A COMPARISON OF PRE AND POST MODERN PORTFOLIO THEORY USING RESAMPLING

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ABSTRACT

This article introduces the Resampling approach to Portfolio modelling, targeted at reducing the effect of estimation error present in any practical implementation of a Portfolio Model. Resampling is a method used in portfolio modelling to try to obtain better out of sample performance for given input model parameters. In the real world, where the possibility of estimating errors for future model forecasts certainly exist, it is necessary to consider the error component in building portfolios. Resampling does this by recombining the input parameters required for a portfolio model. In this paper an application of Resampling is performed using a sample of equities from different stock markets. The results are presented for Tracking Error Minimization, Mean Absolute Deviation Minimization (MADM) and Shortfall Probability Minimization Models. The innovation in this study lies in the comparison made with different portfolio models. Unlike previous studies, the evidence shows that Resampling applied to the Markowitz model does not generate better out of sample performance. However, the benefits of Resampling applied to the Post Modern Theory model are remarkable.

KEYWORDS: Technical Analysis, Post Modern Portfolio Theory, Resampling.

JEL: G11

INTRODUCTION

In order to improve the stability of the out-of-sample performance of a portfolio generated by a process of optimization, economic theory and operational practice have developed techniques for improving the estimation of portfolio model parameters. In this article we introduce Resampling and examine the effectiveness of this technique when applied to the Markowitz model and certain Post Modern Theory models. Resampling is a technique that is based on the simulation of equity returns using Monte Carlo methods. A well-understood fact of asset allocation is that the traditional portfolio optimization algorithm is too powerful for the quality of the input data. The problem is that optimizers are extremely sensitive to minor changes in assumptions. Even small changes in optimization inputs often lead to large changes in optimized portfolios. The Markowitz Model optimizer tends to select the assets with the most attractive features (high returns and low risk and/or low correlations) and to avoid the ones with the worst features when inputs are measured with error. In this sense the optimization algorithm always results in a corner solution and tends to "error-maximize" investment information. This results in portfolios that are too close to a given set of inputs and that are unlikely to perform well in the future. Small changes in the inputs typically result in very different optimal portfolios. As a result users often include many constraints in order to stabilize the optimization. In this sense financial operators should never run an optimizer only once and be satisfied with the results. They should run the optimizer a number of times with differing assumptions. By doing so it is possible to incorporate uncertainty into an optimization algorithm.

Recently, a new concept called "Resampled Efficiency" has been introduced into the asset management world to help deal with estimation errors. The first author involved in diffusing this new statistical approach has been Michaud (1998). Also Markowitz, the father of Modern Portfolio theory, has dealt with this topic in a recent study (Markowitz, Usmen, 2003). Resampling methods are widely used in modern statistics. Monte Carlo simulation is used to compute many statistically similar alternatives. In a portfolio context, risk and return input data estimates are never known with perfect certainty. Resampling efficient frontier optimization generalizes classical mean-variance optimization, incorporating the investor's level of uncertainty in the estimates. In this sense Resampled Efficiency provides a wide range of alternative optimal portfolios that are consistent with risk-return estimates. Resampling does this by generating hundreds of efficient frontiers using small changes with regard to initial input parameter estimates.

In a strict sense Resampled Efficiency can be seen as an averaging process that processes all the alternative efficient frontiers into a new efficient frontier and a set of optimized portfolios. On the other hand each portfolio on the Resampled efficient frontier is the result of averaging a number of statistically equivalent efficient portfolios. The resulting process is more stable and produces more reliably effective optimal portfolios because they are optimal with regard to the many ways in which assets and markets may perform in the period the investment is held.

It is helpful to be clear about the reasons why a financial operator, or even a simple investor would consider a Resampling approach as his way of portfolio management. It is also beneficial to identify the contribution of Resampling to final performance with regard to different portfolio models. The remainder of the paper is organized as follows. After a brief literature review in Section 1, Section 2 provides the theoretical underpinnings of the Resampling methodology. In Section 3 a brief description of some models of the Post Modern Theory are provided, and the mathematical structure of objective functions of all the models are presented. In Section 4 data and results of the application are explored, to identify the effectiveness of Resampling. The applications involve a comparison between a Markowitz approach and Post Modern Portfolio theory approach, and the effectiveness of Resampling. Section 5 provides some concluding comments.

LITERATURE REVIEW

Classical optimizers assume 100% certainty in information input, however investment information is typically uncertain. Markowitz mean-variance efficiency is a cornerstone of modern finance for asset management. Given the presumption that rational investors make investment decisions based on risky assets' expected return and risk; with risk measured as variance, a portfolio is considered mean-variance efficient if it has the minimum variance for a given level of portfolio expected return, or if it has the maximum expected return for a given level of portfolio variance. The quadratic programming optimization algorithm, which is the classical way to apply Markowitz's methodology, is too sensitive to the quality of input parameters. The result produces a maximization of the estimation error. If unchecked, this phenomenon skews the optimizer towards extreme weights that tend to perform poorly when applied to real data. In the case of the investment behaviour of portfolio optimizers, the evidence came more than twenty years ago, in a series of papers authored by two financial economists, J.D. Jobson and Bob Korkie (1981). They showed mathematically and statistically that optimizers have, on average, little investment value and that equal weighted portfolios are often far superior to optimized portfolios.

