

JOB SECURITY AND PERSONAL INVESTMENT PORTFOLIO

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ABSTRACT

This paper incorporates human capital into the well-established portfolio theory by allowing for job security in personal portfolio choice. Our model predicts that young people hold more cash to hedge against risk associated with human capital (layoff risk). As people age, layoff risk decreases, and consequently, they invest in more risky assets – stocks in their portfolios. However, as people approach retirement, their human capital diminishes, and they become more risk averse. Hence, they hold more cash again. Our model provides a plausible explanation for the observed investment behavior of people who reveal humped shape stock holdings over the life cycle. Our results suggest that financial advisors should take into account different levels of job security when giving financial advice to different individuals.

JEL: G11; G23

KEYWORDS: Job security, personal portfolio, human capital

INTRODUCTION

In the past decade, self-directed retirement plans (e.g. 401(K)s, IRAs, etc.) have become the norm for people saving for their retirement. However, many participants in these retirement plans know little about personal investment. As such, professional financial advice is important in helping participants make sensible choices. Professional financial advisors popularly advise young people to invest in stocks, and gradually increase their investment in fixed income as they approach retirement. For example, Fidelity Investments and the Vanguard Group both recommend that the fraction of assets invested in equities should increase with one's wealth and decline with one's age.

However, empirical evidence shows that such professional advice contradicts people's investment behavior. The observed investment pattern is a humped shape for stocks. That is, people's investment in stocks gradually increases from a young age to middle age, and then tapers off. Investment in cash is the opposite: it gradually decreases from a young age to middle age, and then goes up.

An important question is: what causes professional advice to deviate from people's investment behavior? A possible reason is that such advice takes little consideration of human capital and its associated risk. Human capital is a major component of personal assets. Especially, it may be the only asset for most young people who are just starting their career. If this crucial element is ignored in making portfolio choices over the life cycle, the portfolio may not be optimal.

In contrast to previous studies, our study provides an alternative to quantify human capital and its associated risk by incorporating job security into portfolio theory to explore people's investment behavior. We first model human capital (income from job) as a financial asset. Next, we derive the covariance of the financial asset proxy for human capital with the mutual fund that the person would invest in. In our model, the risk associated with human capital is layoff risk, which is a major risk related to personal portfolio choice. Hence, how to diversify or hedge away this risk is an important issue in personal portfolio choice over the life cycle. By incorporating this human capital risk into portfolio

choices, we provide an optimum strategy for making personal investment decisions. The optimum strategy shows that people hold less risky assets at a young age to hedge against high human capital risk (layoff risk). As people age, human capital risk declines and people hold more risky assets. However, as people approach retirement, their human capital diminishes and risk aversion increases, resulting in an increase in the holding of risk-free assets. Our results thus provide a plausible explanation for the puzzle of why people's investment behaviors do not follow the pattern recommended by professionals.

This paper is organized as follows: Section 2 reviews the relevant literature. Section 3 develops a discrete model that incorporates job security into personal portfolio selection and presents the major results of the model. Section 4 extends the model and further discusses its implications. Section 5 concludes the paper.

LITERATURE REVIEW

Since Markowitz (1952, 1959) laid the groundwork for optimizing investment portfolios based on the tradeoff between risk and return, where risk is proxied by the volatility of the return on the underlying assets, professional financial advisors have used portfolio theory extensively to advise personal portfolio choices. A known rule of thumb is that the percentage of one's portfolio to invest in equities should be 100 minus one's age (see, Bodie and Crane, 1997). However, studies show that such professional advice is problematic. For example, Bodie (2002) points out that such advice is logically flawed and dangerously misleading because it does not allow for insurance against a market decline. In addition, empirical evidence shows that such professional advice contradicts people's investment behavior. For instance, the Survey of Consumer Finances (see Table 1) shows that people do not follow investment advice professionals claim to be optimal (Heaton and Lucas, 2000). The observed investment pattern is a humped shape for stocks. People's stock holdings increase from the young (<35) to the middle aged (<64) and then decrease as retirement approaches (>64). Furthermore, Heaton and Lucas find that the share of stock holdings is negatively related to the level and variability of the growth rate of proprietary income. Similarly, a survey on TIAA-CREF participants by Bodie and Crane (1997) shows that controlling for the effects of age and wealth, substantial differences still exist among individuals in the fraction of their total assets invested in equity. Factors reflecting the value and riskiness of human capital help explain these differences.

