	$+60/(1+0.01/2)/(1+0.01)^{12}=$	**+52.98**
15.12.2008	D	15		+60	$+60/(1+0.01/2)/(1+0.01)^{15}=$	**+51.42**
15. 3.2009	E	18		+60	$+60/(1+0.01/2)/(1+0.01)^{18}=$	**+49.91**
15. 6.2009	F	21		+60	$+60/(1+0.01/2)/(1+0.01)^{21}=$	**+48.44**
30. 6.2009	G	22		–120	$-120/(1+0.01)^{22}=$	**–96.41**
30. 6.2010	H	34		–120	$-120/(1+0.01)^{34}=$	**–85.56**
Total	**X**	**X**	**–1 000**	**+240**		**+238.07**

Basic Value of Tax Shield

To determine the basic value of the tax shield, we consider the total (but not discounted) cash amount. The tax liability can be decreased by the basic value of the tax shield. In case of tax-deductible payment, or other transactions entitling the company to claim income tax base reduction, the basic value of the tax shield is calculated as a product of the value of the income tax-base reducing item and the income tax rate:

$$TS = TBRI \times t, \tag{1}$$

where TS = basic value of tax shield,
$TBRI$ = value of the income tax-base reducing item,
t = income tax rate.

If the tax liability decrease is claimed on the basis of entitled income-tax relief, the basic value of the tax shield equals the relief value:

$$TS = relief, \tag{2}$$

where $relief$ = value of income-tax relief.

Present Value of Tax Shield

Next we focus on calculation of the present value of the tax shield. The independent variables are the monthly discount rate i_m, and number of months from the moment of tax shield claim origination till the end of first tax period n. Other parameters reflect the model's assumptions suggested in the following illustrative example.

$$PV(TS) = TS \times \left\{ \frac{1}{(1+i_m)^{n+6}} + \frac{1}{4 \times (1+\frac{i_m}{2})} \times \left(\begin{array}{l} \frac{1}{(1+i_m)^{n+8}} \\ + \frac{1}{(1+i_m)^{n+11}} \\ + \frac{1}{(1+i_m)^{n+14}} \\ + \frac{1}{(1+i_m)^{n+17}} \end{array} \right) - \frac{1}{2} \times \left(\begin{array}{l} \frac{1}{(1+i_m)^{n+18}} \\ + \frac{1}{(1+i_m)^{n+30}} \end{array} \right) \right\}, \qquad (3)$$

where $PV(TS)$ = present value of tax shield,
n = number of months from the moment of tax shield claim origination till the end of tax period,
i_m = monthly discount rate,
= i / 12,
i = annual discount rate.

A Note on Discount Rates

The issue of discount rate determination is not addressed in our article. In fact, its precise determination is not possible from a theoretical perspective because determination of the discount rate depends on the purpose of the financial model, and varies with each individual investor's expectations. However in this research after-tax cash flows shall be discounted by a tax-adjusted discount rate. Financial models reflecting tax shields do employ after-tax cash flows.

DEPENDENCE OF TAX SHIELD ON DISCOUNT RATE

Next, we investigate the dependence of $PV(TS)$ and discount rate. Figure 1 shows dependence between $PV(TS)$ and the discount rate. Individually plotted curves represent the present value of the tax shield for selected number of months between the moment of tax shield claim origination and the end of the tax period (i.e. in our case till the end of calendar year). We define this difference in time periods as the "tax shield realization span." Curve [*12*] demonstrates the dependence of $PV(TS)$ on the discount rate when the tax shield claim originates on 1st January of respective year, i.e. 12 months prior to the tax-period's end. Similarly, e.g. curve [*0*] represents the dependence when the tax shield claim originates at the year (thus, tax period) end. All curves plotted can be characterized by a quadratic equation in common form: $Cx^2 + 2Dy + 2Ex + F = 0$, where the coefficient C is lower the longer the tax shield realization span. Therefore, we can state that a tax shield claim that originates at the tax period's beginning can be approximated by a linear function. Such situation is demonstrated by curve [*12*].