With regard to sample variance and covariance matrix estimate error reduction, a solution that has been proposed is to shrink the sample covariance matrix by changing its most extreme elements with more moderate values. Ledoit and Wolf (2004) proposed an improved estimator of the covariance matrix based on the statistical principal of shrinkage. The idea is to find an optimal linear combination of the sample covariance matrix and a highly structured estimator, which assumes that the correlation between the returns of any two stocks is always the same. In their empirical study, Ledoit and Wolf demonstrated that shrinkage approach provided a significantly higher realized information ratio of the active manager compared to the sample covariance matrix.

An alternative solution is the Resampled Efficiency described in Michaud (1998), invented and patented by Richard Michaud and Robert Michaud. The method is based on Resampling optimization inputs. This is a Monte Carlo simulation procedure to create alternative optimization inputs that are consistent with the investor's level of certainty in the estimates. Some authors wondered if there is a general theoretical justification to the Algorithm at the base of Resampling approach. To shed some light on this question, it is helpful to realize that the algorithm can be considered a special case of the statistical technique of bagging. Bagging is an acronym for "bootstrap aggregating'' and was invented by the statistician Leo Breiman (1996). The general situation is as follows. A predictor (or estimator) θ is computed on observed data. Due to the nature of this predictor, small changes in the data set can lead to significant changes in the predictor values: thus the predictor is unstable. As a consequence, it is deemed unreliable for practical use. Bagging aims to remedy this situation as follows. From the observed data a Resampling approach is conducted via bootstrap and, in this way, the predictor on the bootstrap data is computed, resulting in θ^*. Call the bagged estimator θ^*. This process is repeated many times and the resulting values θ^* are 'aggregated' by averaging them. The hope is that θ^* has a better out-of-sample performance than the original estimator θ. But Breiman (1996) proves that his result is no universal guarantee.

Generally speaking Resampled Efficiency is always preferable to a Mean Variance approach because investors are never 100% certain of their estimates. Again, generally speaking, Resampled Efficiency optimized portfolios are less risky as they are optimal, relative to the many ways in which assets and markets may perform, in the investment period. In the case of long-only constraints, Resampled Efficiency leads to more-diversified portfolios, which, as presented by Michaud (1998), are well known to beat simple Markowitz portfolios in out-of-sample tests in a way that is statistically significant. Michaud's portfolios tend to be more diversified and more stable over time than asset allocations produced by traditional optimizers. A recent study by Markowitz and Usmen (2003) found that the investment performance of Resampled Efficiency optimized portfolios (Michaud 1998) is superior to that of Markowitz (1952) mean-variance (MV) optimized portfolios, also in the case in which sophisticated Bayesian estimates of risk and return are introduced. The Bayesian procedure in Markowitz and Usmen (2003) is a very sophisticated use of Bayesian, Monte Carlo, and numerical analysis methods for optimization input estimation. In a test of 10 scenarios, Markowitz found that the Resampling methodology is superior to a traditional optimizer 10 out of 10 times. In a second test, Markowitz compared the entire efficient frontier generated by the traditional optimizer with one generated by the Resampling methodology. In this case Michaud's methodology added 57 basis points over the traditional optimizer in the first test but just 12 basis points in the second test.

Ledoit and Wolf (2004) in their work performed a test on Resampling methodology effectiveness and they found that the extent to which the result can be generalized is unclear. Portfolio Resampling carries with it some unwanted features. In particular results exhibit deteriorating Sharpe ratios (caused by higher volatility) and additionally, the efficient frontiers may exhibit turning points (a move from concave to convex). Resampling can be combined with shrinkage. However, the two authors cannot find evidence that this combination offers any further improvement beyond 'pure' shrinkage.

A theoretical investigation of Resampled Efficiency is provided by Scherer (2002). In this setting, the portfolio manager aims at an optimal 'absolute' portfolio rather than at outperforming an index. The main findings of Scherer (2002) are two. Firstly, that in the absence of lower and upper bounds on the vector of portfolio weights, Resampling produces results close to those you could obtain from a sample covariance matrix. Secondly, that in the presence of lower and upper bounds on the vector of portfolio weights, Resampling leads to more diversified (active) portfolios compared to the sample covariance matrix. As a result, Resampling improves out-of-sample performance.

According to Ledoit and Wolf (2004) Markowitz efficiency is not the relevant benchmark for Resampled Efficiency whereas Bayesian alternatives, which have a strong foundation in decision theory, are. Therefore, a significant field for future research is, above all, how Resampling can be compared or even integrated with Bayesian alternatives. In fact Bayesian estimation and other methods for improving the reliability of risk and return estimates are not mutually exclusive with Resampled Efficiency. In the future, best practice may require more sophisticated statistical estimation procedures and Resampled Efficiency.

RESAMPLING

Resampling is an operational solution belonging to the heuristic category of asset allocation approaches. It is based on Monte Carlo simulation and a stochastic interpretation of the return-variance approach. The underlying idea is as follows: input parameters for portfolio optimization are derived from representative historic returns of a single realized event of the underlying stochastic process implying that history permits the observation of only one path of return evolution.

