

# LUMP-SUM QUOTA BONUSES AND OTHER VERTICAL RESTRAINTS WITH COURNOT RETAILERS

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## ABSTRACT

*Lump-sum quota bonuses are a specific type of quota bonus that provides a lump-sum transfer from a manufacturer to a retailer when the retailer's sales exceed a pre-determined quota. This paper explores whether lump-sum quota bonuses and two other vertical restraints, two-part tariffs and resale price maintenance, can resolve the double marginalization problem when the market size is uncertain. It emerges that only lump-sum quota bonuses can always resolve the double marginalization problem in our two-state case.*

**JEL:** L42, L13, D89

**KEYWORDS:** Vertical Restraints, Lump-Sum Quota Bonuses, Cournot Competition, Uncertain Market Size

## INTRODUCTION

Perhaps because of a lack of knowledge of the market, manufacturers may not effectively promote their own product, as occurred when Google failed to effectively market the Nexus One. As an alternative to self-promotion, many manufactures sell products through more than one retailer. For example, publishers sell books through bookstores, of which the leaders in the United States include Borders, Barnes and Noble, Amazon.com, and the Follett Corporation (Datamonitor, 2010). The vertical relationships between manufacturers and retailers have been addressed in the literature. Rey and Vergé (2008) provide a good overview of relevant academic studies and legal matters.

This paper addresses a special case in which few retailers control retail markets. In addition to the book market mentioned, the cable television in the U.S. is another example. In each area, there exist only few cable systems operators to carry programming. In this special case, the double marginalization problem (DMP) raises. Market power of retailers induces a pricing inefficiency in vertically related markets, and hence, profits of manufacturers and of channels are eroded. Sprengler (1950) firstly addresses the DMP. Waterman and Weiss (1996) verify the DMP in the U.S. cable television market. Manufacturers often use vertical restraints, which is the restraints imposed by manufacturers on retailers, to regulate retailers' behavior and resolve the DMP. Without market uncertainty, several vertical restraints can resolve the DMP. However, it may not be true when market size is uncertain.

Two-parts tariffs (TTs) and resale price maintenance (RPM) are often discussed in the literature. In addition to the above vertical restraints, this paper address lump-sum quota bonuses (LSBQs), which is only mentioned in very few papers. LSQB, which are similar but not exactly identical to other vertical restraints, are a specific type of quota bonuses, which involve making a lump-sum transfer from a manufacturer to a retailer when the retailer's sales exceed a predetermined quota. For example, car manufacturers often offer some "volume bonus incentives" to dealers. In this paper, we show that only LSBQs can resolve the DMP when market size is uncertain. Hence, we spend more paragraphs in discussing LSBQs than other vertical restraints.

We discuss related literature in the following section. The model and the impacts of LSQB on retailers' best response functions are presented in the section after the next section. If a manufacturer simply maximizes its profits, none of vertical restraints discussed in this paper can be counted upon to always resolve the DMP. However, when a manufacturer maximizes channel profits, LSQB is superior to TTs and RPM. These discussions can be found in the following sections. The final section provides

concluding comments. Proofs of all propositions or lemmas can be found in the appendix.

## LITERATURE REVIEW

Several aspects of vertical relations have been addressed in the literature. Mathewson and Winter (1985) study vertical relations from the principal-agent perspective. Mathewson and Winter (1984) posit that vertical restraints can be used to ease externalities among retailers. In Mathewson and Winter (1984), these externalities are due to advertising by retailers. Klein and Murphy (1988) focus on exclusive territory, a specific form of vertical restraint. They claim that vertical restraints can influence retailer behaviors indirectly by lowering their short-term gains. In contrast, this paper examines whether LSQBs and other vertical restraints can resolve the double marginalization problem given an uncertain market size.

Although LSQBs and slotting allowances are both forms of payments from manufacturers to retailers, they differ in two ways. First, with LSQBs, retailers receive payments from manufacturers only when the amount that they sell exceeds a certain threshold, which creates the quantity-fixing effect. In contrast, retailers can usually have slotting allowances as long as they sell the manufacturer's product. Shaffer (1991) and Kuksov and Pazzal (2007) adopt this setup in their models. Secondly, LSQBs are generally offered when manufacturers are relatively dominant, whereas slotting allowances are more prevalent when retailers are relatively dominant. LSQBs are also very similar to the all-units discounts discussed in Kolay et al. (2004). Unlike with all-units discounts, however, the wholesale price is not altered when the sales exceed the quota.

In practice, quota bonuses are frequently used to stimulate sales either by subordinate sales-persons or by contracted retailers. The role of quota bonuses in motivating salesperson effort has been analyzed in the literature (Raju and Srinivasan, 1996; Oyer, 2000), but no study has addressed the role of quota bonuses in the context of vertical relations or considered the issue of the DMP in particular. The DMP, which is addressed in this paper, often occurs in a decentralized supply chain. When selling products through retailers, manufacturers suffer from the imperfect competition that occurs among retailers. To maximize their profits, retailers raise retail prices. Hence, both the quantity sold and the manufacturer's profits decrease. The same problem also occurs with intermediate goods. To solve this channel coordination problem, a manufacturer can apply vertical restraints to retailers, including TTs, RPM, and exclusive territory agreements (Rey and Tirole, 1986). Note that we exclude exclusive territories from the discussion in this paper because we introduce imperfect quantity competition among multiple potential retailers into the model. Like Rey and Tirole (1986); Gal-Or (1991); Kolay et al. (2004), we compare different vertical constraints in a context in which the manufacturer faces uncertain demand. We simply assume that manufacturers and retailers have common *ex ante* knowledge of the probability distribution of an unknown market size. Furthermore, due to contract costs, manufacturers and retailers neither renegotiate retail contracts nor redesign the vertical restraints when retailers know the actual market size. It emerges that if the manufacturer maximizes the channel profits, only LSQBs can ensure monopoly profits for the whole channel in our two-state case. When the aim is to resolve the DMP, the LSQB scheme is indeed superior to the other two vertical restraints in the case of an uncertain market size.