In addition, we also note that a curve extreme (maximum) can be found for each curve representing a different time span remaining to tax period's end and characterized by parabolic function, with the maximum located at individual parabolas' vertexes. Such a maximum represents the largest $PV(TS)$, and each of the discussed functions is maximized at a different level of discount rate. The closer the tax-shield claim origination is to the tax-period's end, the higher the parabola-maximizing discount rate.

Figure 1: Dependence of Present Value of Tax Shield on Discount Rate

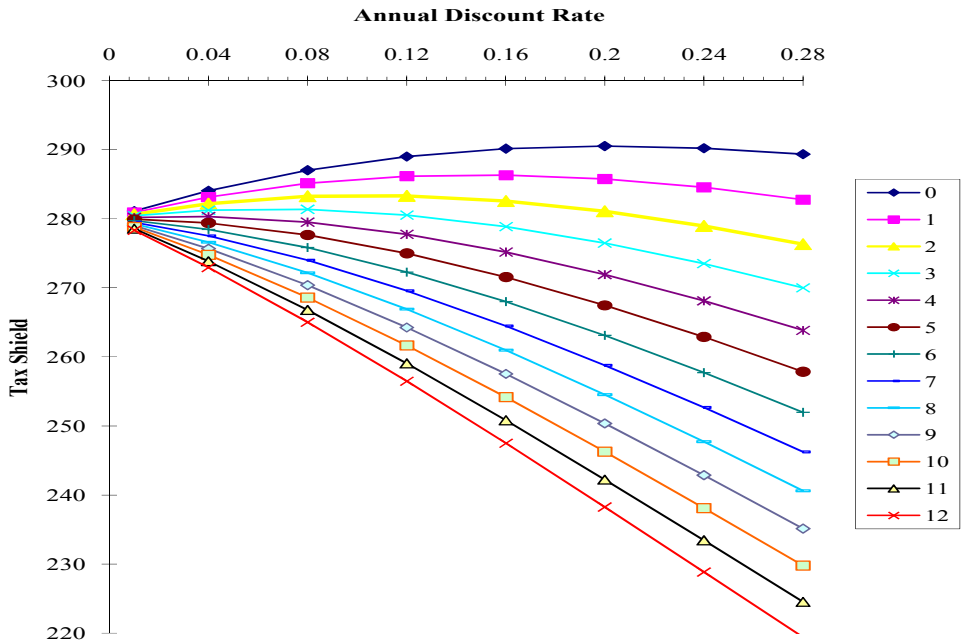
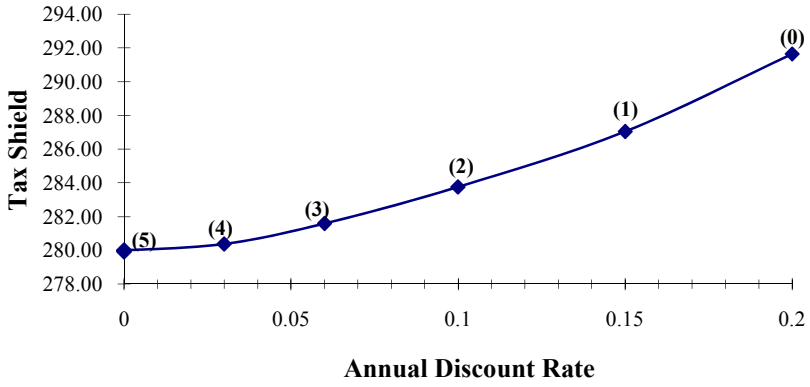


Figure 2 shows the relationship between the maximum present value of the tax shield and the discount rate. Numbers in brackets above the individual plotted points mark the number of months from the origination of tax shield claim till tax period's end.