The first applications of Resampling, in financial theory, were due to the CAPM model as later developed by Markowitz. With Markowitz, the problem of efficient frontier instability ex post mainly depends upon an excessive concentration of efficient portfolios. Resampling promotes a logic that allows for more diversified portfolios incorporating, in contrast with Markowitz optimization, the erratic component in the portfolio construction phase.

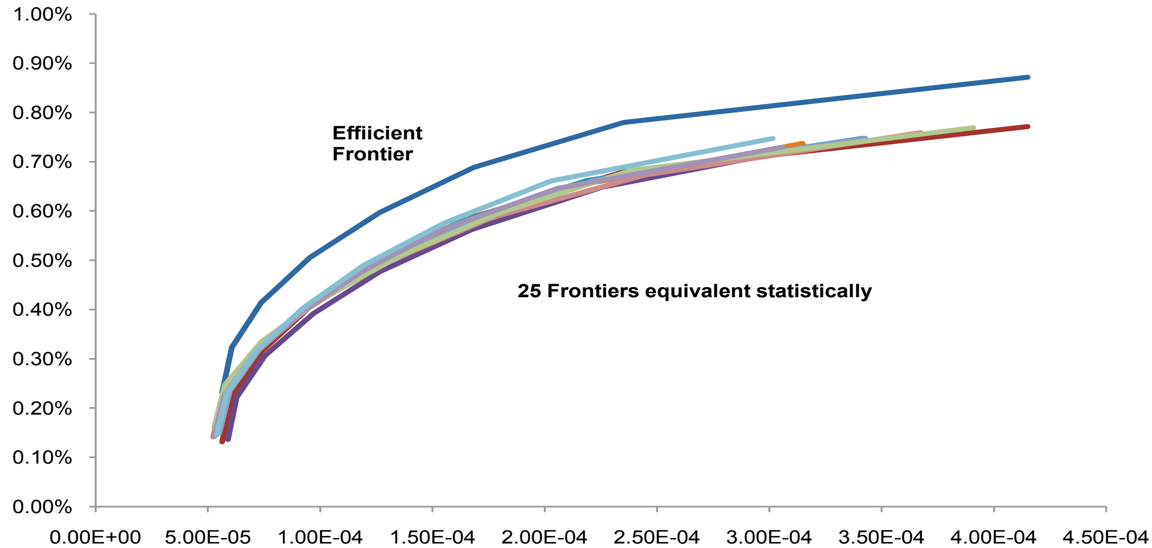
We can articulate the Resampling process in different phases:

1. A vector E (r) is estimated in n components (in which n is the number of securities under consideration), which include estimates of expected returns from the securities under consideration. In addition a vector σ is estimated with *n* standard deviations, and the matrix *n x n* of correlation between asset returns.
2. Starting from the parameters estimated in the first round, a Monte Carlo simulation is performed in which $E(r)_{sim}$ (that is a vector of simulated returns), σ_{sim} (simulated vector of standard deviations), and finally the simulated correlation matrix, ρ_{sim} are identified. The simulation process is not entirely random; in fact it is expected to "tie" the simulated parameters to historic parameters that have been observed. Assume that for security A we have E (r) = 3% and σ = 10%. Therefore, the generation of simulated trajectories of returns is bound by the assumptions made about the standard deviation and the mean value. The process of Resampling therefore requires the joint extraction of N returns, to be repeated a number of times H, from a multivariate normal distribution with parameters (mean, variance and covariance) coinciding with historical values. Of course, with increasing numbers of simulations, values obtained converge to the mean value and estimated standard deviation. For example, after 1000 simulations 2.83% has been obtained as the average value and 10.27% as the standard deviation. The number of simulations is a function of the degree of confidence about the observed estimates. Selecting the input obtained in phase 3, it moves to a process of optimization, such as Markowitz, from which we get a frontier statistically equivalent to that which we would have obtained if we had estimated optimized inputs in the first phase.
3. Repeating phase 2 H times, we obtain statistically equivalent H frontier. It is noted that, for construction, all statistically equivalent portfolios are located below the efficient frontier. If the weighting vector of a mean variance efficient portfolio is optimal for the original set of inputs it is natural that the allocations of portfolios simulated, reassessed with the same information set, are not so optimal (see Figure 1).

For each of the H simulations, it is possible to identify Z portfolios statistically equivalently close to the frontier. Each portfolio was given a ranking in a list. This produces for each ranking position, H portfolios all with the same average return but with different standard deviation and different composition.

