EVALUATION OF MULTI-ASSET VALUE AT RISK: EVIDENCE FROM TAIWAN

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ABSTRACT

Under the internal model approach (IMA) stipulated by Basel II, financial institutions are allowed to develop and employ proprietary internal models to evaluate various risk. However, the flexibility to develop a proprietary model leads to the question of which computing method delivers the most accurate and reliable estimates of value at risk (VaR). This research employs the new backtesting method proposed by Pérignon and Smith (2008) to determine the best method for computing integrated value at risk. It tests three major VaR computation methods — historical simulation, Monte Carlo simulation, and variance-covariance methods. The portfolio on which VaR is computed includes equities, government bonds, foreign exchange, and index options, all of which are commonly traded by financial institutions. The empirical analysis indicates that historical simulation is the best VaR computation method, which is consistent with the result of Pérignon and Smith (2008).

JEL: G11; G28; G32

KEYWORDS: Value-at-Risk (VaR), Backtesting, Unconditional Coverage Test, Internal Model Approach (IMA)

INTRODUCTION

Under the internal model approach (IMA) stipulated by Basel II, financial institutions are allowed to develop and employ proprietary internal models to evaluate various risk. However, the flexibility to develop a proprietary model leads to the question of which computing method delivers the most accurate and reliable estimates of value at risk (VaR). An improper risk assessment model leads to severe consequences. An overstated VaR results in retaining excessive and inefficient amounts of capital, and an understated VaR results in retaining insufficient capital to deal with crises. Thus, judging the accuracy of a VaR model is extremely important.

The univariate unconditional coverage test proposed by Kupiec (1995) is the conventional method for the backtesting of a VaR model. However, Pérignon and Smith (2008) proposed a new backtesting framework — the multivariate unconditional coverage test — to improve the backtesting procedure. The multivariate unconditional coverage test focuses on the left tail of the loss distribution and is a multivariate generalization of Kupiec's unconditional test. This paper employs the backtesting method proposed by Pérignon and Smith (2008) to confirm the best method for computing the integrated VaR of a portfolio containing different asset categories. Three major VaR computation methods: historical simulation, Monte Carlo simulation, and variance-covariance methods are tested. Moreover, four different volatility estimation approaches are used in the calculation of the variance-covariance method. The portfolio on which VaR is computed includes equities, government bonds, foreign exchange, and index options, all of which are commonly traded by financial institutions.

This paper is organized as follows. The Literature Review section reviews the literature on VaR. The Data and Methodology section presents the data and explains the methodology employed for the empirical analysis of integrated VaR methods. The Results section presents the results and analyzes the VaR computation methods employed. Finally, a conclusion is drawn and suggestions are made for the direction of future research in the Concluding Comments section.

LITERATURE REVIEW

VaR is the maximum loss expected to occur within a fixed period at a given confidence level. According to Hull and White (1998a), at the confidence level $1-\alpha$, in the next T days, the maximum loss will not exceed Q dollars, where Q denotes VaR. Additionally, Jorion (2007) defines VaR as follows: given the confidence level $1-\alpha$, if the portfolio profit and loss is ΔL, the relationship between VaR and the confidence level is

$$\text{Prob}(\Delta L < -VaR) \leq \alpha \tag{1}$$

The three major VaR computation methods are historical simulation, Monte Carlo simulation, and variance-covariance methods. Historical simulation uses actual historical data to predict future price changes. It employs historical observations of risk factors (e.g., stock prices, interest rates, and exchange rates) to model the probability distribution of the portfolio's value in the future and calculates the VaR. This method assumes that future price fluctuations of assets in the portfolio will be the same as historical fluctuations. Duffie and Pan (1997) discuss the effect of a fat-tailed distribution of returns on VaR, with examples applied to equity, foreign exchange, and commodity markets. Hull and White (1998b) also find that the returns on many financial assets tend to exhibit a fat-tail distribution. They point out that if, instead, a normal distribution is applied to the returns, the VaR is likely to be underestimated. The historical simulation method needs not assume a normal distribution of returns, but can incorporate the statistical information provided by the data itself. Boyle (1977) first applies the Monte Carlo simulation method to option pricing. Subsequently, this method has been widely used in derivative pricing and risk management. The method assumes that movements in asset prices are stochastic, and on this basis, the probability distribution of portfolio returns is constructed to calculate the VaR. Monte Carlo simulation is based on the law of large numbers. As the number of experiments increases, the simulation result is more accurate. Variance-covariance method assumes that asset returns follow the normal distribution to simplify the VaR calculation. The variance-covariance VaR of a portfolio is calculated as follows:

$$VaR_p = Z_\alpha \times \sigma_p \times V \times \sqrt{t} \tag{2}$$

where $\sigma_p^2 = \left[w_1 \cdots w_n\right] \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$, σ_{ij} is the covariance between assets i and j, n is the number of assets in the portfolio, w_i is the weight of asset i, where i = 1, 2, …, n, V is the market value of the portfolio, and t is the holding period.

