# WHAT HAS WORKED IN OPERATIONAL RISK?

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## ABSTRACT

*Financial institutions have always been exposed to operational risk – the risk of loss, resulting from inadequate or failed internal processes and information systems, from misconduct by people or from unforeseen external events. Both banking supervision authorities and banking institutions have recently showed their interest in operational risk measurement and management techniques. This newfound prominence is reflected in the Basel II capital accord, including a formal capital charge against operational risk, based on a spectrum of three increasingly sophisticated measurement approaches. The objective of this paper is to increase the level of understanding of operational risk within the financial system, by presenting a review of the literature on the modelling techniques proposed for approach such risk in financial institutions. We perform a comprehensive evaluation of commonly used methods, with a view to compare the performance of different estimators and quantitative estimation methods, for implementation of operational risk measurement. We find that there is currently high variability in the quality and quantity of disclosure on operational risk so, as our conclusion, we try to offer instructive and tractable recommendations for a more effective operational risk measurement.*

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## INTRODUCTION

Operational risk has always existed as one of the core risks in the financial industry. Although there is no agreed upon universal definition of operational risk, the Risk Management Group (RMG) of the Basel Committee have recently developed a standardized definition of operational risk. It is commonly defined as the risk of loss resulting from inadequate or failed internal processes, people, and systems, or from external events (e.g. unforeseen catastrophes). This definition includes legal risk, but excludes strategic and reputational risk (Basel Committee, 2004, 2005 and 2006; Coleman, 1999). While firms in general are beginning to more explicitly discuss the importance of operational risk, the new Basel Capital Accord explicitly requires the financial services industry to manage that risk. Particularly Hiwatashi (2002) argues that banks have already begun to consider operational risk because of advances in information technology, deregulation, and increased international competition. The growth of e-commerce, changes in banks' risks management or the use of more highly automated technology, have led, regulators and the banking industry to recognize the importance of operational risk in shaping the risk profiles of financial institutions. In this paper we discuss operational risk and its applications to financial services firms.

Our main focus is a review of the literature and the issues in this critical area in international corporate finance. It is somewhat ironic that while the major focus of regulators and institutions in the financial services sector over recent years has been on developing models for measuring and managing credit risk, most of the large losses in financial institutions over this time have been sourced to operational risk. Large operational losses as a result of accounting scandals, insider fraud, and rogue trading, to name just a few, have received increasing attention from the press, the public, and from policymakers. Considering the size of these events and their unsettling impact on the financial community, as well as the increase in the sophistication and complexity of banking practices, an effective operational risk management and measurement system, becomes increasingly necessary. In the banking world, large financial institutions have experienced more than 100 operational loss events in excess of $100 million each over the past decade. Rosengren (2002) reports examples of operational risk that have imposed significant costs on firms. First, damage to physical assets and disruption of the business are important considerations, including the $27 billion publicly announced insurance exposure to the 9/11 attack on the World Trade Center. In the same event it is assumed that the loss of Bank of New York totalled $140 million. Second, internal fraud and criminal behaviour also impose costs, such as the losses to Allied Irish banks of $690 million in rogue trading. Third, losses that result from dealings with clients, products, and businesses must also be considered. For examples, he cites the $2 billion settlement of the class action lawsuit by Prudential Insurance caused by its improper sales practices and the $400 million paid by Providian Financial for its unfair sales and collection practices and the $484 million settlement due to misleading sales practices at Household Finance. More the $9 billion loss of Banco National due to credit fraud in 1995, the $2.6 billion loss of Sumimoto Corporation due to unauthorized trading activity in 1996, the $1.7 billion loss and subsequent bankruptcy of Orange County due to unauthorized trading activity in 1998 and the $1.2 billion trading loss by Nick Leeson causing the collapse of Barings Bank in 1995.

A survey of the Basel Committee of 89 banks and one year of data (2001) shows 47000 loss events (relating to operational risk in general) totalling €7.8 billion. In their 2001 Annual Reports, Deutsche Bank and JPMorgan Chase disclosed economic capital of €2.5 billion and $6.8 billion for operational risk, respectively. The loss distribution of operational risk is heavy-tailed, i.e. there is a higher chance of an extreme loss event (with high loss severity) than the asymptotic tail behaviour of standard limit distributions would suggest. The tails of the distribution are of particular interest due to their potentially devastating effects, yet, they are also stochastically hard to get by. The paper is organized as follows. In Section 2 we describe Basel II Background. In Section 3 we provide a short overview of the actual Basel II operational risk (OR) approaches. In the next session reviewing the existing literature we describe some of current practices of ORM (Operational Risk Management), including an analysis of quantitative measurement approaches. In the lasts section, we summarize our findings.

## LITERATURE REVIEW

### Basel II

After more than seven years in the making, the New Basel Capital Accord on global rules and standards of operation for internationally active banks has finally taken effect. The latest revision of the Basel Accord represents the second round of regulatory changes since the original *Basel Accord* of 1988. In a move away from rigid controls, the revised regulatory framework is geared towards achieving a greater sensitivity to risk (both in supervisory authorities as well as in supervised institutions), and to achieve a better link, between the regulatory capital that banks need to retain and the risks that are part of a bank's business. At the end of 2006, the *Basel Committee on Banking Supervision* issued the final draft implementation guidelines for new international capital adequacy rules (*International Convergence of Capital Measurement and Capital Standards* or short "Basel II") to enhance financial stability through the convergence of supervisory regulations governing bank capital. As for credit risk, the Basel Committee does not believe in a "one-size-fits-all" approach to capital adequacy and proposes three distinct options for the calculation of the capital charge for operational risk. The Basel Committee was established by the central-bank Governors of the Group of Ten countries at the end of 1974.