Table 1: Portfolio Shares Relative to Liquid Assets by Age

Assets	Age <35	35-49	50-64	>64
1989				
Stocks	0.394	0.465	0.454	0.360
Bonds	0.154	0.137	0.176	0.196
Cash	0.452	0.398	0.370	0.444
1992				
Stocks	0.474	0.534	0.540	0.420
Bonds	0.114	0.130	0.160	0.199
Cash	0.412	0.336	0.300	0.381
1995				
Stocks	0.550	0.590	0.600	0.494
Bonds	0.113	0.116	0.119	0.138
Cash	0.337	0.294	0.281	0.368

Source: Heaton and Lucas (2000), *Journal of Finance* 60, p. 1163-1198.

While the empirical evidence shows that human capital is an important factor in portfolio choices, the challenge is how to quantify human capital and risk associated with it. For example, Bodie et al. (1992) use the earning power from labor to proxy for human capital. Their model indicates that the fraction of an individual's financial wealth optimally invested in equity should "normally" decline with age. Lupton (2001) and Merton (1971) develop models to find that constant labor income substitutes for bonds in a financial portfolio. Gomes and Michaelides (2005) develop a life-cycle model with calibrated uninsurable

labor income risk and moderate risk aversion. They show that the model can simultaneously match stock market participation rates and asset allocation decisions conditional on participation. Cocco et al. (2005) solve numerically for optimal portfolio and savings decisions considering an investor facing mortality risk, borrowing and short-sale constraints, and receiving labor income. Their model shows that an investor reduces her proportional stock holdings in the process of aging, and that labor income loses importance as an investor ages. Polkovnichenko (2007) considers stochastic uninsurable labor income and predicts a much more conservative portfolio when there is a slim chance of a severe income shock. Bodie and Treussard (2007) suggest that if a safe fund matched to the investor's time horizon is included in addition to the target-date funds (TDF), investors who differ from the natural TDF holder in their risk aversion or exposure to human capital risk would realize optimal strategy. By allowing for the mutual relation between labor supply (earnings) and portfolio choice over a life-cycle, Gomes et al. (2008) show that variable labor supply materially alters preretirement portfolio choice by significantly raising optimal equity holdings. Post retirement, however, the optimal equity share increases as households spend down their financial assets, leaving bond-like pension benefits to increasingly dominate household resources.

Although previous studies consider human capital in constructing portfolios, their theoretical prediction of people's investment behavior is not completely consistent with the empirical evidence (e.g. Heaton and Lucas, 2000). In this paper, we provide an alternative approach to quantify human capital and its associated risk in personal portfolio selection by considering job security. We contribute to the literature in that our model prediction of people's investment behavior is consistent with the observed pattern. As such, our results provide insightful implications to professional financial advisors that they should consider different levels of job security when they advise clients on personal portfolio choices.

MODEL

Modeling of Human Capital

In this study, we analyze people's investment behaviors by studying an employee of a corporation. The corporation issues common stocks that trade on the market. We define personal human capital as the present value of the individual's future income. We assume the person has already optimized her human capital. Namely, the person has invested in her education and other trainings such that the marginal cost of education is equal to the expected marginal gain in her income. Therefore, the average return on human capital is higher than the marginal return. The direct consequence of this argument leads to the following lemma:

Lemma 1: The expected average rate of return on personal human capital, R_L^P , is greater than the rate of return on any mutual fund with the same level of risk, R_M .