Beder (1995) applies eight VaR methods to three hypothetical portfolios: treasury bills, equities together with their options, and a combination of the two asset types. The results demonstrate that the methods employed produce significantly different VaRs. They also imply that the VaR is strongly related to the parameters, data, assumptions, and methodology adopted. Pritsker (1997) examines six VaR computation methods that are applied to foreign exchange options, in order to investigate the trade-off between accuracy and computational time. He uses the Monte Carlo simulation method with full repricing as the benchmark. Pritsker demonstrates that when the trade-off between accuracy and computational time is considered, the delta-normal Monte Carlo simulation method is the most suitable for a portfolio containing options. Engel and Gizycki (1999) use data from Australian banks, collected over the past 10 years, to compare four VaR methods: variance-covariance, historical simulation, Monte Carlo simulation, and extreme value models. The first three methods tend to be adopted by the majority of financial firms. They show that of the four methods, historical simulation produces the most reliable results.

According to Jorion (1996) and Lopez and Walter (2001), backtesting is a statistical framework employed to evaluate the effectiveness of a VaR method. It examines the difference between actual loss and that estimated from a VaR method to determine whether the model bears up statistically. If a VaR model fails, its assumptions, parameters, calculation, and test procedure need to be reviewed. The conventional method to backtest the effectiveness of a VaR method is the univariate unconditional coverage test referred to Kupiec (1995). However, the results produced by the univariate test when different confidence levels are employed are often contradictory. The multivariate unconditional coverage test proposed by Pérignon and Smith (2008) provides a solution to this problem. Their method focuses on the left tail of the distribution of losses and is a more generalized model compared with the Kupiec's univariate test.

DATA AND METHODOLOGY

This paper conducts an empirical analysis for the integrated VaR methods. The portfolio for VaR calculation contains equities, bonds, foreign exchange, and equity index options. The initial value of the portfolio is NTD $40 million, and the four asset categories are equally weighted, as shown in Table 1. Historical data, including the TAIEX Index, ten year government bond prices, NTD/USD exchange rates, and TAIEX Index option (put option), are obtained from the Taiwan Economic Journal (TEJ) database. Weekly data are based on Friday closing prices from 1/4/2002 to 12/12/2009, totaling eight years or 417 weeks. Historical simulation, Monte Carlo simulation, and variance-covariance methods are employed to compute the VaRs over the eight-year period studied. Four volatility estimations obtained by RiskMetrics, generalized autoregressive conditional heteroscedasticity (GARCH), GARCH with Student-t distribution (GARCH-t), and AR(1)-GARCH(1,1) approaches are used in the variance-covariance method. Finally, all the VaRs are tested by both univariate (Kupiec, 1995) and multivariate (Pérignon and Smith, 2008) coverage tests.

Table 1: The Portfolio for VaR Computation

Asset Categories	Weight	Amount
Equity (TAIEX Index)	25%	NT $10,000,000
Bond (10 yr Government Bond)	25%	NT $10,000,000
Foreign Exchange (USD Deposit)	25%	US $300,000*
Option (TAIEX Index Option)	25%	NT $10,000,000
Portfolio	100%	NT $40,000,000

This table lists the weights and amounts of the four asset categories which compose the portfolio. By using the average foreign exchange rate of $33 NTD/USD for calculation, the amount of USD deposit equals to about NTD $10,000,000.

Developed by J.P. Morgan (1996), the RiskMetrics VaR calculation employs an exponentially weighted moving average (EWMA) to estimate the variance-covariance matrix. Considering that the impact of more recent volatility on return is greater, EWMA assigns a higher weight to recent data to capture short-term fluctuations. According to Zangari (1995), EWMA volatility can be expressed as

$$\sigma_t^2 = (1-\lambda)\sum_{i=1}^{\infty} \lambda^{i-1} r_{t-i}^2 \tag{3}$$

where λ is the decay factor and $0 < \lambda < 1$. Fleming, Kirby, and Ostdiek (2001) state that $\lambda = 0.94$ will result in more accurate predictions for daily VaR. According to J.P. Morgan's system (1996), the best setting of λ is to assume $\lambda = 0.94$ for daily data and $\lambda = 0.97$ for monthly data.