The Committee does not possess any formal supranational supervisory authority, rather, it formulates broad supervisory standards and guidelines and recommends statements of best practice in the expectation that individual authorities will take steps to implement them through detailed arrangements - statutory or otherwise - which are best suited to their own national systems. In this way, the Committee encourages convergence towards common approaches and common standards without attempting detailed harmonisation of member countries' supervisory techniques. After the third and final round of consultations on operational risk, from October 2002 to May 2003, the *Operational Risk Subgroup* (AIGOR) of the *Basel Committee Accord Implementation Group* establishes various schemes for calculating the operational risk charge in a continuum of increasing sophistication and risk sensitivity - *Basic Indicator Approach* (BIA), *Standardized Approach* (TSA), and *Advanced Measurement Approaches* (AMA). A standardised classification matrix of operational risk into eight Business Lines (BLs) and seven Event Types (ETs) has also been defined, in order to encourage greater consistency of loss data collection within and between banks. In other words Basel II capital adequacy approach move from a crude Basic Approach, based on a fixed percentage of Gross Income - the indicator selected by the Committee as a proxy of banks' operational risk exposure - passing through an intermediate Standardised Approach (SA), which extends the Basic method by decomposing banks' activities and, hence, the capital charge computation, into eight underlying business lines, to the most sophisticated approaches, the Advanced Measurement Approaches (AMA), based on the adoption of banks' internal models. BIA requires banks to provision a fixed percentage (15%) of their average gross income over the previous three years for operational risk losses, whereas SA sets regulatory capital to at least the three year average of the summation of different regulatory capital charges (as a percentages of gross income) across BLs in each year.

The most sophisticated AMA approach, allows banks to use their internal loss experience, supplemented with other elements such as the experience of other banks, scenario analysis, and factors reflecting the business environment and the quality of the bank's internal controls, as the basis for estimating their operational risk capital requirements. Banks were allowed to choose a measurement approach appropriate to the nature of banking activity, organizational structure, and business environment subject to the discretion of national banking supervisors, (*supervisory review* - Pillar 2 of Basel II). In the U.S. the implementation of New Basel Capital Accord underscores the particular role of operational risk as part of the new capital rules. On February 28, 2007, the federal bank and thrift regulatory agencies published the *Proposed Supervisory Guidance for Internal Ratings-based Systems for Credit Risk, Advanced Measurement Approaches for Operational Risk, and the Supervisory Review Process (Pillar 2) Related to Basel II Implementation* (based on a previous advanced notices on proposed rule-making in 2003 and 2006).

These supervisory implementation guidelines permit qualifying banking organizations to adopt *Advanced Measurement Approaches* (AMA) for operational risk as the only acceptable method of estimating capital charges for operational risk for a certain class of financial institutions. So for the most part of institutions worldwide, operational risk, in addition to credit and market risk, is a determinant of minimum capital requirements. About capital adequacy ratio, the *minimum* amount of capital that regulators require a bank to hold, both under Basel I and Basel II amount to 8% of risk weighted assets. What has changed under Basel II, basically, is the way how this 8% are derived. The calculation of the ratio is now more risk sensitive and takes into account the increased sophistication of banking business and in particular best practices developed over time in the banking industry. Consequence of the foregoing statement, this calculation includes, as a new element in the formula to arrive at 8%, an explicit charge for operational risk. Thus, there are now three areas of risk that are related to the minimum capital requirement – 1) credit risk (which was the focus of the original 1988 Accord), 2) market risk of trading activities (which was introduced in a 1996 amendment to the Accord) and 3) operational risk.

$$\frac{Tier1+Tier2+Tier3}{CR+12.5(MR)+12.5(OR)} =\geq 8\% \qquad (1)$$

Too little capital puts banks at risk, while too much capital prevents banks from achieving the required rate of return on capital.

Basel II and OR Approaches

The BIA is the simplest approach and can be applied by all banks that either do not qualify for or are not obliged by their regulator to use one of the more sophisticated approaches. In the BIA, operational risk capital is calculated as a fixed percentage of a financial institution's annual three year average positive Gross Income (*GI)*:

$$K_{BIA} = [\sum(GI_{1\ldots n} * \alpha)]/n \quad (2)$$

whereby $GI_{1..n}$ denominates the amount of GI in those years over the three year horizon, in which the financial institution's GI was positive and α denominates the scaling factor, which is currently set at 15% (BCBS, 2006). The Standardised Approach (SA) is relatively more advance compared to the Basic Indicator Approach (BIA). The Standardised Approach (SA) is better able to reflect the differing risk profiles across bank business activities. A financial institution that uses the SA is required to map its overall annual GI into eight business lines. The BCBS identifies the following business lines and their respective betas (2006) (Table 1).