The rationale behind Lemma 1 is that if $R_L^P < R_M$, the opportunity cost of *not* working is small and one would find it advantageous to use the profits from the mutual fund's gains to pay for more trainings and education, which contradicts our assumption that the person has already optimized her human capital. We next define risk of human capital as layoff risk. We model the value of human capital as a jump process, which is characterized as the layoff probability, λ , and the recovery rate, δ . If a person is not laid off in period 1, the value of her human capital will be her human capital in the next period, H_1 plus her wage income, I_1 . Her rate of return on human capital is

$$R_{Lu} = \frac{(I_1 + H_1) - H_0}{H_0}. \quad (1)$$

However, if she is laid off, she can only recover a portion of the value of her original human capital due to income losses over the jobless period, loss of benefits, and settling in a job with less income. Her human capital loss would be $(I-\delta)(I_1+H_1)$. Her rate of return on human capital is

$$R_{Ld} = R_L | t^* \in \{0, t_1\} = \frac{\delta(I_1 + H_1) - H_o}{H_o} = \delta(1 + R_{Lu}) - 1. \tag{2}$$

Combining equations (1) and (2), we can express the expected rate of return on her human capital as

$$R_L^p = E_t^p(R_L | F_0) = R_{Lu} - (1 - \delta)\lambda - O(\lambda R_{Lu}) \tag{3}$$

where $O(z)$ stands for the term with the same or higher order of z . The variance of her human capital return can be computed as

$$\begin{aligned} \text{var}(R_L) &= \left[(R_L | t^* > t_1) - R_L^p \right]^2 (1 - \lambda) + \left[(R_L | t^* \in \{0, t_1\}) - R_L^p \right]^2 \lambda \\ &= (1 + R_{Lu})^2 (1 - \delta)^2 (1 + \lambda) \lambda. \end{aligned} \tag{4}$$

Equation (4) indicates that the higher the layoff risk and the lower the recovery rate, the more volatile the human capital returns.

Covariance between the Firm’s Stock and the Mutual Fund

Since human capital is an important component in a personal portfolio, we have to understand the covariance between human capital and the person’s other assets in the portfolio to set up an optimal portfolio. Unfortunately, there is no direct link between these two asset processes. Therefore, we first compute the covariance between the stock of the firm that hires the person and the mutual fund that the person may choose for her portfolio. Next, we link human capital to the stock of the employer. In this way, the relation between human capital and the mutual fund is built and the covariance between the two is computed.

We denote x as a publicly traded stock, S , (or a mutual fund M). The stock price can either move up from x_o to a new level $u_x x_o$ or down from x_o to a new level $d_x x_o$. Let $u_x = e^{\sigma_x \sqrt{\Delta t}}$ and $d_x = e^{-\sigma_x \sqrt{\Delta t}}$, where σ is the volatility of the firm’s stock (or mutual fund) and Δt is the length of the period (see Cox, Ross, and Rubinstein, 1979). The probability of an up movement in price, π , is given by

$$\pi_x = \frac{e^{\mu_x \Delta t} - d_x}{u_x - d_x}. \tag{5}$$

There are four possible states, $u_S u_M$, $u_S d_M$, $d_S u_M$, and $d_S d_M$ with probability x_1 , x_2 , x_3 , and x_4 respectively, (see Figure 1). Thus, the covariance between the expected rate of return of the stock and that of the mutual fund is

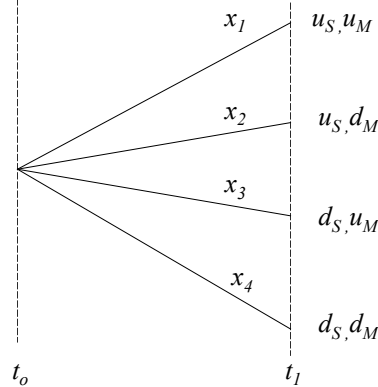
$$\begin{aligned} \text{cov}(R_S, R_M) &= \sigma_{SM} = (u_S - e^{\mu_S \sqrt{\Delta t}})(u_M - e^{\mu_M \sqrt{\Delta t}})x_1 + (u_S - e^{\mu_S \sqrt{\Delta t}})(d_M - e^{\mu_M \sqrt{\Delta t}})x_2 \\ &+ (d_S - e^{\mu_S \sqrt{\Delta t}})(u_M - e^{\mu_M \sqrt{\Delta t}})x_3 + (d_S - e^{\mu_S \sqrt{\Delta t}})(d_M - e^{\mu_M \sqrt{\Delta t}})x_4 \end{aligned} \tag{6}$$