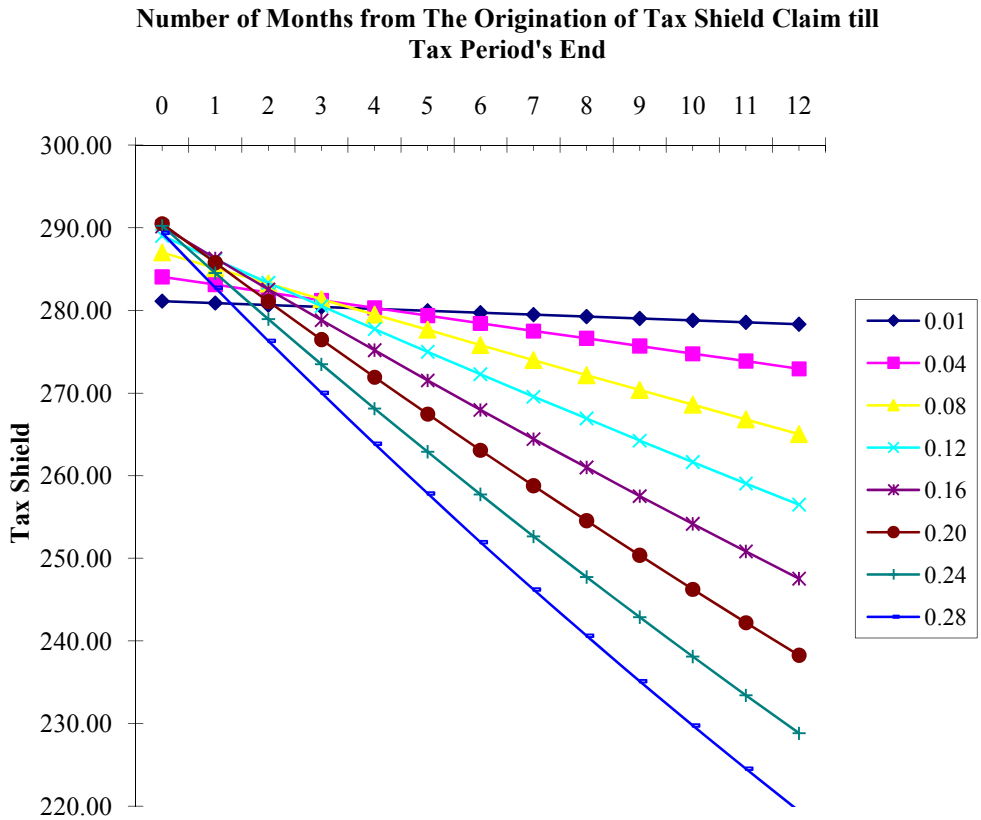
Figure 2 Maximum Present Value of Tax Shield in Dependence on Discount Rate



Relationship between the Present Value of the Tax Shield and the Tax Shield Realization Span

The dependence between *PV*(*TS*) and the tax shield realization span is explored in this section. The relationship is depicted in Figure 3 for various discount rates.

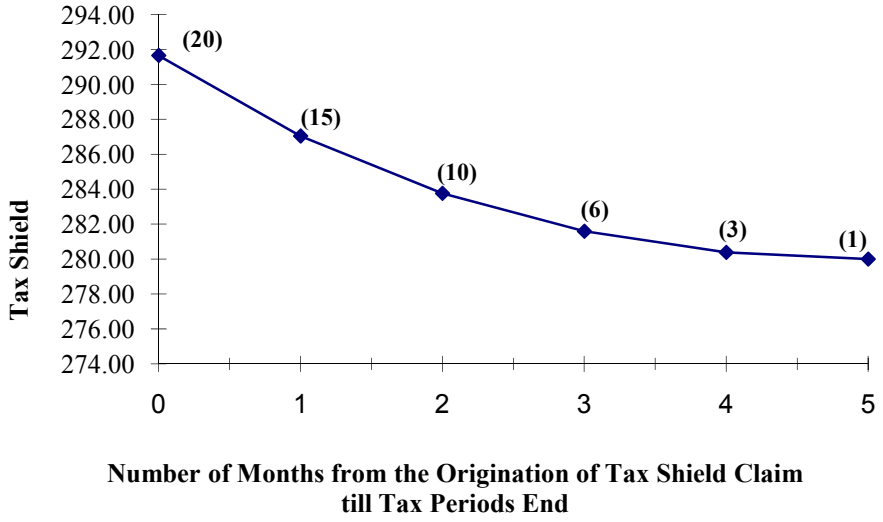
Figure 3: Dependence of Tax Shield Present Value on the Moment of Tax Shield Claim Origination