Table 1: Business Lines and Betas Factors

| Business Lines | Beta Factors |
|---|---|
| Corporate finance ($\beta_1$) | 18% |
| Trading and sales ($\beta_2$) | 18% |
| Retail banking ($\beta_3$) | 12% |
| Commercial banking ($\beta_4$) | 15% |
| Payment and settlement ($\beta_5$) | 18% |
| Agency services ($\beta_6$) | 15% |
| Asset management ($\beta_7$) | 12% |
| Retail Brokerage ($\beta_8$) | 12% |

*Table 1 shows business lines and betas*

Every business line has its own beta to indicate embedded risk. A financial institution's total operational risk capital is calculated as the sum of operational risk capital calculated for each business line.

$$K_{tsa} = \{\sum_{years\ 1-3} max[\sum(GI_{1-8} * \beta_{1-8}), 0]\}/3 \quad (3)$$

A financial institution's total operational risk capital is then the sum of operational risk capital calculated for each business line. As it is well-known this methodology assumes implicitly that aggregate losses are perfectly correlated. AMA banks use internal risk measurement systems and rely on self-assessment via scenario analysis to calculate regulatory capital that cover their total operation risk exposure (both EL – Expected Loss and UL – Unexpected Loss) over a one-year holding period at a 99.9% statistical confidence level. Although the application of AMA is in principle open to any proprietary model, the most popular methodology is by far the Loss Distribution Approach (LDA). *Loss Distribution Approach* (LDA) is based on an annual distribution of the number and the total loss amount of operational risk events and an aggregate loss distribution, by modelling the loss severity and loss frequency separately and then combining them via a Monte Carlo simulation or other statistical technique to form an aggregate loss distribution (see e.g. Frachot *et al.*, 2001, or Cruz, 2002). Under the Loss Distribution Approach, the bank estimates, for each business line/risk type cell, the probability distribution functions of the single event impact and the event frequency for the next (one) year using its internal data, and computes the probability distribution function of the cumulative operational loss. Following the usual LDA methodology, the aggregate loss is naturally defined as a random sum of individual losses:

$$L = \sum_{n=1}^{N} X_n = X_1 + \cdots . + X_n \quad (4)$$

where *L* is the aggregate loss, *N* is the annual number of losses (i.e. frequency of events) and *Xn* are loss amounts (i.e. severity of event). Accordingly aggregate losses result from two distinct sources of randomness (i.e. frequency and severity) which both have to be modelled. In essence the LDA model assumes the three following assumptions within each class of risk:

i. N and $(X_1, X_2 \dots \dots)$ are independent random variables.
ii. $X_1, X_2$,...is a set of independent random variables,
iii. and $X_1, X_2$ follow the same marginal distribution.

The first assumption means that frequency and severity are two independent sources of randomness. Assumptions 2 and 3 mean that two different losses within the same homogeneous class are independent and identically distributed. Modelling the severity usually involves the application of parametric distributions such as the lognormal, Weibull, Pareto distributions, Lognormal-gamma, Exponential, Gamma or Loglogistic. Meanwhile, the frequency distribution is commonly modelled by Poisson, Binomial, and Negative Binomial distributions (de Fontnouvelle, Rosengren, and Jordan (2004) and Dutta and Perry (2006)).

(i) The Log-normal distribution:

$$f(x_i; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(x)-\mu}{\sigma}\right)^2} \qquad \forall\, x \in R_+, \sigma > 0, \mu \geq 0 \tag{5}$$

(ii) The Pareto distribution:

$$f(x_i; a, b) = \frac{ab^a}{x^{a+1}} \qquad \forall\, x \geq b, a > 0, b > 0$$
(6)

(iii) The Weibull distribution:

$$f(x_i; \alpha, \beta) = \frac{\alpha}{\beta} \times \left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \qquad \forall\, x \in R_+, \beta > 0, \alpha \geq 0 \tag{7}$$

(iv) The Exponential distribution:

$$h(x) = \frac{1}{\lambda} \exp\left(-\frac{x}{\lambda}\right) I_{[o,\infty)}(x) \tag{8}$$

where the scale parameter $\lambda > 0$. The exponential distribution is a one parameter distribution used to model process with a constant time to failure per unit of time. The distribution is memoryless in that $P(X > s + t | X > t) = P(X > s)$ for all s,t $\geq 0$.

(v) The Gamma distribution:

$$h(x) = \frac{1}{\lambda^{\alpha} \beth(\alpha)} x^{\alpha-1} \exp\left(\frac{-x}{\lambda}\right) I_{[o,\infty)}(x) \tag{9}$$

where $\alpha$ and $\lambda$ are positive, and $\beth(\alpha)$=$\int_0^{\infty} t^{\alpha-1} e^{-t} dt$ denotes the gamma function. It can be shown that if $\{X_x\}$ are a sequence of independent exponentially distributed random variables with common parameter , then Y= $\sum_{t=1}^{n} X_t$ is distributed with $\alpha = n$ and common parameter $\lambda$. The exponential distribution is a

special case of the gamma function for $\alpha = 1$ The chi-square distribution with k degrees of freedom is also a special case of the gamma distribution for $\alpha = 2k$ and $\lambda = 2$.
(vi) The Loglogistic distribution:

$$h(x) = \frac{\eta(x-\alpha)^{n-1}}{[1+(x-\alpha)^{\eta}]^2} I_{[o,\infty)}(x) \tag{10}$$

Also sophisticated semiparametric distributions have been proposed. The generalized Champernowne distribution (GCD) is described in Champernowne (1936 and 1952) developed by Buch-Larsen, Nielsen, Guillen and Bolance (2005) in their semiparametric approach to better curve fitting in LDA. Use of the GCD coupled with a transformation approach can be found in papers by Gustafsson, Nielsen, Pritchard and Roberts (2006), Buch-Larsen (2006), Guillen, Gustafsson, Nielsen and Pritchard (2007), Clements, Hurn and Lindsay (2003), Buch-Kromann, Englund, Gustafsson, Nielsen and Thuring (2007) and Gustafsson and Nielsen (2008).