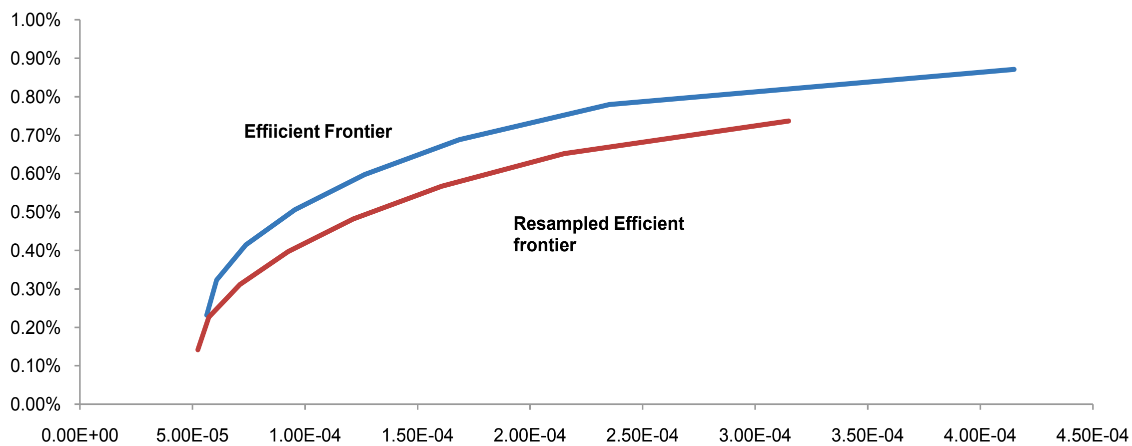
Taking the average weight that each security has in the portfolios with a certain ranking for the H simulations, we obtain a portfolio (portfolio 1 for example) named "Resampled"; repeating the same process for all other positions in the ranking will produce the Resampled frontier (see Figure 2).

Figure 1: Efficient Frontier and 25 Statistically Equivalent Efficient Frontiers



In this figure we compare efficient frontier and a set of statistically equivalent frontier. It is noted that, for construction, all statistically equivalent portfolios are located below the efficient frontier.

Figure 2: Efficient Frontier and Resampled Efficient Frontier



In *this figure we compare efficient frontier and Resampled efficient frontier. It is noted that, for construction, all Resampled efficient frontier are located below the efficient frontier.*

The frontier Resampled is nothing but an average of the weights of the portfolios of the efficient frontiers, which in turn were generated from input obtained from the simulation processes. The Resampling frontier will be more diversified relative to the efficient frontier. This is unlike Markowitz, in which the process of optimization leads to the identification of a particular dominant asset compared with others. From the composition of the H statistically equivalent frontiers, the same asset can also be dominated by other assets, and as a consequence creates less concentrated portfolios. The reasoning above shows a limit of the Markowitz approach known as "counter-intuition". Resampling and Bayesian approaches differ in the way they take into account estimation risk. In the first case, the intervention is directed to the most exposed input to this source of risk, namely the vector of average empirical yields. Otherwise, in the second case no correction is made, as there is the conviction to repair.

POST MODERN PORTFOLIO THEORY

It is possible to divide portfolio models, at least chronologically, into two families: the 'traditional models' (CAPM, Markowitz), which constitute the modern theory of portfolio choices, and the so-called 'post-modern' models (MADM, TEV, Elton-Gruber, SPM). All the quantitative modelling of first generation portfolio theories have in common an assumption of normality in the frequency distribution of portfolio returns. That assumption has proved forced, and does not correspond to the distributions of security returns observed in real markets. One of the assumptions on which the portfolio theory of G. Sortino is based, is recognition of the non-normal distribution, resulting in a greater use of processes that seek to express a better stability of parameters, and a more correct weightings distribution. A brief account, of some models of the so called Post Modern Theory, follows.

The Tracking Error Minimization Model (TEM) is a parametric model based on two factors: the expected return and the variance of the differential between the performance of the portfolio and the performance of the benchmark, which is the square of the Tracking Error Volatility. The objective is to seek the weight to assign to each asset in the portfolio. This is done to obtain the minimum portfolio tracking error with the constraints that the expected returns to be achieved are equal to or below a preset level, and that the weightings of the activities are positive and have sums equal to one. A generalization of the structure of the constraints is also permitted, in the sense that the presence of arbitrary linear constraints on the structure of the portfolio or lower (upper) bound is permitted. The objective function to minimize is:

$$\textit{Min variance} \sum(\omega_i \cdot r_i) - \sum(\chi_i \cdot r_i) \tag{1}$$

where:

χ_i = *fraction of the benchmark portfolio held in asset* i.

$\sum(\chi_i \cdot r_i) = r_t$ *(benchmark return)*

ω_i = *asset* i*'s weight by optimization process*

The Mean Absolute Deviation minimization Model (MADM) is a non-parametric model, based on the idea of finding a benchmark, against which a predetermined over-performance is required. It seeks to achieve a certain return while at the same time, not departing too much from the chosen benchmark. The distance from the benchmark is adopted as the risk measure. This risk is measured as the absolute median difference calculated over a predetermined period of time. The goal is to find the weight to assign in the portfolio to each security, with the condition of minimum absolute mean deviation. The security return, or better the portfolio return, expected to be achieved is equal to or less than a value set in advance. Moreover, the weights of all activities must be positive and of sum equal to one. It also permits the presence of arbitrary linear constraints in the structure of the portfolio. The model does not take into account hypotheses on the shape of the distribution of returns. The only implicit assumption is that the return distribution, observed in the past, remains in the future. The objective function to minimize is:

$$Min_{\omega} \sum \left| \sum(\omega_i \cdot r_i) - \sum(\chi_i \cdot r_{it}) \right| \tag{2}$$

where:

r_i = *asset* i*'s return*

ω_i = *asset* i*'s weight by optimization process*

χ_i = *fraction of the benchmark portfolio held in asset* i.

