MODELS OF FINANCIAL IMMUNIZATION: BEHAVIOR ON THE SPANISH PUBLIC DEBT MARKET

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ABSTRACT

Financial immunization is a passive management strategy for portfolios comprising fixed income financial assets that aims to eliminate from such portfolios any risk arising from uncertainty concerning the future performance of interest rates. Some effort has been made to employ different models of immunization to get this objective. The purpose of this paper is to simulate the behavior of different models of financial immunization, based on information concerning the Spanish public debt market, with a view to conducting a comparative analysis.

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INTRODUCTION

The concept of duration attracted the attention of the business community and turned out to be an analytical as well as practical tool, which is widely used. Originally discovered more than 50 years ago, duration was defined to better reflect the length of a payment stream. A short time later, it was independently derived in an investigation into the elasticity of the price of a bond with respect to the interest rates. Soon thereafter, duration was rediscovered in the context of the immunization of portfolios comprising fixed income financial assets.

Immunization may be defined as the protection of the nominal value of a portfolio against interest rate changes. It has been shown that under idealized conditions this objective can be attained by equating the length of the investment horizon with some measure or measures of the time pattern of cash flows associated with the portfolio. This measure is commonly referred as duration.

The initial basis for strategies of financial immunization was the measure of duration introduced by F. Macaulay in 1938. In view of the limitations of the immunization model based on this measure, and in a bid to achieve greater interest risk coverage, a number of approaches have led to a good many models being proposed, which may be classified into three groups:

- Unifactorial models based on the use of single duration immunization measures.
- Models based on dispersal measures. These seek to minimize the dispersal of bond portfolio cash flows in relation to the investment horizon.
- Multifactorial models based on the simultaneous use of a set of immunizing measures of duration.

The purpose of this paper is to explore empirically the potential for improved immunization using those models. In the next section, we realize a literary review of the analyzed models. Later we describe the database used to develop the empirical work. Finally, we show the results of this study and the reached conclusion.

LITERATURE REVIEW

The financial immunization theorem, proposed in 1971 by Fisher and Weil, proved that a strategy to protect investors with fixed income portfolios from unexpected changes in interest rates during an investment horizon could be devised. This strategy consists on equalizing the duration of the portfolio to this time-period. To reach this conclusion, these authors start from two very restrictive assumptions: the fulfillment of the unbiased expectations hypothesis and the limitation of the possible displacements from the yield curve to variations in parallel.

The casuistry around the interest rates' behaviour is more complex. With the objective of trying to overcome the previously commented limitations, different immunization models have been proposed based on the use of several duration measures.

The unifactorials models of immunization are based on the use of unique duration measures to reach the proposed objectives. In the same way as the model of Fisher and Weil, this models assume the fulfilment of the unbiased expectations hypothesis and each of them suppose that the yield curve can only move following a concrete behaviour pattern. Related with these models, there are proposals carried out by authors like Bierwag (1977), Bierwag and Kaufman (1977), Khang (1979) and Bierwag, Kaufman, Schweitzer and Toevs (1983). The best part of these models is their simplicity, reason for which they are broadly used in the professional field. Their main problem is that they do not allow capturing as a whole the movements of the yield curve. For this reason, they do not provide a complete cover of the interest rates risk.

With the objective of reaching a higher precision in the measurement of the interest rate risk, the multifactorial models substitute the unique duration measure for a vector or group of durations. The proposals carried out that follow this position are diverse: Cooper (1977), Prisman and Shores (1988), Reitano (1991), Ho (1992), Klaffky, Ma and Nozari (1992), Dattatreya and Fabozzi (1995), Willner (1996) and Nawalkha and Chambers (1997).

As alternative to these models, or as a complement to them, different authors have proposed to establish the strategies of financial immunization through the minimization of some measures that quantifies the cash flows dispersion of the portfolios regarding the investment horizon. The objective pursued with the financial immunization would be easily reached if coupon zero bonds, whose terms of maturity coincides with the investment horizon, exist. These models want to achieve the approximation to the coupon zero bonds. See Fong and Vasicek (1984), with the measure M^2 and Nawalkha and Chambers (1996) with the M-absolute measure. The problem of these models is that they summarize the risk of immunization in a unique measure, making hard the disintegration of the risk associated to each yield curve movement for its treatment in an individualized way.