(vii) The GCD distribution:

$$f(x;\alpha,M,c) = \frac{\alpha(x+c)^{\alpha-1}((M+c)^{\alpha}-c^{\alpha})}{((x+c)^{\alpha}+(M+c)^{\alpha}-2c^{\alpha})^2} \forall\, x \in R_{+}, \alpha > 0, M > 0, c \geq 0 \tag{11}$$

The historical experience of operational risk events suggests a heavy-tailed loss distribution, whose shape reflects highly predictable, small loss events left of the mean with cumulative density of EL. As loss severity increases, higher percentiles indicate a lower probability of extreme observations with high loss severity, which constitute UL. While banks should generate enough expected revenues to support a net margin after accounting for the *expected* component of operational risk from predictable internal failures (EL), they also need to provision sufficient economic capital to cover the *unexpected* component (UL). If we define the distribution of operational risk losses as an intensity process of time *t*, the cumulative distribution function of EL reflects a high expected conditional probability of small losses over time horizon *T*, so that

$$EL(T-t) = E[P(T) - P(t) | P(T) - P(t) < 0] \tag{12}$$

UL captures losses larger than EL below a tail cut off (or threshold value) E$[P_{\alpha}(T) - P(t)]$.beyond which residual losses occur at a probability of α or less. The specification of UL (less EL) concurs with the concept of *Value-at-Risk* (VaR), which estimates the maximum loss exposure at a certain probability bound for a given aggregate loss distribution. Thus, we can write

$$UL(T-t) = VaR_{\alpha}(T-t) - EL(T-t) \tag{13}$$

UL estimates are more sensitive to the shape of the loss distribution than EL, due to the low probability of extreme losses. Losses in excess of UL are commonly denoted as extreme losses with cumulative density $1 - VaR_{\alpha}(T-t)$ which is also frequently referred to as residual risk “in the tail”.

The regulatory capital requirement (or Capital-at-Risk) is the sum of expected loss (EL) and unexpected loss (UL) for a one year holding period and a 99.9 percent confidence interval. In other words according to the Committee the bank must be able to demonstrate that the risk measure used for regulatory capital purposes reflects a holding period of one-year and a confidence level of 99.9 percent. The Committee proposes to define the Capital-at-Risk (CaR) as the “unexpected loss", given by:

$$CaR_1(\alpha) = inf\{x \in R | F(x) \geq \alpha\} - \int_0^{\infty} xf(x)\,dx \tag{14}$$

The total loss $L$ of the bank is then the sum of aggregate losses for each business line x loss type class. Let $H$ be the number of classes (where $H$ = 7x8 in the Basel II context). Therefore:

$$L = \sum_{h=1}^{H} L_h \tag{15}$$

where $L_h$ is the aggregate loss corresponding to the $h$ class.
The rare incidence of severe operational risk losses, however, does not mesh easily with the distributional assumptions of conventional VaR. The fat-tailed behaviour of operational risk defies statistical inference that characterizes loss severity, therefore conventional VaR is a rather ill-suited concept for risk estimation and warrants adjustments that explicitly account for extremes at high percentiles. The development of internal risk measurement models has led to a spread consensus that generalized parametric distributions, such as the g-and-h distribution or various limit distributions under *extreme value theory* (EVT), can be applied to satisfy the quantitative AMA standards for modelling the fat-tailed behaviour of operational risk under LDA (see Embrecht, Kiauppelberg, and Mikosch (1999) for a detailed mathematical treatment, also Reiss and Thomas (2001), Vandewalle et al. (2004), Stephenson (2002), and Coles et al. (1999) for additional information on the definition of EVT). EVT is a particularly appealing statistical concept to help improve LDA under AMA, because it delivers a closed form solution of operational risk estimates at very high confidence levels without imposing additional modelling restrictions if certain assumptions about the underlying loss data hold.

The multivariate extreme value distribution can be written as $G(x) = exp\left\{-(\sum_{i=1}^{n} y_i)A\left(\frac{\frac{y_1}{\sum_{i=1}^{n} y_{1,\dots,}y_n}}{\sum_{i=1}^{n} y_i}\right)\right\}$ for $x = (x_1, \dots, x_n)$ where the $i$-th univariate marginal distribution $y_i = y_i(x_i) - \left(1 + \xi_i \frac{(x-\mu_i)}{\sigma}\right)^{-\frac{1}{\xi_i}}$ is generalized extreme value, with $1 + \xi_i \frac{(x-\mu_i)}{\sigma} > 0$, scale parameter $\sigma_i > 0$ location parameter $\mu_i$, and shape parameter $\xi_i$. If $\xi_i = 0$ (*Gumbel* distribution), then *yi* is defined by continuity. The dependence function *A(.)* is defined on simplex $S_n = \{\omega \epsilon R_+^n : \sum_{i=1}^{n} \omega_i\}$ with $0 \leq \max(\omega_{1,} \dots, \omega_n) \leq A(\omega) \leq 1$ for all $\omega \;\; = (\omega_1 \dots, \omega_n)$.

GEV and GPD are the most prominent parametric methods for the statistical estimation of the limiting behaviour of extreme observations under EVT. GPD is an exceedance function that measures the residual risk of a sequence of extremes beyond a predefined threshold for regions of interest, where only a few or no observations are available (Vandewalle et al., 2004). Balkema and de Haan (1974) and Pickands (1975) state that, for a broad class of distributions, the values of the random variables above a sufficiently high threshold $U$ follow a Generalized Pareto Distribution (GPD) with parameters $x$ (the tail index) and $b$ (the scale index, which is a function of $U$). The GPD can thus be thought of as the conditional distribution of $X$ given $X > U$ (see Embrechts *et al.*, 1997, for a comprehensive review). Its probability distribution function (pdf) can be expressed as:

$$F(y; \xi, \beta) = 1 - \left(1 + \frac{\xi}{\beta} y\right)^{\frac{-1}{\xi}} \tag{16}$$

where the "threshold excess" $y$ is simply the difference $x$-$U$.