$\sum(\chi_i \cdot r_{it}) = r_{bt}$ = *benchmark return*

Remaining with models designed to optimize performance against a benchmark, the Shortfall Probability Minimization (SPM) model aims to reduce the probability of occurrence of a portfolio underperformance against a benchmark. The probability of shortfall is estimated over time, by relating the number of periods in which there was a shortfall to the total over the time periods preselected.

The aim of the model of minimizing the shortfall probability is therefore to find the weight assigned to each financial instrument so that in a given timeframe the shortfall frequency is minimal. This occurs with the constraint that the expected return is equal to or less than a value to be assigned, and the sum of the weightings is equal to one. Other restrictions can also be imposed on the linear weights of financial assets. In this model, as with the previous, the only assumptions on the distribution of returns made are that, they are the product of a stationary process. In this way the past contains useful information for future distribution. The objective function to minimize is:

$$\min_{\omega} \sum_{t=1}^{M} \frac{I_t}{m} \tag{3}$$

Where:
It = dichotomic variable, which assumes value equal to 1, in the event that at time t, shortfall occurs, otherwise it assumes value equal to 0.
m = number of sample periods in the time domain considered.

DATA AND RESULTS

The database studied consists of blue chip firms from three financial indices: the Eux50 (DJ Euro Stoxx 50) index, the U.S. Sp100 index and finally the SPMIB40 index. For each security the weekly series consisting of 383 observations is considered. The relative time period: from 07/01/2000 to 04/05/2007 is evaluated relative to an of sample period from 21/11/2003 to 04/05/2007. For each index a "rolling" estimate logic was used, or rather, the different outputs of the various models and different techniques used were calculated by moving forward one period at a time.

As shown in the Section 1, the Resampling uses Monte Carlo simulation which uses a generation of random numbers, within the range 0 and 1, of a uniform distribution. Each number obtained, by each random generation in the interval [0-1], is associated with an area within a standardized normal distribution, N~ (0,1), which corresponds to a particular value on the x-axis. For any N ~ (μ, σ), it is possible to identify that value on the x-axis which generates this area (by inverse function of a normal distribution). In this way, there will be a match between random number and x-axis value of a normal distribution. In practice for each security that is a component of the index considered, an average value and standard deviation of historical returns is calculated. Parameters obtained, for every security, represent input data for a N ~ (μ, σ) distribution.

Operationally, for each security, 200 random numbers and then 200 returns were considered. For example in the case of the Eux50 index, the definition of the simulated return matrix refers to a matrix *200 x 50* (50 being the number of securities within the index) for each period (in total there are 181 out of sample periods). The repeating of the return simulation is considered with the simulated returns repeated 100 times. From the simulated returns, a matrix of simulated returns is computed, from which a frontier is obtained that is statistically equivalent. The frontier is created using the mean value for each one of the securities from the simulated return matrix and the covariance of the time series of simulated returns.

In the application run on the dataset under consideration, 100 statistically equivalent frontiers (and thus 100 arrays of *200 x n* random numbers, where *n* is the number of securities included in the index) were generated. From 100 frontiers simulated a single frontier was produced. The procedure involved calculating, for 10 expected returns, the average of the weightings (relative to the fixed returns) of 100 statistically equivalent frontiers.

This article proposes a comparison, with reference to the Resampling technique, between the Markowitz model and Post Modern Theory models; specifically: TEV, MADM, and SPM. Unlike the Markowitz model, for Post Modern Theory models, the statistically equivalent frontiers have not been calculated. Instead for every simulated return matrix referring to a specific period (100 for each period), the optimal weight vector was calculated. With reference to the selected period, the optimal weighting vector is given from the weighting average of the optimal vectors obtained from each simulated matrix.

For each index considered (Eux50, Spmib40, Sp100) eight models are calculated. Statistics for each model are described as follows: Compound return, Rap measures (Sharpe ratio, Sortino Index, Information Ratio), Risk measures (Tem, Tev, Standard Deviation, Down Side Risk); test of statistical significance for the difference between mean returns of models and benchmark, and test of statistical significance for equality between variance returns of models and benchmark. At the top of the second and third column of each table is the name of the adopted model:

- Markowitz application of the Markowitz model, using the conditions that investors cannot hold securities in negative amounts and cannot short sell. In this case the best portfolio is the one lying on point of tangency between the efficient frontier and the Security Market Line.
- Resampling Markowitz, application of Resampling to the Markowitz model;
- MADM Resampling, application of Resampling to the Mean Absolute Minimization model (MADM);
- TEV Resampling standing for application of Resampling to the Tracking Error Minimization Volatility (TEV) model;
- SPM Resampling standing for Resampling in Shortfall Probability Minimization model (SPM) context.