and $x_1 + x_2 = \pi_S$, $x_1 + x_3 = \pi_M$, $x_1 + x_2 + x_3 + x_4 = 1$, which yield

$$x_1 = \frac{\sigma_{SM}}{\left(e^{\sigma_S \sqrt{\Delta t}} - e^{-\sigma_S \sqrt{\Delta t}} \right) \left(e^{\sigma_M \sqrt{\Delta t}} - e^{-\sigma_M \sqrt{\Delta t}} \right)} + \pi_S \pi_M. \tag{7}$$

If the high order terms of Δt are ignored, equation (7) is reduced to $x_1 = \sigma_{SM} + \sigma_S \sigma_M$. The reduced equation (7) shows that the probability of the stock and mutual fund going up simultaneously depends on the covariance between the expected returns of the two and the volatility of each one.

Figure 1: Movement of Stock and Mutual Fund Returns



This figure shows that the probability of both the stock and the mutual fund going up is x_1 , the probability of the stock going up and the mutual fund going down is x_2 , the probability of the stock going down and the mutual fund going up is x_3 , and the probability of both the stock and the mutual fund going down is x_4 .

Covariance between Human Capital and the Mutual Fund

To link human capital to the stock of the employer, we assume that the layoff would occur if the stock of the firm were down. If x_o goes down to dx_o , there is a probability $\lambda/(1-\pi_S)$ that the firm will lay off employees. The covariance between the return on human capital and that on the mutual fund is

$$\begin{aligned} \text{cov}(R_L, R_M) = \sigma_{LM} = & (u_M - e^{\mu_M \Delta t})(R_{Lu} - R_L^P)x_1 + (d_M - e^{\mu_M \Delta t})(R_{Lu} - R_L^P)x_2 \\ & + (u_M - e^{\mu_M \Delta t})(R_{Lu} - R_L^P)x_3 \left(1 - \frac{\lambda}{1 - \pi_S}\right) + (u_M - e^{\mu_M \Delta t})(R_{Ld} - R_L^P)x_3 \frac{\lambda}{1 - \pi_S} \\ & + (d_M - e^{\mu_M \Delta t})(R_{Lu} - R_L^P)x_4 \left(1 - \frac{\lambda}{1 - \pi_S}\right) + (d_M - e^{\mu_M \Delta t})(R_{Ld} - R_L^P)x_4 \frac{\lambda}{1 - \pi_S}. \end{aligned} \quad (8)$$

Equation (8) can be simplified as $\sigma_{LM} = \alpha \sigma_{SM}$, where $\alpha = \frac{(1 + R_{Lu})(1 - \delta)\lambda}{e^{\sigma_S \sqrt{\Delta t}} - e^{\mu_S \Delta t}}$. The simplified equation

(8) shows that the covariance between the return of human capital and that of the mutual fund is determined by the covariance between the return of the firm's stock and that of the mutual fund.