The Weibull, Gumbel and Frechet distributions can be represented in a single three parameter model, known as the Generalised Extreme Value distribution (GEV). GEV identifies the possible limiting laws of the asymptotic tail behaviour associated with the order statistics of i.i.d. normalized extremes drawn from a sample of dependent random variables. Its pdf can be expressed as:

$$G(\xi,\alpha,\sigma)\begin{cases}\exp\left(-\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\xi}}\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)>0,\xi\neq 0\right)\\ \exp\left(-\exp-\left(\frac{x-\mu}{\sigma}\right)\right)\quad x\in R,\xi=0\end{cases} \tag{17}$$

The *Peak-over-Threshold* (POT) method is the most popular technique to parametrically fit GPD based on the specification of a threshold, which determines how many exceedance shall be permitted to establish convergence of asymptotic tail behaviour between GEV and GPD. Alternately Degen et al. (2006) proffer the *g-and-h* distribution as another generalized parametric model to estimate the residual risk of extreme losses. The g-and-h family of distributions was first introduced by Tukey (1977) and represents a strictly monotonically increasing transformation of a standard normal variable Martinez and Iglewicz (1984) show that the g-and-h distribution can approximate probabilistically the shapes of a wide variety of different data and distributions.

The loss distribution for a certain loss type is characterized by frequency and severity. The frequency distribution describes the number of losses up to time *t* and is represented by a counting process *N*(*t*). The most popular distribution is the Poisson distribution. In the simplest case the aggregate loss up to time *t* simply follows a compound Poisson process of the form:

$$Y_t(x)=\sum_{t=1}^{L}\quad\sum_{r=1}^{N_l(t)}x_{\tau,r} \tag{18}$$

and is generated by adding up severities $x_{\tau,r}$ of all loss types $l=\{1,\dots,L\}$ over time $\tau$ up to *t*.

## WHAT HAS WORKED AT ALL

Institutions face many modelling choices as they attempt to measure operational risk exposure. In order to understand the inherent nature and exposure of operational risk that a financial institution faces, we analyze various approaches that could be used to measure operational risk under the *Loss Distribution Approach* (*LSA*). The LDA has three essential components, a distribution of the annual number of losses (frequency), a distribution of the amount of losses (severity), and an aggregate loss distribution that combines the two.

*Frequency distribution and aggregate loss distribution:* For short periods of time, the frequency of losses is often modelled either by a homogenous Poisson or by a (negative) binomial distribution. The choice between these distributions may appear important, as the intensity parameter is deterministic in the first case and stochastic in the second (see Embrechts *et al.*, 2003). However, as the prudential requirement involves measuring the 99.9% OpVaR over a yearly period, this issue is only marginally relevant: Chapelle et al. (2005) evidence suggests that the mere calibration of a Poisson distribution with constant parameter *l* corresponding to the average number of observed losses during a full year provides a very good approximation of the true frequency distribution.

*Modelling Severity:* One of the most significant choices is which technique to use for modelling the severity of operational losses. There are many techniques being used in practice, and for policy makers an important question is whether institutions using different severity modelling techniques can arrive at very different (and inconsistent) estimates of their exposure. There is no commonly agreed-upon definition of what constitutes a heavy-tailed distribution. However, one such definition can be based upon a distribution's maximal moment, which is defined as *sup* $\{r : \mathrm{E}(x_r) < \infty\}$. Therefore , the majority of the distributions used in finance and actuarial sciences can be divided into these three classes, according to their tail-heaviness: first, light-tail distributions with finite moments and tails, converging to the Weibull curve (Beta, Weibull); Second, medium-tail distributions for which all moments are finite and whose cumulative distribution functions decline exponentially in the tails, like the Gumbel curve (Normal, Gamma, LogNormal); third, heavy-tail distributions, whose cumulative distribution functions decline with a power in the tails, like the Frechet curve (T-Student, Pareto, LogGamma, Cauchy).

To model the severity distribution, K. Dutta and J. Perry (2006) review two different techniques: parametric distribution fitting and Extreme Value Theory (EVT). In parametric distribution fitting, the data are assumed to follow some specific parametric model, and the parameters are chosen (estimated) such that the model fits the underlying distribution of the data in some optimal way. EVT is a branch of statistics concerned with the study of extreme phenomena such as large operational losses. Jobst (2007) parametric risk estimates of i.i.d. normalized maxima at the required 99.9th percentile implied capital savings of up to almost 97% compared to a uniform measure of operational risk exposure.

According to P. de Fontnouvelle et al. (2004) loss data for most business lines and event types may be well modelled by a Pareto-type distribution, as most of the tail plots are linear when viewed on a log-log scale. Second, the severity ranking of event types is consistent across institutions. Clients, Products and Business Practices is the highest severity event type, while External Fraud and Employment Practices are the lowest severity event types.

It is commonly accepted that lognormal and Weibull distributions fit operational loss data reasonably well over a large part of the distribution but can diverge in the tail due to underestimation of large sized losses. Conversely applying a Pareto distribution to the data gives a good fit to the tail (where there is sufficient data to allow this judgement) but a less good fit elsewhere. The ideal which we would seek is therefore to choose a distribution that performs well in the tail but also uses some of the better quality information available at smaller loss values to inform tail behaviour. J. Gustafsson, et al. (2008) aim to show that the GCD has the potential to be a good estimator across the full dataset. Chapelle et al. (2005) establish that the Generalized Champernowne Distribution (GCD) demonstrates a great flexibility and is therefore an appropriate choice for the severity side in LDA on operational risk data. The reason for investigating is that the GCD has an interior maximum that resembles a lognormal distribution and converges asymptotically to a Pareto distribution for extreme losses. This is a favourable feature when modelling operational losses. In the papers by Buch-Larsen et al (2005), Gustafsson et al (2006) and Guillen et al (2007) it is assumed that this distribution is more flexible and therefore more appropriate than the common lognormal or Weibull distributions.