In the first column of the tables shown below you can find, the total return, that is the maturity yield (compound return) of the model, calculated by assuming a starting capital of 100 compound interest calculated with weekly observations. We have calculated the model return for each period, using the hypothesis of investing in stocks suggested by the best vector of the model weights. The weekly return of the model was calculated by multiplying the best weights vector with real returns of stocks. This renders a different performance (actual/realized) from the performance estimated by the model. This difference is commonly called a discrepancy, similar in concept to the definition of instability of efficient portfolios. In this analysis no transaction costs have been taken into account. Using the Markowitz model the Resampling technique did not overperform with regard to the reference model (that is the Markowitz model without Resampling), for all the three considered indexes even though the compounded return is high, respectively 94% against 128%, for the Sp100 index, 21% against 31% for the Ex50 index, and 93% against 183% for the Spmib40 index.

The application of the Resampling technique to Post Modern Theory produced benefits (except in the case of the TEV model for the Sp100 index) in terms of compound return as determined by comparing the reference model to the ones the Resampling technique is applied to. In fact the MADM returned 42% for the US Sp100 index and 43% for MADM Resampling. The same results were produced for the Ex50, 34% index without Resampling against 38% in cases where the technique was applied, and 95% for the MADM through Resampling for the SpMib40 index against 162% for the simple MADM. The TEV model gave, for the three indexes, Sp100, Ex50 and Spmib40, 74%, 64% and 140% respectively, against 68%, 100% and 212% of the TEV with Resampling. The benefits achieved, through the Resampling technique for the SPM model, were even higher; in fact according to the index considered, the best performance was from a minimum of about 78% to a maximum of over 196%.

An analysis of risk adjusted performance showed that the difference observed for the compound returns (in the use of the model) decreased. For example, in the use of the Sharpe indicator, the model excess return is compared to return standard deviation. So we can deduce that the models which have produced the best returns, are also the riskiest ones as defined by the standard deviation. It is symptomatic to observe how the difference in basis points, between the Markowitz and the TEV models, for the Spmib40 index, equivalent to over 40 points, might persuade the choice of the first model. On the contrary, if we analyse performances in the light of the Sharpe, the choice will lead to the second model. See the first and third panels in Table 3.

In fact, if we consider Table 3 again, we can observe how the Markowitz model standard deviation of 3.6%, is higher compared to the TEV model standard deviation of 2.7%. Yet if we analyse the results in the light of the Sharpe index, again Resampling technique has not improved the simple model performance. With regard to the Resampling method applied to post modern models, we get improvements in the Sharpe index in 100% of the cases against 89% improvements for the compound returns (8 cases out of 9).

The Information Ratio value is shown in Table 1 to 3 for each model related to the period 2004-2007. We cannot compare models in the case of Information Ratio, with different starting benchmarks, as the information ratio related to each model is influenced by the effects of its own reference benchmark. The Resampling technique, applied to post-modern models, enables us to improve the reference models 7 times out of 9, or in the 78% of cases, (Tables 1 to 3). In the case of the Markowitz model (once again) there are no benefits.

Next the Sortino index which uses as a measure of the Downside risk, is considered where the minimum acceptable return is represented by the risk free return. The Sortino index shows the percentage of over performance per unit of Downside Risk. In all cases the Resampling method improves the Markowitz model (first panel of tables 1 to 3). For Post Modern Theory models it improves about 56% of all cases (5 times out of 9), with particular difficulty with the MADM model and the Sp100 index. With regard to risk measures, such as Downside risk and Standard Deviation, the Resampling method improves almost exclusively in the TEV model.

Table 1 - Sp100 Model Results

	Sp100		
	Markowitz	**Markowitz Resampling**	**Difference**
Total Return	128.24%	94.61%	33.62%
Sharpe	13.07%	11.96%	1.12%
Sortino	1.76%	2.02%	-0.26%
Informatio Ratio	13.36%	12.43%	0.93%
Tev	3.24%	2.62%	0.62%
Standard Deviation	3.67%	3.12%	0.55%
Dsr	2.66%	2.32%	0.34%
	MADM	**MADM Resampling**	**Difference**
Total Return	42.16%	42.76%	-0.60%
Sharpe	5.88%	7.68%	-1.80%

	Sp100		
Sortino	1.63%	1.62%	0.01%
Informatio Ratio	5.32%	7.44%	-2.12%
Tev	3.36%	3.27%	0.09%
Standard Deviation	3.84%	3.78%	0.06%
Dsr	2.88%	2.89%	-0.02%
	TEV	**TEV Resampling**	**Difference**
Total Return	74.36%	68.86%	5.50%
Sharpe	9.51%	8.82%	0.69%
Sortino	1.90%	1.79%	0.11%
Informatio Ratio	9.86%	8.99%	0.87%
Tev	2.77%	2.88%	-0.11%
Standard Deviation	3.36%	3.47%	-0.11%
Dsr	2.46%	2.62%	-0.15%
	SPM	**SPM Resampling**	**Difference**
Total Return	-23.86%	53.99%	-77.84%
Sharpe	-2.55%	8.12%	-10.67%
Sortino	1.42%	2.22%	-0.81%
Informatio Ratio	-3.39%	7.49%	-10.89%
Tev	4.50%	2.53%	1.97%
Standard Deviation	4.15%	2.91%	1.24%
Dsr	3.31%	2.10%	1.20%

In this table we report results, for the applications of the Markowitz Model and Post Modern Theory Models for Sp100 Index over a sample period from 21/11/2003 to 04/05/2007.