From the figure, it is noted that the relationship is linear for any selected discount rate. The largest *PV*(*TS*) logically occurs with the minimal tax shield realization span, i.e. when the claim is realized on the last day of tax period (in our case on 31st December). At this point, the *PV*(*TS*) is paradoxically even larger than the basic value. This paradox is, however, merely ostensible. This result is caused by the existence of income tax deposits, and by discounting of individual tax payments. With increasing time spans, the *PV*(*TS*) declines, to a minimum at 12 months prior to tax-period's end, i.e. on 1st January in our case.

The intersection of lines representing *PV*(*TS*) for various discount rates is an interesting graphical finding. In this case, the tax-shield claim origination moment falls on the last day of tax period and the maximum *PV*(*TS*) is reached for a discount rate of 20 %. Conversely, should the tax shield claim originate 12 months prior to tax-period's end, the maximum *PV*(*TS*) is reached for the lowest discount rate (in our illustration for 1 % annual discount rate). In other words, as depicted in Figure 4 for any discount rate we are able to find a time span between the moment of tax-shield claim origination and tax-period's end, such that the present value of tax shield is at a maximum. Numbers in brackets in the figure mark discount rates, which maximize the tax shield present value.

Figure 4: Maximum Present Value of Tax Shield in Dependence on the Moment of Tax Shield Claim Origination



COEFFICIENTS OF PRESENT VALUE OF TAX SHIELD

With respect to the presented research, it is clear that the present and basic value of the tax shield can differ. However, the question of whether the difference is significant enough to necessitate adjustments financial models remains open. That is, if the difference is large enough to necessitate use of the present rather then basic value. We introduce a function $\alpha(i;n)$ produces an absolute deviation of the difference between TS and $PV(TS)$ as a percentage of the basic tax shield value TS for a given discount rate i, and a given tax shield realization span n.

$$\alpha(i;n) = \frac{|TS - PV(TS)|}{TS} \times 100, \qquad (4)$$

where $\alpha(i;n)$ = absolute value of a deviation between the present and basic value of tax shield.

Figure 5 shows a map of the function's values $\alpha(i;n)$ for various discount rates i, and various time spans, n, For the sake of clarity, we have used discount rates up to 24 % even though such rates are unrealistic in current economic conditions.

It follows from the figure and the following Table 2 that the deviation between the present and basic value of the tax shield does not represent more than 2.5 % of the basic value in most cases. With an annual discount rate between 0.01 and 0.12 approximately 71.15 % of all cases do not exceed the 2.5% level. In the interval of 0.01 to 0.12 deviations exceeding 10 % do not occur at all, and in the interval of 0.01 to 0.24 only in 10.9 % of all cases exceed the 2.5% level. The largest deviations occur in cases when tax shield claims originate at the tax-period's beginning. In cases when the claims originate at the tax-period's end, the deviation values are rather negligible.