Gustafsson, et al. (2008) considers the question of the appropriate severity distribution estimators for Loss Distribution Analysis (LDA) of operational risk data. They compare the performance of four severity distributions. The capital requirements when using the GCD (both for VaR 99.5% and TVaR 99.5%) is right between the capital requirements when using the light tailed distributions (lognormal and Weibull) and heavy tailed Pareto. This leads authors to conclude that the GCD is suitable for use in LDA, its three parameter configuration making it more flexible than other estimators in this study and therefore better at capturing the whole of the data generating distribution.

Jobst (2007) identified GEV, GPD, and the g-and-h distribution as feasible measurement approaches to assess the generalized parametric specification of the fat-tailed limiting behaviour commonly associated with large operational risk losses. In their effort to derive a consistent measure of operational risk across several U.S. banks, Dutta and Perry (2006) find that GPD tends to overestimate UL in small samples, contending its adequacy as a general benchmark model. To evaluate how well the model fits the observed loss data, J.M. Netter and A.B. Poulsen (2010) calculate Quantile-Quantile plots for both the OpRisk Analytics and OpVantage databases. These plots compare the predicted quantiles of the fitted loss distributions with the actual quantiles of the empirical loss distributions. The fit of both Quantile-Quantile plots does deteriorate towards the tail of the loss distribution. Overall, the results based on U.S. data indicate that the logit-GPD model provides a good estimate of the severity of the loss data in external databases. In addition, the estimated loss severity is quite similar for the two databases examined.

Jobst (2007) found that AMA-compliant risk estimates of operational risk under both EVT and the g-and-h distribution generated reliable and realistic estimates of UL. More, in a simulation study of generic operational risk based on the aggregate statistics of operational risk exposure of U.S. banks, both GPD and GHD generate reliable and realistic AMA-compliant risk estimates of UL. In the effort to curb parameter uncertainty of GPD, they introduced the concept of the “threshold-quantile surface” as an integrated approach to illustrate the contemporaneous effect of the threshold choice, the estimation method, and the desired statistical confidence on the accuracy of point estimates and upper tail fit. Author found that the selection of the right percentile level rather than the threshold choice seemed to matter most for robust point estimates of aggregate operational risk. Estimation uncertainty increased significantly at high levels of statistical confidence beyond the 99.7th percentile or threshold quantiles that classified less than 0.5% of all losses as exceedances for the parametric GPD-based upper tail fit. More the GHD distribution outperformed both GEV and GPD in terms of the goodness of upper tail fit. In fact, the g-and-h distribution *underestimated* actual losses in all but the most extreme quantiles of 99.95% and higher, when EVT-based estimates *overstated* excess elongations of asymptotic tail decay. Authors' findings suggest a symbiotic association between EVT and the g-and-h distribution for optimal point estimation depending on the percentile level and the incidence of extreme events. Moreover parametric risk estimates of i.i.d. normalized maxima at the required 99.9th percentile implied capital savings of up to almost 97% compared to an uniform measure of operational risk exposure.

De Fontnouvelle et al. (2004) fit a set of distributions to the LDCE (*Loss Data Collection Exercise*) data via Maximum Likelihood. In general, the heavy-tailed distributions (Burr, LogGamma, LogLogistic, Pareto) seem to fit the data quite well. The reported probability values exceed 5% for many business lines and event types, which suggests that we cannot reject the null that data are in fact drawn from the distribution under test. Moscadelli (2004) shows that the Extreme Value model, in its severity representation (Peaks Over Threshold-Generalised Pareto Distribution, POT-GPD), provides an accurate estimate of the actual tail of the BLs at the 95th and higher percentiles; this is confirmed by the results of three goodness-of-fit tests and a severity VaR performance analysis. In light of its supremacy in the estimate of the loss tail-severity distribution, the Extreme Value model, in its Peaks Over Threshold - Point Process representation (POT-PP), is also used to estimate the loss tail-frequency distribution, that is to derive the probability of occurrence of the large losses in each BL. Owing to the higher frequency of losses, *Retail Banking* and *Commercial Banking* are the BLs which absorb the majority of the overall capital requirement (about 20 per cent each), while *Corporate Finance* and *Trading & Sales* are at an intermediate level (respectively close to 13 per cent and 17 per cent) and the other BLs stay stably under 10 per cent. Moreover, the results show the very small contribution of the expected losses to the total capital charge: on average across the BLs, they amount to less than 3 per cent of the overall capital figure for an international active bank, with a minimum value of 1.1 per cent in *Corporate Finance* and a maximum of 4.4 per cent in *Retail Banking*. Moreover, one of the main remarks coming out of this paper is that, if the aim of the analysis is to estimate the extreme percentiles of the aggregated losses, the treatment of these two components within a single overall estimation problem may reduce the estimate error and the computational costs. As the paper makes clear, the EVT analysis requires that specific conditions be fulfilled in order to be worked out, the most important of which are the i.i.d. assumptions for the data.