Morgan (1976) finds that the variance of stock returns may change over time. That is, heteroscedasticity is present. Engle (1982), Bollerslev (1986), and Engle and Manganelli (2004) believes that residual variance is usually unstable in time series data of asset prices. The time series data often has two characteristics: non-normal distributed and volatility clustering. Engle (1982) proposes an autoregressive conditional heteroscedasticity (ARCH) model to explain this phenomenon. In the ARCH model, the conditional variance of the time series data is a function of past residuals, and heterogeneous variance is emphasized. This differs from the traditional assumption that volatility is time independent. Bollerslev (1986) extends the ARCH model to a GARCH model, which makes the setting of variables more flexible.

The GARCH model assumes that conditional variance of return is affected not only by the past residual but also by past conditional variance. The simplest form of this model is GARCH(1,1), expressed as

$$\sigma_t^2 = \omega + \alpha\, r_{t-1}^2 + \beta\, \sigma_{t-1}^2 \tag{4}$$

A large α indicates that volatility will disappear slowly, and a small β indicates that volatility will react fast to market fluctuations. If $\alpha + \beta = 1$, volatility will grow at a constant rate, and the model will not be convergent. Thus let $\alpha + \beta < 1$ and $E(r_{t-1}^2) = \sigma_t^2 = \sigma_{t-1}^2 = \sigma^2$, then the volatility will converge to $\dfrac{\omega}{1-\alpha-\beta}$. Bollerslev, Chou, and Kroner (1992) believe that the volatility of asset return in financial markets is predictable. Their empirical study also shows that GARCH(1,1) can fully describe heteroscedasticity in the variance of asset returns.

This paper also adopts the AR(1)-GARCH(1,1) model to estimate variance. Let the asset return of time t be R_t, where $t = 1, 2, \cdots, T$. Then, the AR(1)-GARCH(1,1) model can be expressed as

$$\begin{aligned} R_t &= \gamma_0 + \gamma_1 R_{t-1} + \varepsilon_t \\ \varepsilon_t &= \nu_t \sigma_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{aligned} \tag{5}$$

where $\nu_t \overset{i.i.d.}{\sim} N(0,1)$, $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$, and $\alpha_1 + \beta_1 < 1$. When applying this model to financial markets, we generally set $\alpha_1 \geq 0.7$.

Coverage tests are used to identify the most accurate estimation method when comparing the performance of VaR computation methods. Let N_v be a random variable that denotes the number of periods elapsed till the first VaR failure is recorded. If p is the probability of a VaR failure at any given time, the probability of observing the first failure in period n is given by

$$\Pr(N_v = n) = p(1-p)^{n-1} \tag{6}$$

N_v follows a geometric distribution with an expected value of $1/p$. Given a realization for N_v, Kupiec (1995) constructs a likelihood ratio (LR) test to determine whether the potential loss estimates are consistent with the null hypothesis. The LR statistic for testing the null hypothesis $p = p^*$ is given by

$$LR(n, p^*) = -2\ln[p^*(1-p^*)^{n-1}] + 2\ln[(1/n)(1-1/n)^{n-1}] \tag{7}$$

Under the null hypothesis, $LR(n, p^*)$ has a chi-square distribution with one degree of freedom. The univariate test uses only a single VaR and a confidence level. While this test remains the reference test in financial risk management, it displays low statistical power.

Rather than using a single VaR, Pérignon and Smith (2008) suggests an alternative testing procedure, a multivariate coverage test, which uses a series of VaR with different coverage probabilities. Considering K VaRs with different coverage probabilities, indexed by i in descending order, i.e., p_1 > p_2 > …> p_K. These VaRs become more extreme as i increases, i.e., VaR(p_1) < VaR(p_2) < …< VaR(p_K). Associated with each of the K VaR numbers is an indicator variable for losses falling in each disjoint interval. That is,

$$J_{it} = \begin{cases} 1, & if\ -VaR(p_{i+1}) < R_t \le -VaR(p_i) \\ 0, & otherwise \end{cases} \tag{8}$$

where $i = 1, 2, \cdots, K$ and R_t denotes the portfolio return. J_{it} are Bernoulli random variables. Thus, J_{it} = 1 with probability θ_i = $p_i - p_{i+1}$. We collect the K elements θ_i into a K-dimensional parameter vector θ = $(\theta_1, \ldots, \theta_K)^T$.