As higher is the complexity of the models proposed to eliminate the interest rates risk, better should be the results obtained in the covertures. Nevertheless, the setting in practice of these models is more complex and more expensive, being necessary to analyze if the results obtained with them improve significantly those reached applying the simplest models.

METHODOLOGY AND DATA

The Spanish Public Debt Market: Models Tested

In this paper, basically, we examined immunization models in all three of the abovementioned categories.
As regards unifactorial models, we simulated the performance of three models based respectively on shifts in the temporal structure of additive, multiplicative and maturity-related multiplicative interest rates.
To verify the importance of cash flow dispersal in bond portfolios in relation to the investment horizon, for each of the three models contrasted and for each of the terms assumed, we composed three types of portfolios:

- Bullet portfolios, comprising, of the bonds available, the two with durations closest to the investment horizon only.
- Barbell portfolios, also comprising two bonds, but in this case the ones with the longest and the shortest duration of those available.
- Ladder portfolios, comprising all available bonds under the criterion of maximum diversification.

In view of the importance some authors (Bierwag, Fooladi and Roberts (1993)) give to the inclusion in portfolios of the bond maturing closest to the investment horizon, we tested the additive duration model including in the portfolios the two bonds with a duration longer and shorter than the investment horizon but with maturity closest to it.

The second group of models tested was based on the use of methods of dispersal. We formed immunized portfolios by applying the model proposed by Fong and Vasicek (1983) which aims, out of all the possible portfolios with a specific duration, calculated on the basis of an additive shift in the term structure of interest rates, to find the one with the lowest cash flow dispersal in relation to the investment horizon, measured by the M^2 variable.

Finally, we formed immunized portfolios by applying one of the multifactorial models proposed. We replaced the single measure of duration with a set of durations, each of which quantifies the risk of bond prices in the event of a specific shift of the term structure of interest rates. To define such shifts we analyzed the movements in the term structure of interest rates in Spain between January 1991 and August 1997, applying the principal component analysis; we defined the immunization measures of duration based on the results obtained. Displacements during that period may be explained basically by 3 factors. The first factor explains 72.83% of all shifts, the second 24.17% and the third 2.32%. Three cases were analyzed; account being taken of one factor, two factors and three factors in each respectively, the criterion of maximum diversification in portfolio formation being followed.

For the models mentioned so far, simulation was conducted without considering the possibility of taking short-term positions in spot trading on Spanish Public Treasury bonds and securities, since such operations cannot be conducted in the Spanish public debt market. Nevertheless, such positions could be taken by using derivatives, thereby facilitating substantial changes in the results obtained by the portfolios. To analyze this possibility, we simulated the performance of two groups of multifactorial immunization models on the assumption that it is possible to take up short-term positions on the spot market. The lack of information on futures markets, and the sheer number of adjustments in portfolios necessary to meet the initial margins, and the mark to market persuaded us to use this hypothesis.These models were: the first of the multifactorial models seen above, based on the shifts in the term structure of interest rates over a long period in Spain, taking account once again of 1 to 3 factors, and the polynomial duration model proposed by Prisman and Shores (1988) and Nawalkha and Chambers (1997), which, starting from the possibility of defining the term structure of interest rates from a polynomial, defines the conditions immunized portfolios should comply with as follows:

$$D_j = \frac{\sum_{t=1}^{n} t^j FC_t e^{-h(0,t)t}}{\sum_{t=1}^{n} FC_t e^{-h(0,t)t}} = m^j \qquad (1)$$

Where:
D_j: Immunization duration factor j

FC_t: cash flows generated by the portfolio at each moment t, t=1, 2, 3, …n
h (0,t): spot interest rate in term t, in an instant

We formed immunized portfolios following this model with 3, 4 and up to 5 durations respectively, following the criterion of maximum diversification in forming portfolios. Prisman and Shores (1988) and Nawalkhan and Chambers (1997) demonstrated that establishing immunization strategies on the basis of minimizing M^2 eliminates the risk of immunization from parallel changes in the term structure of interest rates, minimizing the risk of a multiplicative variation of it occurring. A multifactorial model of two factors is therefore implicitly being applied, based on additive and multiplicative shifts in the term structure of interest rates.Table 1 shows the most important characteristics of the models tested.