Also Hübnera, et al. (2005) find a reasonable statistical fit using the EVT POT method for most of the institutions. However they show that good fit does not necessarily mean a distribution would yield a reasonable capital estimate. This issue is especially of concern for the EVT POT approach, which gave the most unreasonable capital estimates with the most variation of all of the methods across the enterprise, business line, and event type levels. Also, the capital estimates for these institutions are highly sensitive to the threshold choice. With respect to the capital estimates at the enterprise level, only the g-and-h distribution resulted in realistic, consistent and least varying capital estimates across institutions at the enterprise, business line, and event type levels. In the paper it shows that the g-and-h distribution results in a meaningful operational risk measure in that it fits the data and results in consistently reasonable capital estimates. More, in spite of many researchers have conjectured that one may not be able to find a single distribution that will fit both the body and the tail of the data to model operational loss severity; the g-and-h distribution imply that at least one single distribution can indeed model operational loss severity without trimming or truncating the data in an arbitrary or subjective manner.

<u>OR Approaches</u>

Jobst (2007) evidence from a cursory examination of balance sheet data of U.S. commercial banks suggests a significant reduction of economic capital from AMA-based self-assessment of operational risk. The standardized measure of 15% of gross income under BIA and TSA of the New Basel Capital Accord would result in a capital charge that grossly overstates the economic impact of even the most extreme operational risk events in the past, such as the physical damage to assets suffered by the Bank of New York in the wake of the 9/11 terrorist attacks.
Sundmacher (2004) show that a financial institution that initially uses the BIA might only marginally benefit from moving to the next higher approach, the TSA. The benefits accruing from a lower capital charge might be offset by the compliance costs associated with the fulfilment of Basel's qualifying criteria for the TSA. Further, the capital-saving in the TSA compared to the BIA will be highly dependent on the business units in which the financial institution generates the bulk of its Gross Income.

The objective of Mongid's paper (2009) is to estimate operational risk capital charge using historical data for 77 rural banks in Indonesia for a three-year period, 2006 to 2008. The study uses three approaches: (i) Basic Indicator Approach (BIA), (ii) Standardized Approach (SA) and (iii) Alternative Standardized Approach (ASA). He found that the average capital charge required to cover operational risk is IDR 154 million (1.5% of asset). When the calculation is conducted using the SA method, he found, on average a requirement of IDR 123 million (1.23% of asset). When the calculation is conducted using the Alternative Standardized Approach (ASA), the capital required was IDR 43 million (0.43% of asset).

A result from the work of Ebnother et al. (2001) is that only a fraction of processes needs to be defined to measure operational risk to a high level of accuracy. Hence, the costs for doing the necessary work to measure operational risk can be significantly reduced if one first concentrates on selecting the important processes. From a practitioners point of view an important insight is that not all processes in an organization need to be equally considered for the purpose of defining accurately the operational risk exposure. Management of operational risks can focus on key issues; a selection of the relevant processes reduces significantly the costs of defining and designing the workflow items (in Ebnother example, out of 103 processes only 11 are needed to estimate the risk figures at a 90 percent level of accuracy). Second, although six risk factors were considered, only 2 of them seem to really matter. Following a similar approach Ebnother et al. (2002) find that 10 processes lead to a VaR of 98% of the VaR calculated from all processes.

<u>Correlation</u>

Sundmacher (2004) empirical findings show that the correlation between two aggregate losses is typically below 5%, which opens a wide scope for large diversification effects, much larger than those the Basel Committee seems to have in mind. In other words, summing up capital charges is in substantial contradiction with the type of correlation consistent with the standard LDA model. It would require allowing frequency and severity to be correlated with one another and within a risk type and business line class, which is a clear departure from the standard LDA model. Author finally proposes the following simplified formula, for the global capital charge:

$$K = EL + \sqrt{\sum_{i,j=1}^{H} \rho_{i,j}(K_i - EL_i) \times (K_i - EL_i)} \tag{19}$$

However, even though this kind of correlation between frequency and severity can make sense in practice, this cannot be done but at the expense of model tractability, and the extended model thus obtained is far out of reach of what current databases and state-of-the-art technology can cope with.
Dependence betweens risks can be modelled either as correlation between frequencies of loss events, or between their severities, or between aggregate annual losses. Frachot et al. (2004) explain that this dependence can be adequately captured in the LDA framework by the frequency correlations, but not by the severity correlations (see also Frachot et al. (2003) for a discussion of this topic). Brandts (2004) directly model the dependence of aggregate losses and propose to use *copulas* in order to combine the marginal distributions of different risk categories into a single joint distribution (see e.g. Genest and McKay (1986) or Nelsen (1999) for an introduction to copulas). In its work he tested 4 families of copulas.

1. The Gaussian copula is naturally related to the Normal distribution. It is expressed as:

$$C_{NORMAL}(u,v) = \Phi_{\rho}\left(\Phi^{-1}(\mathrm{u}), \Phi^{-1}(\mathrm{v})\right) \tag{20}$$

Where $\Phi_{\rho}$ is the bivariate Normal distribution with correlation ρ and Φ is the standard Normal distribution. So, when the marginals are Gaussinan, it produces the multivariate Normal

2. Frank's copula (Frank (1979)) depicts a symmetrical dependence structure. It is expressed as:

$$C_{FRANK}(u,v) = -\frac{1}{\alpha} ln\left(1 + \frac{(\exp(-\alpha u)-1)(\exp(-\alpha v)-1)}{(\exp(-\alpha)-1)}\right), \alpha \neq 0 \tag{21}$$

3. Clayton's copula (Clayton (1978)) models the lower tail dependence. It is given by:

$$C_{CLATTON}(u,v) = max\left([u^{-\alpha} + uv^{-\alpha} - 1]^{\frac{1}{\alpha}}, 0\right), \alpha\epsilon[-1,\infty[\backslash\{0\} \tag{22}$$