We can test the joint hypothesis of this specification of the VaR model, i.e., if the empirical θ significantly deviates from the hypothesized θ_i^*, using a multivariate LR test LR_M. Formally, the LR_M test is given by

$$LR_M = 2\left\{\left[n_0 \ln\left(1 - 1^T \theta^*\right) + \sum_{i=1}^{K} n_i \ln\left(\theta_i^*\right)\right] - \left[n_0 \ln\left(1 - 1^T \theta\right) + \sum_{i=1}^{K} n_i \ln\left(\theta_i\right)\right]\right\} \tag{9}$$

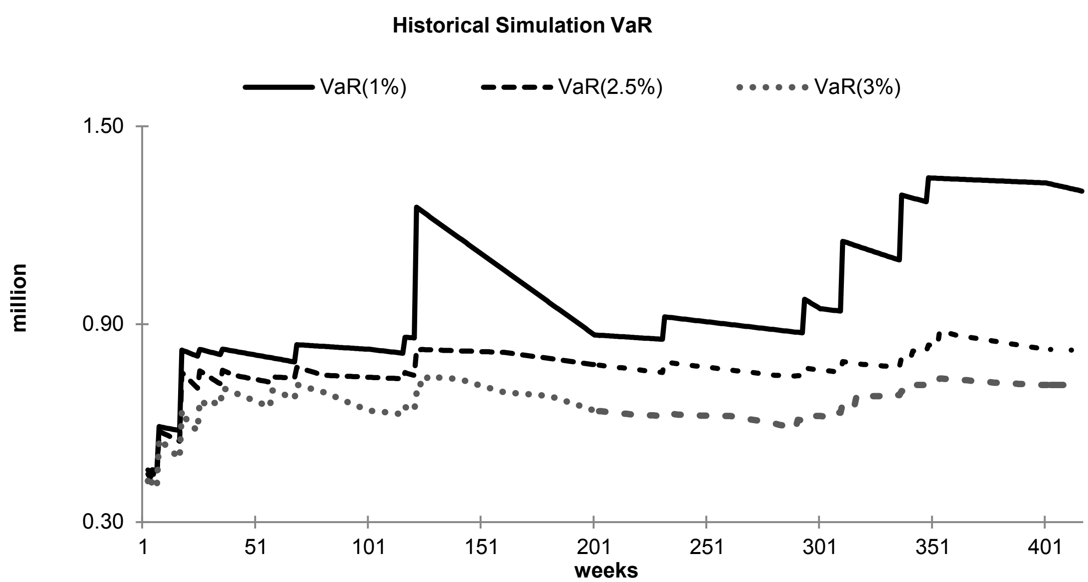
where $n_i = \sum_{t=1}^{T} J_{it}$, $n_0 = N_v - n_i$, and $\theta_i^* = (1/T)\sum_{t=1}^{T} J_{it}$, where θ_i^* is the maximum likelihood estimator of θ with element i.

The LR_M statistic is asymptotically a chi-square distribution with K degrees of freedom. When $K = 1$, the LR_M test is equivalent to the univariate coverage test of Kupiec (1995). In this paper, we employ both the univariate and multivariate coverage approaches to test VaR models.

RESULTS

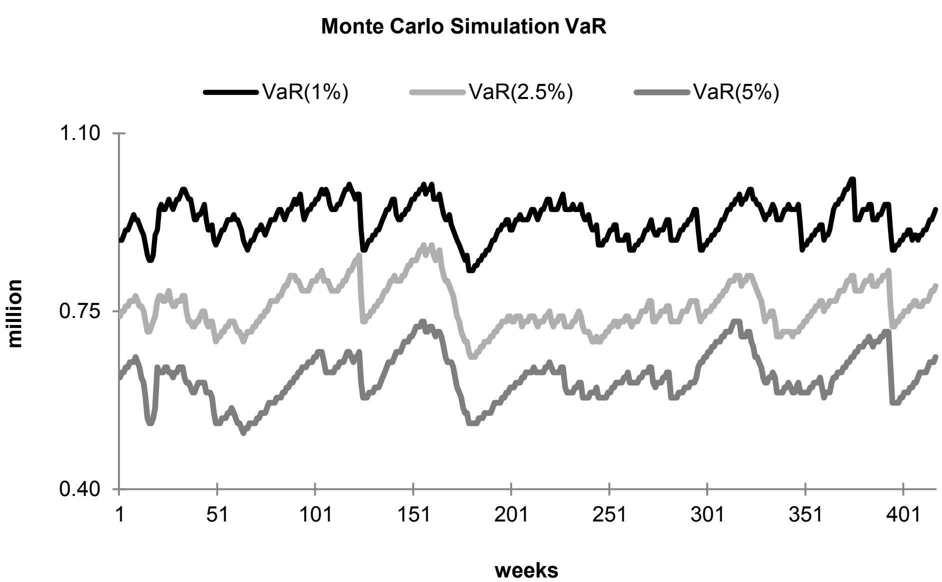
The VaRs are shown in Figure 1 to 6. The historical simulation VaRs in Figure 1 is generally smooth. The smoothness of the curve demonstrates a slow response to market. Campbell (2005) and Pritsker (2006) indicates that one drawback of the historical simulation method is that it cannot react immediately to market fluctuations. However, the variance-covariance methods, including RiskMetrics, GARCH, GARCH-t, and AR(1)-GARCH(1,1), are more sensitive to market fluctuation. Moreover, as shown in Figures 4, 5, and 6, the GARCH-based models are particularly similar in their path.