Personal Portfolio Choice

After figuring out the covariance between the return of human capital and that of the mutual fund, we set up the personal portfolio. We assume that the personal portfolio is composed of human capital, real estate, and the mutual fund (stocks and bonds). The variance matrix of the return on the assets in the portfolio is given by

$$\begin{bmatrix} \sigma_L^2 & \sigma_{LH} & \sigma_{LM} \\ \sigma_{LH} & \sigma_H^2 & \sigma_{HM} \\ \sigma_{LM} & \sigma_{HM} & \sigma_M^2 \end{bmatrix}$$

where $\sigma_i^2, i = L, H, M$ stands for the variance of the return of human capital, real estate, and the mutual fund, respectively. σ_{LH} is the covariance between the return of human capital and that of real estate. σ_{LM} is the covariance between the return of human capital and that of the mutual fund. Lastly, σ_{HM} is the

covariance between the return of real estate and that of the mutual fund. The return and the variance of the portfolio are given by

$$R_p = w_L R_L + w_H R_H + w_M R_M \quad (9)$$

$$\sigma_p^2 = w_L^2 \sigma_L^2 + w_H^2 \sigma_H^2 + w_M^2 \sigma_M^2 + 2w_L w_H \sigma_{LH} + 2w_H w_M \sigma_{HM} + 2\alpha w_L w_M \sigma_{SM}$$

where w_i is the weight of asset i and $w_L + w_H + w_M = 1$.

In such a personal portfolio, the person can adjust her mutual fund more easily than her human capital and real estate. As she adjusts her mutual fund, the variance of the return of mutual fund and the covariance between the return of mutual fund and that of human capital changes accordingly. Simultaneously, the covariance between the return of mutual fund and that of real estate also changes. We can see that this model possesses two characteristics: 1) In a personal portfolio, human capital and the real estate asset are not completely flexible in portfolio adjustment. 2) Because of inflexibility of the two assets, the objective of optimization of the personal portfolio is different from that implied by traditional portfolio theory. In this setting, the mutual fund becomes a tool for the person to hedge against the risk associated with her human capital and real estate.

Let us look at a person's typical life cycle. When the person graduates from college and gets her first job, she starts with only her personal human capital that has the following predetermined risk and return characteristics: $w_L = 1, w_H = w_M = 0$. As time goes by, her portfolio gradually shifts towards her mutual fund and real estate. Eventually, at retirement age, the weights of the three components become $w_L = 0$ and $w_H + w_M = 1$. At retirement, her human capital is zero. She would choose a mutual fund to maximize her utility given her total assets. In other words, her portfolio at retirement depends on her risk tolerance or risk aversion. Given this target portfolio on her retirement, she can choose different paths to reach this target portfolio when she retires. Even though all possible paths lead to the same portfolio, the paths may have important differences among each other. Next, let us examine some possible paths.

One possible path is that a person puts her savings directly into the target portfolio. When she retires, her human capital becomes zero and her portfolio is the target portfolio.

Another possible path is that she invests all her savings in risk-free assets and real estate. As her portfolio risk reduces to the risk level of her target portfolio, she redistributes her savings between risk-free and risky assets so that the risk level remains the same as that of her target portfolio. On retirement, her portfolio will be her target one.

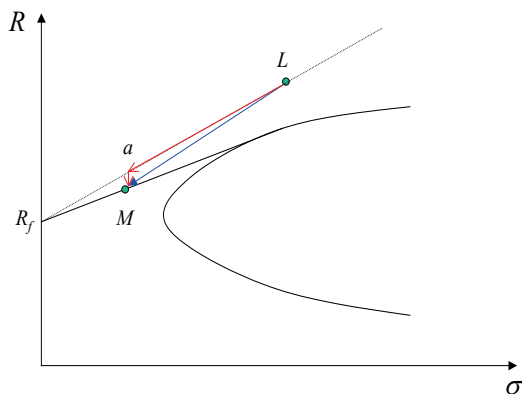
The difference between these two paths is the time at which the target risk level is reached. In the first path, she reaches the risk level exactly on her retirement date, while the second path allows her to reach the risk level before her retirement. We next use an example to show the difference between the two paths.

To simplify, we assume that a person rents an apartment and thus, her real estate asset is zero, $w_H = 0$. Her target portfolio, i.e. her mutual fund at retirement, is point M in Figure 2. Suppose that her target mutual fund is a combination of risk-free assets and a portfolio of risky assets. To reach her target portfolio, she may put her savings constantly into this mutual fund. As time goes by, the weight of her human capital decreases and the weight of her mutual fund increases. At retirement, 100 percent of her assets are in this mutual fund. Consequently, she reaches her target portfolio. Line LM in Figure 2 indicates this possible path.