4. The Gumbel-Hougaard copula (Gumbel (1960) and Hougaard (1986)) focuses on the upper tail dependence. The bivariate version of this copula has the form:

$$C_{0-h}(u,v) = exp\left(-\left[(-ln\,u)^{\alpha}(-ln\,u)^{\frac{1}{\alpha}}\right]\right), \alpha\epsilon[1,\infty[ \tag{23}$$

In Brandts' study, the difference between various copulas is not very significant, probably because of the very low dependence between the business lines under consideration. G.Hübner et al. (2005), aggregated business line (and event types) capital estimates for the g-and-h distribution in two different ways: assuming zero correlation (independence) and comonotonicity (simple sum of individual numbers). They observed that the differences between these two numbers are much smaller than we expected. Also, the diversification benefit of using comonotonicity at the enterprise level was not unreasonably high for the g-and-h distribution. The diversification benefit is much smaller for the summation of capital estimates from event types than from business lines.

Estimation Methods

The MLE method is arguably the most frequently used estimation method in current operational risk capital quantification practice (de Fontnouvelle, Rosengren, and Jordan (2004)). The MLE assigns weights to the observations according to their likelihood. Because of that, the most of the weight gets concentrated in the body of the loss distribution resulting in a poor fitting of the distribution' right tail where the likelihood values are small. The accuracy of the estimates could be improved by exploring alternative estimation methods.

B. Ergashev (2008) compares the performance of four estimation methods, maximum likelihood estimation included, that can be used in fitting operational risk models to historically available loss data. The other competing methods are based on minimizing different types of measure of the distance between empirical and fitting loss distributions. These measures are the Cramer-von Mises statistic, the Anderson-Darling statistic, and a measure of the distance between the quantiles of empirical and fitting distributions. Authors call the last method the quantile distance method. The likelihood statistic is defined as:

$$L(X_\tau|\theta) = \prod_{i=1}^{n} \quad \prod_{j=1}^{N_i(\tau)} f(logX_{i,j}\theta, \tau) \tag{24}$$

The Cramer-Von-Mises statistic is defined as:

$$W^2(\theta) = N(\tau)\int_{-\infty}^{+\infty}[F(x|\tau) - F(x|\theta,\tau)]^2 dF(x|\theta,\tau) \tag{25}$$

This statistic is a measure of “closeness” of the empirical and fitting distributions to each other.
The Anderson-Darling (AD) is another measure of closeness of two distributions. In contrary to the Cramer-Von Mises statistic, this statistic gives more weight to the distance between the tails of the distributions. The AD statistic is defined as:

$$A^2(\theta) = N(\tau)\int_{-\infty}^{+\infty}\frac{[F(x|\tau)-F(x|\theta,\tau)]^2}{F(x|\theta,\tau)\big(1-F(x|\theta,\tau)\big)} dF(x|\theta,\tau) \tag{26}$$

the Quantile Distance (QD) method is based on finding the parameter estimates that minimize the weighted sum of squares of the difference between a set of $k$ quantiles of the two distributions corresponding to the cdf values of $0<p_1<...p_k<1$ .This sum can be defined as:

$$Q^2(\theta,p,\omega) = \sum_{i=1}^{k}\omega_i\,[\widehat{q_i} - q(\theta,p)]^2 \tag{27}$$

Where p=$(p_{1,} \dots \dots, p_k)$ are the quantile levels, $\omega = (\omega_1, \dots . \omega_k)$ are the weights, and

$$\widehat{q_i} = y_{[n\times p_i]}, q_i(\theta,p) = F^{-1}(p_i|\theta,\tau), i = 1 \dots . . k \tag{28}$$

are the quantiles of the empirical and fitting distributions.

Ergashev’s simulation exercise shows that the quantile distance method is superior to the other three methods especially when loss data sets are relatively small and/or the fitting model is unspecified.

**CONCLUSION**

Although the application of AMA is in principle open to any proprietary model, the most popular methodology is by far the Loss Distribution Approach (LDA). It is commonly accepted that light-tail distributions fit operational loss data reasonably well over a large part of the distribution but can diverge in the tail due to underestimation of large sized losses. Conversely applying a heavy-tail distributions to the data gives a good fit to the tail (where there is sufficient data to allow this judgement) but a less good fit elsewhere. The ideal which we would seek is therefore to choose a distribution that performs well in the tail but also uses some of the better quality information available at smaller loss values to inform tail behaviour. There is a spread consensus that generalized parametric distributions, such as the g-and-h distribution or various limit distributions under *extreme value theory* (EVT), as GPD and GCD, can be applied to satisfy the quantitative AMA standards for modelling the fat-tailed behaviour of operational risk under LDA.

More, in spite of many researchers have conjectured that one may not be able to find a single distribution that will fit both the body and the tail of the data to model operational loss severity; the g-and-h distribution and Peaks Over Threshold - Point Process representation (POT-PP), imply that one single distribution can indeed model operational loss severity. For what concern estimation methods, while the MLE method is arguably the most frequently used estimation method in current operational risk capital quantification practice, B. Ergashev (2008) comparing the performance of four estimation methods, shows that the quantile distance method is superior on average. Moreover some authors show that only a fraction of processes needs to be defined to measure operational risk to a high level of accuracy. Hence, the costs for doing the necessary work to measure operational risk can be significantly reduced if one first concentrates on selecting the important processes. While other find that the correlation structure between aggregate losses opens a wide scope for large diversification effects, much larger than those the Basel Committee seems to have in mind. We believe that this study contributes to a better understanding of operational risk management, trying to offer instructive and tractable recommendations for a more effective operational risk measurement.

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