Table 2 - Ex50 Model Results

	Ex50		
	Markowitz	**Markowitz Resampling**	**Difference**
Total Return	31.33%	21.75%	9.58%
Sharpe	6.30%	4.34%	1.96%
Sortino	17.55%	18.30%	-0.74%
Informatio Ratio	-8.13%	-13.83%	5.70%
Tev	1.60%	1.26%	0.34%
Standard Deviation	2.00%	1.89%	0.11%
Dsr	1.46%	1.40%	0.06%
	MADM	**MADM Resampling**	**Difference**
Total Return	34.02%	38.28%	-4.26%
Sharpe	5.92%	7.60%	-1.68%
Sortino	14.01%	17.21%	-3.19%
Informatio Ratio	-5.76%	-6.86%	1.11%
Tev	1.86%	1.46%	0.40%
Standard Deviation	2.52%	2.05%	0.47%
Dsr	1.83%	1.49%	0.34%

Ex50			
	TEV	**TEV Resampling**	**Difference**
Total Return	64.59%	100.42%	-35.82%
Sharpe	11.58%	18.69%	-7.11%
Sortino	16.22%	18.26%	-2.04%
Informatio Ratio	-0.05%	8.51%	-8.57%
Tev	1.55%	1.20%	0.34%
Standard Deviation	2.20%	1.92%	0.28%
Dsr	1.58%	1.40%	0.18%
	SPM	**SPM Resampling**	**Difference**
Total Return	51.84%	248.28%	-196.45%
Sharpe	8.42%	25.78%	-17.36%
Sortino	14.03%	13.43%	0.60%
Informatio Ratio	-1.79%	20.64%	-22.44%
Tev	1.99%	2.06%	-0.08%
Standard Deviation	2.62%	2.65%	-0.03%
Dsr	1.82%	1.91%	-0.08%

In this table we report results, for the applications of the Markowitz Model and Post Modern Theory Models for Ex50 Index over a sample period from 21/11/2003 to 04/05/2007.

Table 3 - Spmib40 Model Results

SpMib40			
	Markowitz	**Markowitz Resampling**	**Difference**
Total Return	183.57%	93.97%	89.60%
Sharpe	16.66%	13.89%	2.76%
Sortino	9.59%	13.33%	-3.75%
Informatio Ratio	11.12%	5.31%	5.80%
Tev	3.20%	2.15%	1.06%
Standard Deviation	3.58%	2.55%	1.03%
Dsr	2.51%	1.80%	0.70%
	MADM	**MADM Resampling**	**Difference**
Total Return	94.59%	162.42%	-67.83%
Sharpe	12.51%	18.91%	-6.40%
Sortino	11.51%	11.68%	-0.17%
Informatio Ratio	5.13%	12.24%	-7.10%
Tev	2.46%	2.36%	0.10%
Standard Deviation	2.93%	2.80%	0.13%
Dsr	2.09%	2.06%	0.03%
	TEV	**TEV Resampling**	**Difference**
Total Return	140.37%	212.29%	-71.91%
Sharpe	17.75%	21.69%	-3.93%
Sortino	12.27%	11.44%	0.83%

	SpMib40		
Informatio Ratio	10.72%	15.59%	-4.87%
Tev	2.21%	2.49%	-0.28%
Standard Deviation	2.69%	2.90%	-0.21%
Dsr	1.96%	2.10%	-0.14%
	SPM	**SPM Resampling**	**Difference**
Total Return	115.39%	199.80%	-84.41%
Sharpe	12.67%	19.38%	-6.71%
Sortino	9.83%	10.36%	-0.52%
Informatio Ratio	6.55%	13.53%	-6.97%
Tev	3.05%	2.76%	0.28%
Standard Deviation	3.47%	3.17%	0.30%
Dsr	2.45%	2.32%	0.12%

In this table we report results, for the applications of the Markowitz Model and Post Modern Theory Models for SpMib40 Index over a sample period from 21/11/2003 to 04/05/2007.

Up till now we have presented heuristic evidence of Resampling processes. Next, we deal with the statistical significance of results obtained from the models presented above. In order to verify the significance of positive track error, between results of different models, we use a statistical test based on the difference between two mean values. Let us consider two populations (X, Y) independent and normally distributed. Let us also consider two samples of *n* independent observations from the two normal populations (X1,…Xn) and (Y1,…,Yn). We want to test:

H0: μx = μy against H1: μx ≠ μy.