Another possible path is that the person puts her savings into risk-free assets first, which counterbalances the risk associated with her human capital (layoff risk). Her path takes line La in Figure 2 to point a , at which the total risk of her portfolio reaches the risk level of her target portfolio. Then she changes her investment strategy by redistributing her savings in both risk-free and risky assets such that her portfolio eventually reaches her target portfolio at retirement, as indicated by line aM in Figure 2. The example suggests that the important difference between the two paths is the time taken to reach her target risk level. In the former path, the risk level of the target portfolio is reached exactly on the retirement date. In the latter path, the risk level can be reached before the retirement date.

The example above shows the important role human capital plays in portfolio choice. We find that if the risk associated with human capital is considered, the portfolio choice is quite different from the one suggested by financial advisors, which invests more risky assets at a young age and gradually reduces risk towards retirement. In reality, a person often starts with a single asset, human capital over the life cycle. Since human capital cannot be traded in the market, the young person is stuck with the risk associated with human capital. Her financial objective is generally how to adjust her relatively flexible assets to achieve her target portfolio. Our finding thus provides a plausible explanation on the puzzle that professional advice is not consistent with empirical evidence. The reason is that such professional advice may not allow for the risk associated with human capital in making a portfolio choice.

Figure 2: Possible Paths to Reach Target Portfolio



This figure shows two possible paths to reach a person's target portfolio (M) that consists of risk-free and risky assets. The path LM indicates that the person constantly puts her savings in a mutual fund. As time goes by, the weight of her human capital (L) decreases and the weight of her mutual fund increases. At retirement, 100 percent of her assets are in the mutual fund and the target portfolio is reached. The other path, La and aM, indicates that the person puts her savings into risk-free assets first, which counterbalances her human capital risk. At point a, the total risk of her portfolio reaches the risk level of her target portfolio (M). Then she redistributes her savings in both risk-free and risky assets such that her portfolio reaches her target portfolio at retirement, as indicated by aM.

EXTENSION AND IMPLICATIONS OF THE MODEL

In Section III, we assume that a person chooses to invest in risk-free assets to hedge her human capital risk at a young age. However, some young people may be less risk averse and invest more in stocks to hedge human capital risk. In this case, the path is represented by line Lb in Figure 3, which shows that an investor can reach the risk level of the target portfolio earlier by investing in portfolio c instead of the risk-free instrument. In addition, the return at point b is higher than that at point a . Therefore, in this case, investing in c is superior to investing in a risk-free instrument. Proposition 1 states the result.

Proposition 1: If there is a portfolio c that satisfies

$$R_c > R_f \text{ and}$$

$$\sigma_M^2 < \frac{2\alpha\sigma_S \left[M\sigma_p^2 - w_H^2\sigma_H^2 - w_H\sigma_{HL}(1-w_H) + A \right] + 2w_H\sigma_{HM} \left[\sigma_L^2(1-w_H) + w_H\sigma_{HL} + A \right] - w_H A}{\sigma_L^2(1-w_H)^2 - \sigma_p^2 + w_H 2\sigma_{HL}(1-w_H) + w_H\sigma_H^2}$$

where

$$A = \sqrt{\sigma_p^2\sigma_L^2 + w_H^2(\sigma_{HL}^2 - \sigma_H^2\sigma_L^2)},$$

then investing in portfolio c is superior to investing in risk-free assets. R_c and R_f denote the rate of return of portfolio c and risk-free assets respectively. Other variables are defined as before. Please see Appendix A for the detailed proof.