Table 1: Immunization Strategies Tested

TERMS TESTED		IMMUNIZATION MODELS	PORTFOLIO STRUCTURE
2 YEARS 3 YEARS 5 YEARS	Without Short-term Positions	**Unifactorial Immunization Models**	Bullet: 2 bonds, ones with durations closest to investment horizon. (BUL) Barbell: 2 bonds, ones with greatest & least duration of those available (BAR) Ladder: maximum diversification. (LAD)
		Additive duration (ADI) Multiplicative duration (MUL) Mat.-related multiplicative duration (MULVT)	
		Multifactorial Models: Main Components	Maximum diversification
		1 Factor (ACP 1) 2 Factors (ACP 2) 3 Factors (ACP 3)	
		Model M^2	2 bonds with minimum M^2
		Additive duration and minimum M^2 (M^2)	
		Unifactorial Model with Bond Maturity	2 bonds with maturity closest to investment horizon
		Additive duration (BONOVTO)	
	With Short term Positions	**Multifactorial Models: Main Components**	Maximum diversification
		1 Factor (ACP CORT 1) 2 Factors (ACP CORT 2) 3 Factors (ACP CORT 3)	
		Multifactorial Models: Polynomial	Maximum diversification
		2 Factors (MULTICORT 2) 3 Factors (MULTICORT 3) 4 Factors (MULTICORT 4) 5 Factors (MULTICORT 5)	

Principal characteristics of the models of financial immunization tested.

As regards maturity terms, we compared the performance of these models for investment horizons of 2, 3 and 5 years.

Establishing financial immunization strategies entails regularly restructuring portfolios to attempt to comply at all times with the conditions necessary to obtain the expected results. We applied the criterion used in similar articles by other authors of restructuring portfolios once a week. Further, intermediate cash flows generated by portfolios (either though coupon payments or the return, on maturing, of the principal of some bonds) were reinvested in the portfolios, thereby maintaining their structure at the moment these cash flows were received. We assumed transaction costs in portfolio restructuring operations did not exist.

Data

To run the simulation we used data published by the Bank of Spain on simple spot transactions between January 1993 and March 2004 of bonds issued by the Spanish Public Treasury. The simulation used average daily trading prices for these operations. In some cases, on specific dates when some bonds were not traded or negotiated, we had to calculate the theoretical price of the bonds involved applying theoretical spot interest rates calculated by the Svensson method. Rather than use all the assets available in the simulation, we only employed those with a reasonable level of liquidity. This we did to prevent bond prices being influenced by the existence of premiums designed to offset lack of liquidity and to avoid problems when restructuring portfolios. To select portfolio-friendly bonds we used a procedure based on monthly asset trading frequency. For each security, we calculated the percentage of days effectively traded with regard to the total number of working days in each month, from January 1993 to March 2004, provided the stock was live. With this parameter, the criterion used in judging a bond or security as having sufficient liquidity, always from a weekly perspective, was for it to have been traded at eighty per cent-plus frequency in at least four months of the preceding semester. The semester was taken as the evaluation period as portfolios were also restructured every six months For our purposes, semesters begin in January and July every year. The idea behind this criterion was to profile portfolios with bonds and securities whose high liquidity levels made them easy to trade.

Target portfolio yield was calculated using the Svensson model (1994) for establishing spot interest rates, based on quotations of public debt securities issued by the Spanish Treasury. To ensure we had a sufficiently large number of portfolios for each strategy, portfolio investment periods overlapped, except in periods of a semester. As a result, we analyzed the results of 19 portfolios considering an investment horizon of 2 years, 17 with a three-year horizon and 13 with a term of 5 years. For the multifactorial model based on the main component analysis to quantify shifts in the term structure of interest rates, our analysis covered 10 portfolios at 2 years, 8 at 3 years and 4 at 5 years. This was because the model was applied based on shifts in the term structure of interest rates in Spain between January 1991 and August 1997, as we began to form portfolios from the latter date.

We used the financial law of compound interest in annual terms to evaluate the yield initially expected from the portfolios and final portfolio yield. We also used 365/365 as a time base for the calculations. We calculated the target portfolio yield using Svensson's model (1994) to determine spot interest rates based on the quotations of the debt securities issued by the Spanish Treasury.