Figure 1: The Historical Simulation VaRs



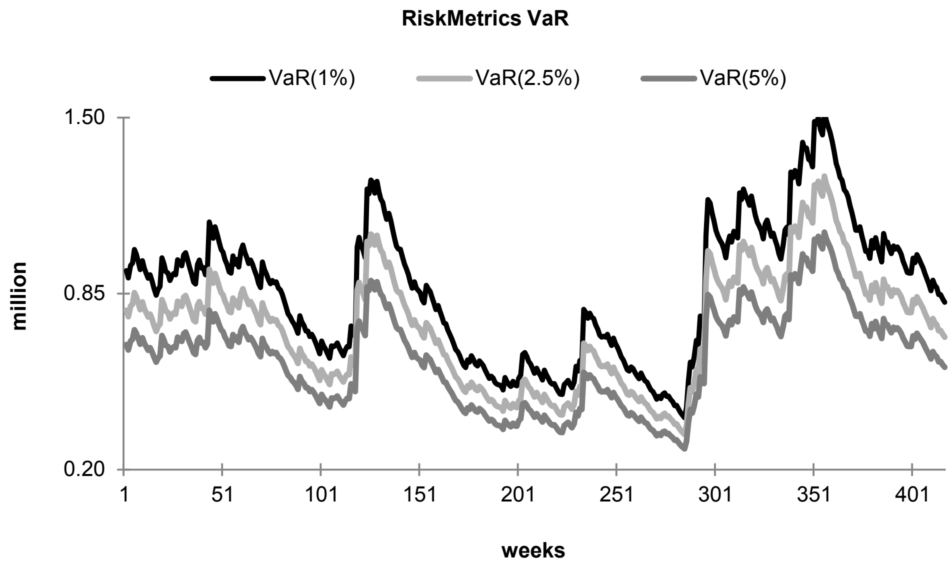
This figure shows the historical simulation VaRs of the portfolio for 417 weeks and 1%, 2.5% and 5% confidence levels. The historical simulation VaR is generally smooth.

Figure 2: The Monte Carlo Simulation VaRs



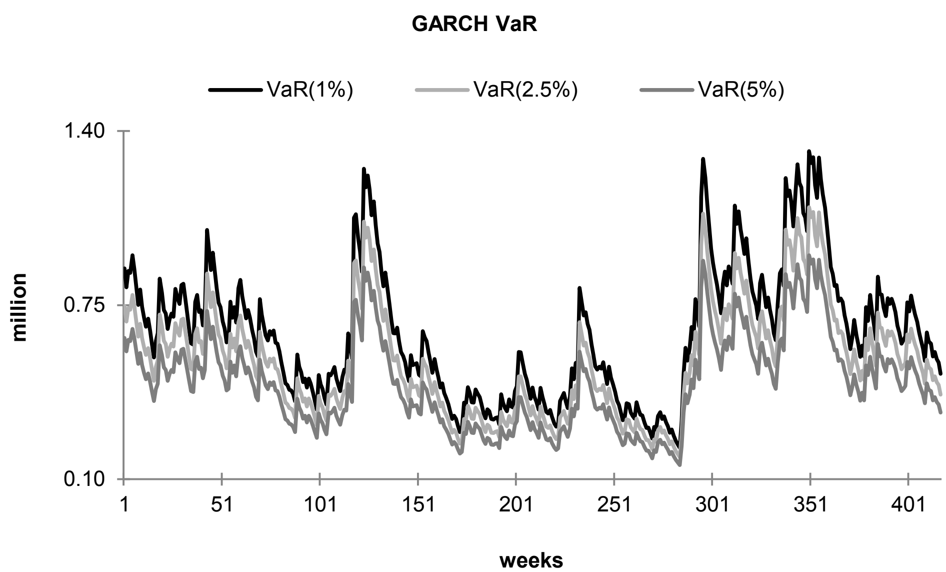
This figure shows the Monte Carlo simulation VaRs of the portfolio for 417 weeks and 1%, 2.5% and 5% confidence levels. The Monte Carlo simulation VaR is more fluctuant compared with the historical simulation VaR.