Figure 5: Function *α(i;n)* Value Map

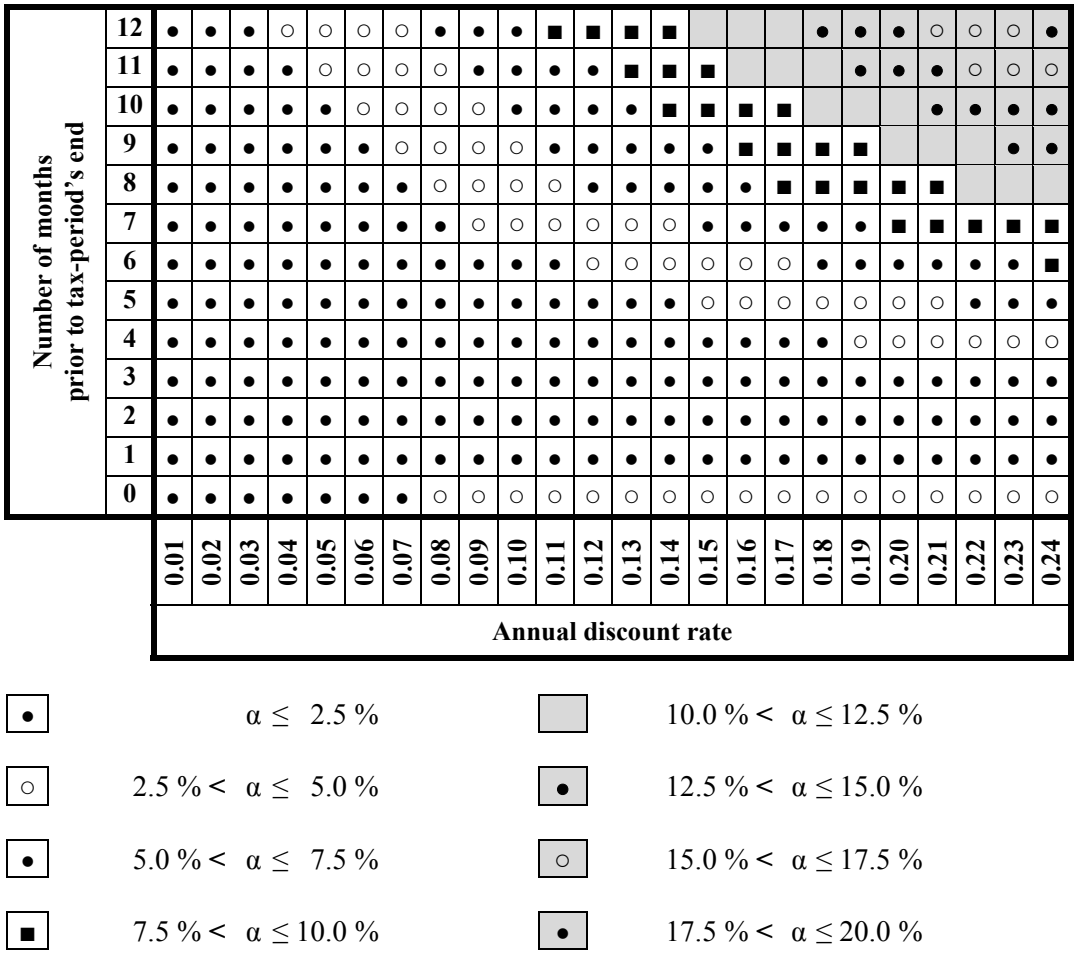


Table 2: Function Value Frequency *α*(*i*;*n*)

Discount ate	Frequency	α ≤ 2.5 %	> 2.5 % α ≤ 5.0 %	> 5.0 % α ≤ 7.5 %	> 7.5 % A ≤ 10.0 %	> 10.0 % α ≤ 12.5 %	> 12.5 % α ≤ 15.0 %	> 15.0 % α ≤ 17.5 %	> 17.5 % α ≤ 20.0 %	Σ
up to 0.24	absolute	155	62	35	26	15	12	6	1	312
	relative	49.7	19.9	11.2	8.3	4.8	3.9	1.9	0.3	100.0
up to 0.12	absolute	111	30	13	2	–	–	–	–	156
	relative	71.2	19.2	8.3	1.3	–	–	–	–	100.0

The question of when the deviations should be considered significant remains. The answer is rather straightforward and intuitive. The deviation is significant enough only when substitution of basic tax shield value with its present value leads to alteration in investment choice, or modification of other financial or capital decision. Thus, the critical deviation is case specific. The critical values can be determined by applied decision-making criteria for various investment alternatives.