To illustrate Proposition I, we still use the example in Section 3.4. Since the person is assumed to rent an apartment and thus, her real estate assets are zero, $w_H = 0$. The results in Proposition 1 can be simplified as

$$\sigma_M^2 < -\frac{2\alpha\sigma_{SM}(\sigma_L\sigma_p + \sigma_p^2)}{\sigma_L^2 - \sigma_p^2}. \tag{10}$$

If layoff risk is lower than the risk of the target portfolio, $\sigma_L < \sigma_p$, the person can invest in risky assets to achieve higher returns. However, the typical case is that layoff risk is higher than the risk of the target portfolio, $\sigma_L > \sigma_p$. In deciding whether to invest in risk-free assets or a portfolio of risky assets, she has to search for a portfolio that satisfies the condition shown in equation (10):

1. Because $\sigma_L > \sigma_p$, for the right-hand-side of the inequality to be positive, σ_{SM} must be negative. This means the correlation between the firm’s stock and portfolio c is negative.
2. The more negative the correlation, the higher the variance for portfolio c . Or, the more risky portfolio c is.

If the person can find a portfolio that satisfies the above condition, she should invest in a risky portfolio instead of risk-free assets. This path allows her to reach the risk level of the target portfolio earlier than the path by investing in the risk-free assets. Figure 3 illustrates this case.

Figure 3: The Path to Reach Target Portfolio Earlier

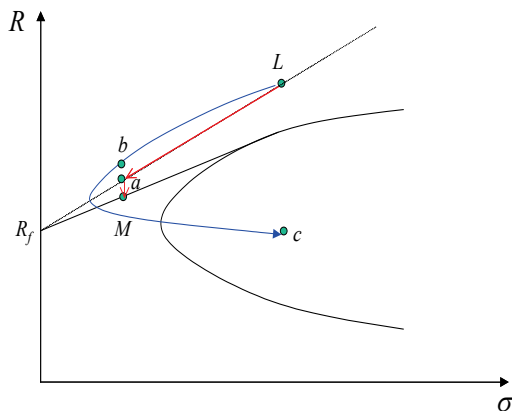


Figure 3 shows a path to reach a person’s target portfolio (M) earlier. Path Lb indicates that the person invests more in stocks to hedge human capital (L) risk because she is less risk averse. At point b , the total risk of her portfolio reaches the risk level of her target portfolio (M). To reach her target portfolio earlier, it is better to invest in portfolio c , rather than in the risk-free assets.

The above example suggests that Proposition 1 provides several intuitive implications. First, the proposition implies that the risk of the mutual fund a person chooses should be negatively correlated with the risk of the stock of the firm hiring her. Since risk of human capital (layoff risk) is often positively correlated with risk of a firm's stock, risk of the mutual fund must be negatively correlated with the risk of human capital. In this way, the risk of human capital can be diversified away by choosing the mutual fund. Second, the proposition provides the same rationale behind the negative correlation between the mutual fund and real estate. Hence, the risk of real estate assets can be hedged by the mutual fund.

Our analysis shows that risk associated with human capital is the most important risk in the early stages of a person's career. Hedging this risk is a key factor in making personal portfolio choices over the life cycle. Our finding thus provides a plausible explanation on the puzzle of household investment behavior that contradicts common wisdom – the holding of stocks decreases with age. The Survey of Consumer Finances by Heaton and Lucas (2000, see Table 1) shows that young people, who are less than 35 years old, have a relatively higher percentage of their investments in cash than middle-aged people, who are between 35 to 64 years old. The stock holding exhibits a humped shape as people age. According to our model, young people hold a large amount of cash to hedge human capital risk. As human capital risk decreases as people age, middle-aged people tend to invest in more risky assets – stocks. However, as people approach retirement, they become more risk averse, thereby reducing the holding of stocks.

CONCLUSIONS

In this paper, we propose a model that incorporates human capital into personal investment portfolio decisions. Human capital is a major source of income for people who do not inherit a large amount of assets. Human capital differs from other assets traded on the market in that the sole owner of human capital is an individual, and human capital cannot be sold or transferred on the market. Thus, individuals have to take the risk associated with human capital by themselves.