Figure 3: The RiskMetrics VaRs



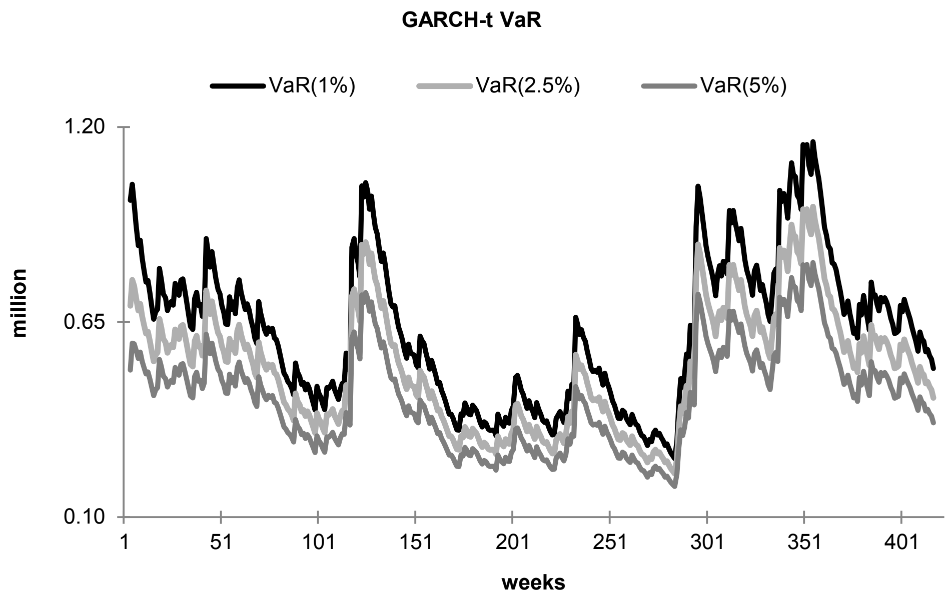
This figure shows the RiskMetrics VaRs of the portfolio for 417 weeks and 1%, 2.5% and 5% confidence levels. The RiskMetrics VaR is volatile.

Figure 4: The GARCH VaRs



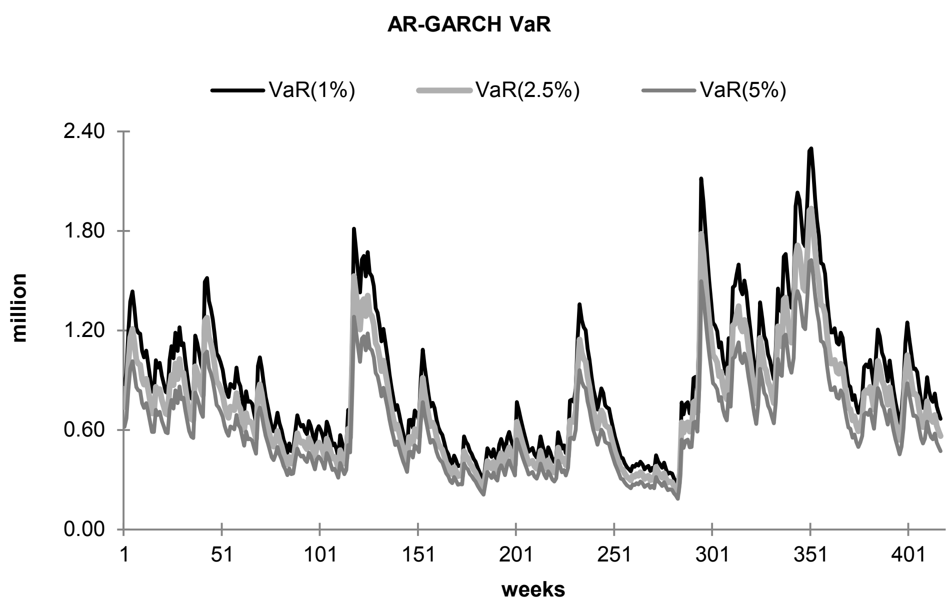
This figure shows the GARCH VaRs of the portfolio for 417 weeks and 1%, 2.5% and 5% confidence levels. The GARCH VaR is much more volatile than the historical simulation, Monte Carlo simulation and RiskMetrics VaRs.

Figure 5: The GARCH-t VaRs



This figure shows the GARCH- t VaRs of the portfolio for 417 weeks and 1%, 2.5% and 5% confidence levels. The GARCH-t VaR is much more volatile than the historical simulation, Monte Carlo simulation and RiskMetrics VaRs.