Interest risk coverage will be more effective the nearer the final yield obtained by the immunized portfolios is to the yield initially planned when they were formed. We analyzed the efficiency of the coverage based on the degree of proximity between achieved yield and the yield initially expected for the portfolios. For this, we used the following statistics:

Corrected Euclidean distance. The Euclidean distance is only useful for comparing different models when analyzing portfolios with the same maturity. So we needed to correct the Euclidean distance (DEC) if we wanted to have a statistic we could use to compare the degree of suitability of immunization models for which a different number of observations are available. This was calculated as follows:

$$DEC = \sqrt{\sum_{i=1}^{N}(x_i - y_i)^2 \times \frac{1}{N}} = DE \times \sqrt{\frac{1}{N}} = \frac{DE}{\sqrt{N}} \tag{2}$$

Where:

- x_i, is the yield obtained for semester i by the immunized portfolio
- y_i, is the target yield for the same period for the target portfolio
- N, the number of observations available (semesters) for each pair of variables (model-target)

we want to test the corrected Euclidean distance is one of the most robust statistics of those used, since it does not permit the possibility of offsetting the positive differences against the negative, between yield achieved and final yield.

The correlation coefficient between target yields and achieved yields. The standard deviation between the yield of the immunized portfolios and the target portfolios. The problem with this measurement is that it allows sign offsetting. Even so, it is useful for determining, on average, the sense they take from the deviation between achieved portfolios and target portfolios.

RESULTS

Although all the statistics introduced gave interesting information, the Euclidean distance was basically the most relevant in our study, since offsetting errors was not permitted. In any case, the greater the precision, the greater the correlation between variables, and the shorter the Euclidean distance.