These hypotheses can equivalently be written as:

H0: (μx - μy) = 0 against H1: (μx - μy) ≠ 0

Consider the standard distribution of the variable $(\overline{X}-\overline{Y})$ we obtain test statistic (with hypothesis that the two populations have identical variance):

$$z=\frac{(\overline{X}-\overline{Y})-(\mu_x-\mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x}+\frac{\sigma_y^2}{n_y}}} \tag{4}$$

under H0: (μx - μy) = 0 we obtain:

$$z=\frac{(\overline{X}-\overline{Y})}{\sqrt{\frac{\sigma_x^2}{n_x}+\frac{\sigma_y^2}{n_y}}} \tag{5}$$

In most of cases the common population variance is not known, so in this case we can refer to unbiased variance estimator $s^{2,}$ defined by:

$$s^2 = \frac{\sum_{i=1}^{n_x}(X_i - \overline{X})^2 + \sum_{i=1}^{n_y}(Y_i - \overline{Y})^2}{n_x + n_y - 2} \tag{6}$$

Under H0 the test statistic is given by:

$$t = \frac{(\overline{X} - \overline{Y})}{\sqrt{\frac{s^2}{n_x} + \frac{s^2}{n_y}}} \tag{7}$$

this has a t distribution with *nx + ny - 2* degrees of freedom. In this case test critical values are given by:

$$(\overline{X} - \overline{Y})\,\mathrm{I} = -t_{\alpha/2}\sqrt{s^2/n_x + s^2/n_y} \tag{8}$$

$$(\overline{X} - \overline{Y})\,\mathrm{S} = t_{\alpha/2}\sqrt{s^2/n_x + s^2/n_y} \tag{9}$$

In the case in which the two sample variances are different, before conducting a test on mean values, we have to verify the hypothesis by a test for variance equality. This is done using a test based on the F statistic with Fisher distribution.

In the case in which test F is statistically significant the variables X and Y, from which the two samples are drawn, may not have the same variance. In this case we have to use two unbiased variance estimators s^2_x and s^2_y defined by:

$$s_x^2 = \sum_{i=1}^{n}\frac{(X_i - \overline{X})^2}{n_x - 1} \tag{10}$$

$$s_y^2 = \sum_{i=1}^{n}\frac{(Y_i - \overline{Y})^2}{n_y - 1}\,. \tag{11}$$

The test statistic in this case will be:

$$t = \frac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}} \tag{12}$$

A test of the difference between the two return means, one from the model and the other the benchmark, deals with a crucial topic in finance. That is, considering two populations X and Y, we try to establish if they have identical means. In other words, we try to understand if, on average, investing in the model or in the benchmark are equivalent. Considering the full period, 21/11/2003 to 04/05/2007 examining model (X) and benchmark (Y) returns, we find that in all cases except two, we cannot reject H0, at the 0.05% significance level. Thus we conclude that on the basis of the samples observed, identical means between model and benchmark returns are present in the majority of cases. For Ex50 index the null hypothesis is rejected at the 0.05% significance level only in one case corresponding to SPM with Resampling. For SpMIb40 index the null hypothesis is accepted in all of the cases considered. For Sp100 index the null hypothesis is rejected at the 0.05% significance level, only in one case corresponding to SPM without Resampling. With regard to the test of variance equality between model and benchmark, only in 11% of all cases we can accept the null hypothesis of equality of variances between the two populations. In the remaining of 89% of the cases, the test suggests a difference between the variances of the two populations.

CONCLUSION

This article examines the approach to Portfolio modelling called Resampling, which reduces the effect of estimation error present in any practical implementation of a Portfolio Model. Resampling is a method used in portfolio modelling to try to obtain better out of sample performance for given input model parameters and in this way better measures of performance. Applying Resampling methods to blue chip portfolios of 3 different stock markets (Italian Equity Market, European stock Exchange and New York Stock Exchange) we compared the results for the Markowitz model and the so called Post Modern Portfolio Theory models (in particular Tracking Error Minimization Model (TEV), Mean Absolute Minimization Model (MADM), and Shortfall Probability Model (SPM)). With regard to the Markowitz model, in terms of compound return, the Resampling method did not over perform the reference model. The application of the Resampling method to Post Modern Portfolio Theory models produced benefits in all cases considered except for one (TEV Resampling for the Sp100 index). In particular benefits were notably evident for the SPM model where the performance improves from 78% to over 196% depending on the index considered.

An analysis of indicators taking into account the risk showed that the Resampling method applied to Post Modern Portfolio Models improved the Sharpe Index in 100% of the cases. However, there were no benefits for the Markowitz model. When we examine the information ratio values the Resampling method produced benefits only for about 78% of the post modern portfolio theory cases. Examining the Sortino index results we obtained contrasting results, whereas there are smaller benefits, for the Post Modern Theory and bigger ones for the Markowitz model. Our conclusion is that there is general evidence to show that Resampling methods produce better stability in the input of the models. As such, they are capable of generating substantial improvements both for compound return and for RAP performance measures, but only for post modern portfolio theory models. When considering Downside Risk and Standard Deviation, the Resampling method only improves the TEV model. So it is evident that this method is useful for obtaining better returns but not proportional stability of the same returns. The statistical significance of the positive tracking errors obtained from the models indicates that these results should be taken with caution. This caution is warranted because the equality tests of average returns, from models with or without Resampling method, lead to rejection of the null hypothesis in 25% of all cases. In short, though the benefits of Resampled Efficiency optimality are not clear, Resampling remains an interesting heuristic to deal with the important problem of error minimization.

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