Figure 6: The AR(1)-GARCH(1,1) VaRs



This figure shows the AR(1)-GARCH(1,1) VaRs of the portfolio for 417 weeks and 1%, 2.5% and 5% confidence levels. The AR(1)-GARCH(1,1) VaR is much more volatile than the historical simulation, Monte Carlo simulation and RiskMetrics VaRs.

Table 2 shows the results of univariate and multivariate convergence tests of different VaR methods. The sample periods are from 52 to 417 weeks. Violation rate is defined as the ratio of the number of violations to the number of observed samples. The performance of historical simulation appears superior regardless of the length of the sample period and the extent of the coverage probability. Neither the univariate nor the multivariate test allows the null hypothesis to be rejected, which indicates that the historical simulation method performs well. As the length of the sample period increases, the violation rate decreases. This shows that increments in sample size aid the accuracy of the model.

Table 2: The Violation Rates and P-Values of Different VaR Models

VaR Model \ Sample period			T = 52	T = 104	T = 156	T = 208	T = 260	T = 313	T = 365	T = 417
Historical Simulation	Univariate Coverage Test	α = 1%	1.92%	1.92%	1.28%	1.44%	1.15%	1.28%	1.10%	1.20%
			(0.5528)	(0.4012)	(0.7344)	(0.5476)	(0.8077)	(0.6356)	(0.8561)	(0.6920)
		α = 2.5%	3.85%	2.88%	2.56%	2.88%	2.69%	2.56%	2.74%	2.64%
			(0.5640)	(0.8062)	(0.9593)	(0.7286)	(0.8444)	(0.9497)	(0.7726)	(0.8581)
		α = 5%	5.77%	5.77%	5.13%	5.29%	5.00%	5.11%	5.21%	5.04%
			(0.8036)	(0.7250)	(0.9417)	(0.8499)	(0.9998)	(0.9279)	(0.8580)	(0.9731)
	Multivariate Coverage Test		-	-	-	-	-	-	-	-
			(0.9225)	(0.8033)	(0.9825)	(0.9460)	(0.9918)	(0.9534)	(0.9936)	(0.9802)
Monte Carlo Simulation	Univariate Coverage Test	α = 1%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
			(0.3066)	(0.1482)	(0.0766)	**(0.0409***	**(0.0222***	**(0.0121***	**(0.0068***	**(0.0038***
		α = 2.5%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
			(0.1047)	**(0.0217***	**(0.0049***	**(0.0012***	**(0.0003***	**(0.0001***	**(0.0000***	**(0.0000***
		α = 5%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
			(0.0209*	**(0.0011***	**(0.0001***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***
	Multivariate Coverage Test		-	-	-	-	-	-	-	-
			(0.1489)	**(0.0137***	**(0.0011***	**(0.0001***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***
RiskMetrics	Univariate Coverage Test	α = 1%	1.92%	3.85%	3.85%	3.85%	3.08%	3.51%	3.29%	3.12%
			(0.5528)	**(0.0262***	**(0.0065***	**(0.0017***	**(0.0069***	**(0.0005***	**(0.0005***	**(0.0005***
		α = 2.5%	5.77%	9.62%	9.62%	7.69%	6.15%	6.71%	6.30%	6.47%
			(0.1956)	**(0.0004***	**(0.0000***	**(0.0001***	**(0.0014***	**(0.0001***	**(0.0001***	**(0.0000***
		α = 5%	5.77%	11.54%	11.54%	9.13%	7.69%	7.99%	8.22%	8.15%
			(0.8036)	**(0.0084***	**(0.0012***	**(0.0137***	(0.0640)	**(0.0251***	**(0.0095***	**(0.0065***
	Multivariate Coverage Test		-	-	-	-	-	-	-	-
			(0.2330)	**(0.0052***	**(0.0003***	**(0.0009***	**(0.0081***	**(0.0003***	**(0.0007***	**(0.0001***
GARCH	Univariate Coverage Test	α = 1%	5.77%	12.50%	12.18%	10.10%	8.08%	8.63%	8.49%	8.39%
			(0.0019*	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***
		α = 2.5%	11.54%	15.38%	14.74%	12.02%	10.38%	10.22%	10.68%	10.55%
			(0.0022*	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***
		α = 5%	17.31%	19.23%	17.31%	15.38%	13.85%	13.74%	13.42%	13.43%
			(0.0012*	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***
	Multivariate Coverage Test		-	-	-	-	-	-	-	-
			(0.0086*	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***
GARCH-*t*	Univariate Coverage Test	α = 1%	5.77%	12.50%	12.18%	10.10%	8.08%	8.63%	8.49%	8.39%
			(0.0172*	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***
		α = 2.5%	11.54%	15.38%	14.74%	12.02%	10.38%	10.22%	10.68%	10.55%
			(0.0022*	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***
		α = 5%	17.31%	15.38%	17.31%	15.38%	13.85%	13.74%	13.42%	13.43%
			(0.0012*	**(0.0001***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***
	Multivariate Coverage Test		-	-	-	-	-	-	-	-
			(0.0086*	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***
AR(1)-GARCH(1,1)	Univariate Coverage Test	α = 1%	5.77%	9.62%	9.62%	7.69%	6.15%	6.71%	6.30%	6.47%
			(0.0172*	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***
		α = 2.5%	5.77%	11.54%	11.54%	9.13%	7.69%	7.99%	8.22%	8.15%
			(0.1956)	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***
		α = 5%	9.62%	14.42%	13.46%	11.06%	9.62%	9.90%	10.14%	10.07%
			(0.1729)	**(0.0003***	**(0.0001***	**(0.0005***	**(0.0023***	**(0.0004***	**(0.0001***	**(0.0000***
	Multivariate Coverage Test		-	-	-	-	-	-	-	-
			(0.0561)	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***	**(0.0000***