Table 2: Summarizing Statistical Results

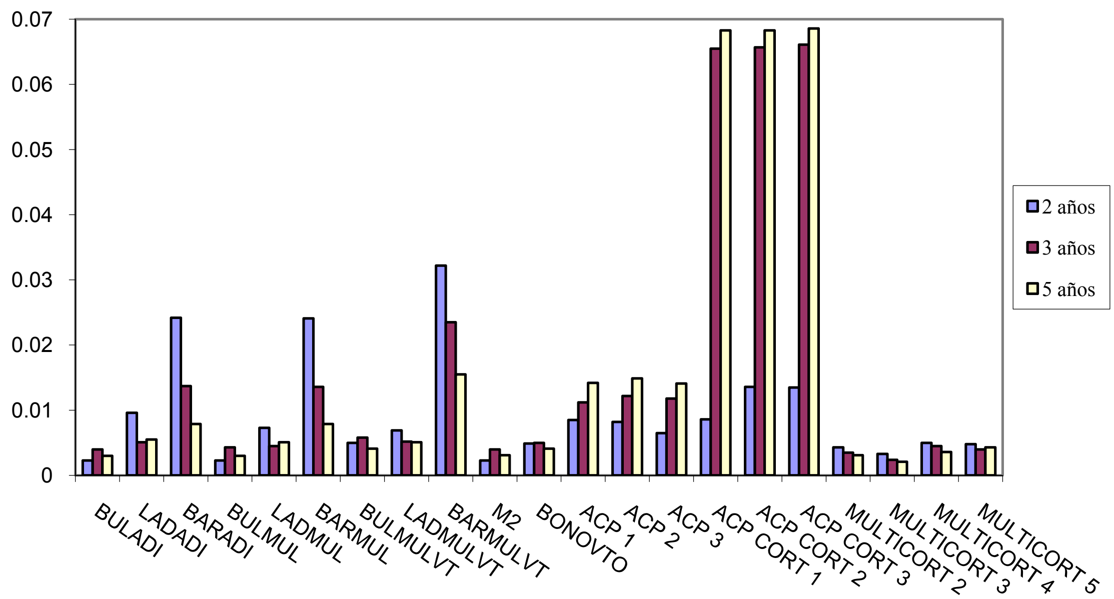
Immunization model	Portfolios Maturing at 2 Years			Portfolios Maturing at 3 Years			Portfolios Maturing at 5 Years		
	Corr[b]	DEC[c]	E(dif)[d]	Corr[b]	DEC[c]	E(dif)[d]	Corr[b]	DEC[c]	E(dif)[d]
BULADI	0.9975**	0.0023	0.109%	0.9927**	0.0040	0.206%	0.9942**	0.0030	0.161%
LADADI	0.9420**	0.0096	-0.125%	0.9822**	0.0051	-0.050%	0.9916**	0.0055	-0.274%
BARADI	0.6602**	0.0242	0.507%	0.8892**	0.0137	0.571%	0.9579**	0.0079	0.153%
BULMUL	0.9975**	0.0023	0.115%	0.9914**	0.0043	0.219%	0.9943**	0.0030	0.158%
LADMUL	0.9661**	0.0073	0.073%	0.9896**	0.0045	0.029%	0.9889**	0.0051	-0.240%
BARMUL	0.6625**	0.0241	0.500%	0.8903**	0.0136	0.559%	0.9582**	0.0079	0.134%
BULMULVT	0.9874**	0.0050	0.227%	0.9856**	0.0058	0.363%	0.9924**	0.0041	0.320%
LADMULVT	0.9709**	0.0069	0.167%	0.9834**	0.0052	0.113%	0.9863**	0.0051	-0.059%
BARMULVT	0.4713*	0.0322	1.360%	0.7659**	0.0235	1.586%	0.9007**	0.0155	1.271%
M^2	0.9975**	0.0023	0.127%	0.9917**	0.0040	0.194%	0.9936**	0.0031	0.155%
BONOVTO	0.9873**	0.0049	0.210%	0.9886**	0.0050	0.277%	0.9909**	0.0041	0.236%
ACP 1	0.7195*	0.0085	0.603%	0.7718*	0.0112	0.599%	0.9274	0.0142	0.586%
ACP 2	0.7898**	0.0082	0.570%	0.1475	0.0122	0.668%	0.9950**	0.0149	0.406%
ACP 3	0.8229**	0.0065	0.413%	0.2264	0.0118	0.569%	0.9954**	0.0141	0.669%
ACP CORT 1	0.8082**	0.0086	0.606%	0.8575**	0.0655	-2.034%	0.9702*	0.0683	-2.941%
ACP CORT 2	0.4297	0.0136	0.814%	0.3026	0.0657	-1.911%	0.8243	0.0683	-3.001%
ACP CORT 3	0.1756	0.0135	0.194%	-0.1843	0.0661	-2.103%	0.3408	0.0686	-3.373%
MULTICORT 2	0.9933**	0.0043	0.135%	0.9966**	0.0035	0.146%	0.9949**	0.0031	0.293%
MULTICORT 3	0.9933**	0.0033	0.085%	0.9966**	0.0024	0.048%	0.9939**	0.0021	0.120%
MULTICORT 4	0.9851**	0.0050	0.131%	0.9900**	0.0045	0.122%	0.9944**	0.0036	0.256%
MULTICORT 5	0.9861**	0.0048	0.129%	0.9912**	0.0040	0.113%	0.9894**	0.0043	0.157%

** Significant at level 0.05 (bilateral)   ** Significant at level 0.01 (bilateral)*
Results of models of financial immunization analyzed

In all, we did 63 analyses: 21 models and 3 different maturity terms for each one. Table 2 shows the results, the most revealing of which were:

First, after conducting the Kolmogorov-Smirnov comparison for all yield distributions analyzed, in no case could we rule out the variables not behaving differently from the normal distribution Secondly, with regard to unifactorial models, it should be noted that addressing bond portfolio structure exclusively, the best performers were bullet or concentrated portfolios, independently of the immunization model chosen and of the portfolio maturity or time horizon, followed by the ladder and barbell portfolios, the barbells performing worst of all. The correlation coefficient was greater for all models and maturities tested, the greater the degree of concentration of portfolio cash flows. The corrected Euclidean distance was lower in bullet than in ladder or barbell portfolios. In light of these results, it is fair to say that ladder and barbell portfolios are more inefficient in achieving the objective of immunization than are bullet portfolios. Figure 1 shows the values of the corrected Euclidean distance for all models analyzed.