Over a life cycle, an individual converts her human capital into income, which supports her consumption and savings. Therefore, part of human capital is gradually transformed into financial assets through personal savings. Most young people starting out on their careers have limited financial assets. Some young people even have negative financial assets, such as education loan liabilities. Their main “asset” is human capital. However, in the early stage of their careers their jobs are less stable than those of older people. As a result, young people hold risk-free assets to hedge against the risk associated with human capital (layoff risk). As time passes, people accumulate more savings, but the present value of the future incomes generated by human capital decreases. People then allocate more financial assets to risky assets to obtain higher returns. As people grow older and head towards retirement, their human capital diminishes and their financial assets are dominant factors in their investment decisions. As predicted by the classical portfolio theory, old people have higher risk aversion to hold more risk-free assets in their portfolios.

Our model thus predicts an investment pattern that is consistent with empirical observation. The model suggests that individuals should take job security into account in making portfolio choices and that financial advisors should tailor their investment advice to different individuals with different levels of job security. Although our model provides insightful implications, it has a few limitations. First, our model does not rigorously consider the variation on risk aversion among different people, which is an important factor in personal portfolio choices. Second, the expected human capital and its expected rate of return are not constant. They are likely affected by various factors, such as the state of the macro economy and policy changes in health insurance. Therefore, stochastic rates of returns (including the one on human capital) are more realistic. These extensions are interesting and worthwhile for future pursuit in this line of research.

APPENDIX

Appendix A: Proof of Proposition 1

The variance of a portfolio that is composed of human capital, real estate, and mutual fund is as follows:

$$\sigma_p^2 = w_L^2 \sigma_L^2 + w_H^2 \sigma_H^2 + w_M^2 \sigma_M^2 + 2w_L w_H \sigma_{LH} + 2w_H w_M \sigma_{HM} + 2\alpha w_L w_M \sigma_{SM}.$$

If investing in portfolio *c* as the mutual fund, we solve for w_M to yield

$$w_{M,c} = \frac{(\sigma_L^2 - \alpha \sigma_{SM})(1 - w_H) + w_H(\sigma_{HM} - \sigma_{HL}) \pm \sqrt{B}}{\sigma_L^2 + \sigma_M^2 - 2\alpha \sigma_{SM}}$$

where

$$B = \left[\sigma_L^2 - \alpha \sigma_{SM}(1 - w_H) - w_H(\sigma_{HM} + \sigma_L^2 - \sigma_{HL}) \right]^2 - (\sigma_L^2 + \sigma_M^2 - 2\alpha \sigma_{SM}) \left[\sigma_L^2(1 - w_H)^2 - \sigma_p^2 + w_H(2\sigma_{HL} + w_H(\sigma_H^2 - 2\sigma_{HL})) \right].$$

The solution with plus sign before \sqrt{B} is not the one we need since we rule out possible negative weight, i.e., shorting an asset. For risk-free asset as the mutual fund, we have

$$\sigma_M^2 = \sigma_{HM} = \sigma_{SM} = 0.$$

Solve for w_M to yield

$$w_{M,R_f} = \frac{\sigma_L^2(1 - w_H) - w_H \sigma_{HL} \pm \sqrt{D}}{\sigma_L^2}$$

where

$$D = \left[\sigma_L^2 - w_H(\sigma_L^2 - \sigma_{HL}) \right]^2 - \sigma_L^2 \left[\sigma_L^2(1 - w_H)^2 - \sigma_p^2 + w_H(2\sigma_{HL} + w_H(\sigma_H^2 - 2\sigma_{HL})) \right].$$

The solution with plus sign before \sqrt{D} is not the one we need. Let

$$w_{M,R_f} > w_M$$

which means that the portfolio needs more investment in risk-free assets than in portfolio *c* to get the same level of total risk. In general, we can solve for σ_M to yield Proposition 1.

By the assumption

$$R_c \geq R_f$$

and by definition

$$R_p = w_L R_L + w_H R_H + w_M R_M$$

for the same weight in mutual fund immediately we have

$$R_{p,c} \geq R_{p,R_f}.$$

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