*This table shows the results of univariate and multivariate convergence tests of VaR methods. The sample periods in this table are from 52 to 417 weeks. Violation rate is defined as the ratio of the number of violations to the number of observed samples. The p-values are listed in the parentheses. * indicates the p-values that allow a rejection of the null hypothesis at the 5% confidence level.*

The Monte Carlo simulation performs well for short sample periods, particularly in the univariate coverage test with 1% coverage probability. As observed from the table, sometimes the univariate and multivariate coverage tests render different conclusions regarding the null hypothesis when employing a

Monte Carlo simulation method. As Table 2 shows, among the four variance-covariance VaRs, the RiskMetrics approach performs well for shorter sample periods involving more recent data. The other three variance estimation approaches, including GARCH, GARCH-t, and AR(1)-GARCH(1,1), also emphasize time-varying variance. Although GARCH-t improves the GARCH method by using the t-distribution to calculate the multiplier Z_α, its performance is not substantially improved according to the test result. Almost all test results of GARCH and GARCH-t show a rejection of the null hypothesis. Moreover, AR(1)-GARCH(1,1) performs better than GARCH and GARCH-t.

In Table 2, the univariate test produces contradictory conclusions for different confidence intervals. For example, in the RiskMetrics approach sampling 260 weeks, the tests with coverage probabilities of 1% and 2.5% reject the null hypothesis, whereas the test with a coverage probability of 5% shows that the null hypothesis should not be rejected. This demonstrates the inadequacy of the univariate coverage test and the importance of the multivariate coverage test, which considers the entire left tail of the distribution.

The empirical test shows that the historical simulation method performs best. The Monte Carlo simulation, RiskMetrics and AR(1)-GARCH(1,1) are second best. The GARCH, and GARCH-t perform poorly. Perhaps because the portfolio contains options, a nonlinear asset, variance-covariance methods, which are more suitable for linear assets, perform poorly, and the historical simulation performs better.

CONCLUDING COMMENTS

VaR is an important measure of risk for the Basel II internal model approach (IMA). The most widely used VaR computation models are historical simulation, Monte Carlo simulation, and variance-covariance method. This gives rise to the question of which computing method produces the most accurate and reliable estimates of VaR. This research employed univariate (Kupiec, 1995) and multivariate tests (Pérignon and Smith, 2008) to evaluate the performance of three VaR models with differing sample periods. The empirical data are drawn from the four major asset categories in Taiwan market for the period 2002–2009. The empirical analysis shows that the historical simulation performs best. Among the four volatility estimation approaches for variance-covariance method, RiskMetrics and AR(1)-GARCH(1,1) perform better than the GARCH and GARCH-t approaches. The presence of nonlinear assets in the portfolio may explain the poor performance of variance-covariance methods. In addition, this research results are consistent with those of Pérignon and Smith (2008). The VaR test results for the most commonly traded financial assets in Taiwan market are presented here. When the selected portfolios differ in terms of the assets included, the period chosen for data collection, or the market involved, the test results tend to vary. There is scope, therefore, for further research that seeks to evaluate which VaR method is the most suitable for portfolios containing other assets such as foreign exchange options and swaps.

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ACKNOWLEDGEMENT

This paper is based on our research presented in the 59th Annual Meeting of the Midwest Finance Association, U.S.A., 2010. We thank Ms. Chih-Hsin Cheng, a graduate student of National Taiwan University, for helping process the empirical data. The empirical results are also used in her master thesis to which Professor Cheng-Kun Kuo is the advisor.

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