Originally, we tested the present value of tax shield in a lease vs. buy analysis. According to our results in most of cases the differences were not significant. However, there were some significant cases. As a certain guideline on whether it is necessary to use the present tax shield value instead of the basic one, we might use the map of function values $\alpha(i;n)$ depicted in Figure 5. Based on this approach, we recommend using the present value of the tax shield in all cases where one can rationally expect that the value difference affects the relevant decision. For simplicity we can use the coefficients of the tax shield present value $k_{TS}(i;n)$:

$$k_{TS}(i;n) = \frac{PV(TS)}{TS}, \text{ or } PV(TS) = TS \times k_{TS}(i;n), \quad (5)$$

where $k_{TS}(i;n)$ = coefficient of present value of tax shield.

In Table 3, coefficients $k_{TS}(i;n)$ for various discount rates i, and for various number of months from the origination of tax shield claim till tax period's end n are presented

Table 3: Coefficients of Present Value of Tax Shield $K_{ts}(i;n)$

		n												
i	**i_m**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0.01	**0.00083**	1.0040	1.0032	1.0024	1.0015	1.0007	0.9999	0.9990	0.9982	0.9974	0.9965	0.9957	0.9949	0.9940
0.02	**0.00167**	1.0078	1.0061	1.0044	1.0028	1.0011	0.9994	0.9978	0.9961	0.9945	0.9928	0.9911	0.9895	0.9878
0.03	**0.00250**	1.0113	1.0088	1.0062	1.0037	1.0012	0.9987	0.9962	0.9938	0.9913	0.9888	0.9863	0.9839	0.9814
0.04	**0.00333**	1.0145	1.0112	1.0078	1.0044	1.0011	0.9978	0.9945	0.9912	0.9879	0.9846	0.9813	0.9781	0.9748
0.05	**0.00417**	1.0175	1.0133	1.0091	1.0049	1.0007	0.9966	0.9925	0.9883	0.9842	0.9802	0.9761	0.9720	0.9680
0.06	**0.00500**	1.0203	1.0152	1.0102	1.0051	1.0001	0.9952	0.9902	0.9853	0.9804	0.9755	0.9706	0.9658	0.9610
0.07	**0.00583**	1.0228	1.0169	1.0110	1.0051	0.9993	0.9935	0.9877	0.9820	0.9763	0.9706	0.9650	0.9594	0.9539
0.08	**0.00667**	1.0251	1.0183	1.0116	1.0049	0.9982	0.9916	0.9850	0.9785	0.9720	0.9656	0.9592	0.9529	0.9465
0.09	**0.00750**	1.0272	1.0196	1.0120	1.0044	0.9970	0.9895	0.9822	0.9749	0.9676	0.9604	0.9532	0.9461	0.9391
0.10	**0.00833**	1.0291	1.0206	1.0121	1.0038	0.9955	0.9872	0.9791	0.9710	0.9630	0.9550	0.9471	0.9393	0.9315
0.11	**0.00917**	1.0307	1.0214	1.0121	1.0029	0.9938	0.9848	0.9758	0.9670	0.9582	0.9495	0.9409	0.9323	0.9238
0.12	**0.01000**	1.0322	1.0220	1.0119	1.0019	0.9919	0.9821	0.9724	0.9628	0.9532	0.9438	0.9345	0.9252	0.9160
0.13	**0.01083**	1.0335	1.0224	1.0115	1.0006	0.9899	0.9793	0.9688	0.9584	0.9481	0.9380	0.9279	0.9180	0.9081
0.14	**0.01167**	1.0346	1.0227	1.0109	0.9992	0.9877	0.9763	0.9650	0.9539	0.9429	0.9320	0.9213	0.9107	0.9002
0.15	**0.01250**	1.0355	1.0227	1.0101	0.9976	0.9853	0.9731	0.9611	0.9493	0.9375	0.9260	0.9145	0.9033	0.8921
0.16	**0.01333**	1.0363	1.0226	1.0092	0.9959	0.9828	0.9698	0.9571	0.9445	0.9321	0.9198	0.9077	0.8958	0.8840
0.17	**0.01417**	1.0368	1.0223	1.0081	0.9940	0.9801	0.9664	0.9529	0.9396	0.9265	0.9135	0.9008	0.8882	0.8758
0.18	**0.01500**	1.0372	1.0219	1.0068	0.9919	0.9773	0.9628	0.9486	0.9346	0.9208	0.9072	0.8938	0.8805	0.8675
0.19	**0.01583**	1.0375	1.0213	1.0054	0.9897	0.9743	0.9591	0.9442	0.9295	0.9150	0.9007	0.8867	0.8728	0.8592
0.20	**0.01667**	1.0376	1.0206	1.0038	0.9874	0.9712	0.9553	0.9396	0.9242	0.9091	0.8942	0.8795	0.8651	0.8509
0.21	**0.01750**	1.0375	1.0197	1.0022	0.9849	0.9680	0.9513	0.9350	0.9189	0.9031	0.8876	0.8723	0.8573	0.8425
0.22	**0.01833**	1.0374	1.0187	1.0003	0.9823	0.9646	0.9473	0.9302	0.9135	0.8970	0.8809	0.8650	0.8494	0.8342
0.23	**0.01917**	1.0370	1.0175	0.9984	0.9796	0.9612	0.9431	0.9254	0.9080	0.8909	0.8741	0.8577	0.8416	0.8257
0.24	**0.02000**	1.0366	1.0162	0.9963	0.9768	0.9576	0.9388	0.9204	0.9024	0.8847	0.8673	0.8503	0.8337	0.8173