Figure 1: Corrected Euclidean Distance between Yields Obtained and Target Yields for All Models and Terms Analyzed



This figure shows the Corrected Euclidean Distance between the target and the yields obtained for the models and terms analyzed

Continuing with unifactorial models, and limiting ourselves exclusively to bullet portfolios (the best performers), the model of financial immunization based on shifts in the term structure of multiplicative-type interest rates according to term of maturity was by far the worst in adjusting. Any of the statistics calculated gave a less favorable value for this model as opposed to the other two. There were very few differences between the additive duration and multiplicative duration models, the results being more favorable to one model or the other depending on the term of portfolio maturity and the statistic employed.

Considering the models analyzed as a whole, portfolio maturity-related results were more satisfactory the greater the investment horizon considered. However, if one concentrates exclusively on the best performers, i.e. bullet portfolios and additive and multiplicative durations, it becomes clear that the time factor did not play a significant role in the decision to choose one immunization model or another. Although the best results were obtained with a two-year portfolio horizon, no clear tendency was discernible here, as while portfolios with a three-year horizon performed less well than two-year ones, five-year portfolios broke the tendency by obtaining better results than the three-year maturity portfolios.

In the third place, two important facts need underscoring with regard to the model based on the minimization of portfolio cash flows in relation to investment horizon M^2. Portfolios constructed using this model achieved very high adjustment levels, comparable to the levels of the best portfolios achieved under the unifactorial model. For a two-year maturity term, the results were virtually identical in unifactorial models with bullet profiles and additive and multiplicative durations and in the M^2 model. For three- and five-year maturity terms, differences between the three models were very low, one or other model performing better depending on the statistic chosen to quantify results.

As occurred in the unifactorial models, portfolio maturity was not particularly representative, it being possible to order them from better to worse as follows: two, five and three years.

In the fourth place, results for portfolios immunized against additive shifts in the term structure of interest rates including bonds maturing closest to the investment horizon were fairly satisfactory, although no better than those obtained by bullet portfolios and additive and multiplicative durations or than the results obtained by portfolios immunized by minimizing cash flow dispersal. In short, the inclusion of the bond maturing closest to the investment horizon did not, in the cases analyzed, guarantee yields closer to those initially forecast.

Fifth, the following may be said about multifactorial models. The results obtained using the model with durations defined based on the historic shifts in the term structure of interest rates, our ACP models, were at best very discreet, and were even worse if the possibility of taking short-term positions on the spot market was taken into account. These results were comfortably surpassed by portfolios based on unifactorial models, when portfolios were "bullet-shaped", by portfolios based on M^2 minimization and even by the portfolio including the maturity bond. A major drawback found on testing these models, and which might help to explain the results, was the lack of observations, as the model construction process entails losing data on the first nine semesters.

Results for the Prisman and Shores model, the ones we named MULTISHORT models, were similar to those achieved with the best unifactorial models and the M^2 model. For a maturity of 2 years, regardless of the number of factors considered, the results did not exceed those of the models mentioned. However, in some cases such results were improved on with three- or five-year maturities.

Immunization as opposed to a larger number of factors did not necessarily mean an improvement in results. Focusing on the Prisman and Shores model, results were more satisfactory when 3 factors were taken into consideration, the portfolios being less well immunized when 4 or 5 factors were involved. Furthermore, for investment horizons of 3 and 5 years, the portfolios best immunized were obtained using this model and considering 3 factors.

To end, some mention must be made of the limitations of this study. The first comes through using Svensson's model for deciding spot interest rates for establishing target yields for immunized portfolio. Some of the differences between real immunized portfolio yields and the expected yields may be explained by the model's possible errors of estimation.
Finally, the number of portfolios considered in the study is very low. Low liquidity on the public debt market prior to January 1993 prevents us from conducting a more exhaustive analysis of the problem.

CONCLUSIONS

The unifactorial financial immunization models facilitating greater proximity between yields achieved by portfolios and target yields are those based on measurements of duration arising from additive or multiplicative changes in the term structure of interest rates and supported in bullet portfolios. In these models, the best results are obtained with portfolios with shorter investment horizons.

For three- and five-year investment horizons, the best results are obtained building portfolios using the Prisman and Shores multifactorial model when three factors are considered. Although slightly poorer, results for an investment horizon of 2 years are remarkably similar to those obtained by bullet-shaped unifactorial models of additive and multiplicative duration. Including the fourth and fifth factors does not improve the results.

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