TAX RATE IRRELEVANCY

In the previous discussion we introduced function $\alpha(i;n)$ to be an absolute difference between the initial and present value as a percentage of the initial value for given discount rate i and for given tax shield realization span, n. Now we analyze effects of different income tax rate levels on the results. It follows from relationship (1) that the basic value is in direct proportion to income tax rate. Rearranging the formula (4) for absolute difference between the basic and present value of tax shields we obtain:

$$\alpha(i;n) = \frac{|TS - PV(TS)|}{TS} \times 100 = \frac{TS|1-y|}{TS} = \frac{TBRI \times t|1-y|}{TBRI \times t} = \frac{TBRI|1-y|}{TBRI}, \quad (6)$$

where:

$$y = \frac{1}{(1+i_m)^{n+6}} + \frac{1}{4 \times (1+\frac{i_m}{2})} \times \left(\begin{array}{l} \frac{1}{(1+i_m)^{n+8}} + \frac{1}{(1+i_m)^{n+11}} \\ + \frac{1}{(1+i_m)^{n+14}} + \frac{1}{(1+i_m)^{n+17}} \end{array} \right) - \frac{1}{2} \times \left(\begin{array}{l} \frac{1}{(1+i_m)^{n+18}} \\ + \frac{1}{(1+i_m)^{n+30}} \end{array} \right). \quad (7)$$

It is obvious that the difference between the basic and present value of tax shields is independent of the income tax rate level. Adopting analogical procedure and mathematical adjustments the coefficient is also independent of the level of income tax.

CONCLUSIONS

In this paper we examine the relationship between the basic value of a tax shield and its present value. We discuss the importance of the issue and computations showing the economic significance of the difference. We argue that the present value of the tax shield should be used in managerial decision making. The findings of the paper can be summarized as follows:

The real moment of tax shield realization is the moment, when the tax shield is reflected in taxpayer's cash flows in form of tax payment. In relation to time span between the tax shield claim origination and tax period's end, holding other variables equal, the maximum present value of the tax shield PV(TS) is reached when the claim originates at the end of tax period. Extending the time span the tax shield decreases the present value.

Ceteris paribus, the maximal present value of tax shield PV(TS) is located at the parabola vertex, i.e. the tax shield present value grows until a certain level of discount rate is reached; after reaching this level the present value decreases. In most of the cases, the absolute value of the deviation between the present and basic value of the tax shield α(i;n) does not exceed 2.5 % of the basic value at reasonable discount rate levels. The maximum deviation is reached with the longest possible time span between the tax shield claim origination and tax period's end, i.e. 12 months.

We recommend using the present value of the tax shield instead of the basic value in all cases where we can expect that the difference in values would influence a financial decision. For simplicity, coefficients of present value of tax shield kTS(i;n) can be used as a measure of the difference in values.

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