### The Model

There exists one monopoly manufacturer, which produces one product and sells the product via potential retailers. This paper addresses the issue of the distribution of profits within the channel. Hence, we exclude the case in which multiple manufacturers compete with homogeneous or heterogeneous products though the case is more general and may generate different interests. Further, there are two potential Cournot retailers. That is, the two potential retailers in the retail market simultaneously choose quantity to compete with each other. The theorem of independent Nash equilibria in Bergstrom and Varian (1985), of which Cournot competition is a typical example, is still valid here. The assumption of two potential retailers can simplify the analysis without loss of generality. In our model, the manufacturer has a better position in the distribution of channel profits because of the status of monopoly. If the number of potential retailers is reduced to one, the retailer will become a monopsony. The negotiation power of the retailer must be considered then. The manufacturer's production cost is zero, and retailers sell the product at no distribution

cost. This paper does not address the issue of asymmetric information of retailers' distribution cost. The inverse market demand is  $\alpha - Q$ , where  $\alpha > 0$ , and  $Q$  is the quantity supplied by retailers in the market. Unlike in O'Brien and Shaffer (1992), we do not introduce retailers' heterogeneity in the model. Hence, no retailers are discriminated; all are offered the same retail contract once the latter is designed. If the manufacturer is allowed to offer some retailers preferential treatment, it is easy for the manufacturer to manipulate the equilibrium in stage of the retailers' competition, and it becomes difficult to investigate the influence of different vertical restraints. Further, the DMP emphasized in this paper is caused by the imperfect competition among retailers. Preferential treatment to specific retailers can enhance the retailers' market power. Hence, the DMP cannot be mitigated by preferential treatment.

Here is the timing of our model. First, the manufacturer designs the retail contract, including the wholesale price and vertical constraints. The retail contract can be considered the take-it-or-leave-it offer from the manufacturers to the potential retailers. Both potential retailers must agree to the retail contract and accept all vertical constraints in the equilibrium. Finally, the two potential retailers decide the amount of products purchased from the manufacturers according to the terms in the retail contract. If any potential retailer decides to purchase nothing from the manufacturer, the potential retailer will be considered to quit the retail market. Two potential retailers  $r_1, r_2$  are Cournot duopoly in the retail market, competing with quantities  $q_i, i = 1, 2$ , where  $q_i$  is the amount purchased from the manufacturer and sold by  $r_i$ . Hence, the market quantity  $Q = q_1 + q_2$  and the market price  $P = \alpha - Q$ , given  $Q < \alpha$ .

However, the market size in the retail market can be affected by many factors, such as the business cycle, and fluctuates all the time. It is assumed that market size  $\alpha$  is unknown to both the manufacturer and the potential retailers until the moment when potential retailers decide the amount of purchased. All the manufacturer and potential retailers know is the probability distribution of  $\alpha$ .  $\alpha$  may be  $\alpha_H$  or  $\alpha_L$ ,  $\alpha_H > \alpha_L$  when the retail contract is designed and agreed. The probability that  $\alpha = \alpha_H$  is  $p$ ,  $p \in (0, 1)$ . In this one-period model, it is also assumed that the retail contract cannot be renegotiated due to contract costs or negotiation costs. Hence, the information of the actual market size does not affect the design of the retail contract.

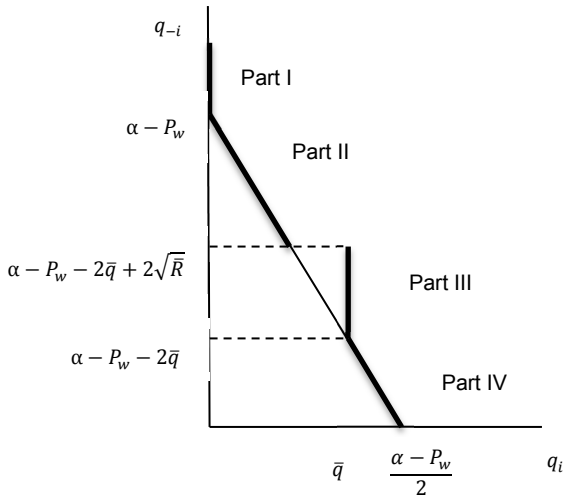
We investigate a specific form of LSQBs in this paper and identify how LSQBs introduce more competition into the retail market. Consider the following LSQB scheme. Once any retailer's sales equal or exceed a given sales threshold  $\bar{q}$ , the retailer can obtain a lump-sum bonus  $\bar{R}$ . Under LSQBs, the manufacturer needs to decide the amount of  $\bar{q}$  and  $\bar{R}$  in addition to  $P_w$ . Unlike the all-units discounts in Kolay et al. (2004), the LSQB scheme in this paper does not allow the wholesale price to vary, even if sales exceed  $\bar{q}$ .

If the manufacturer can only set the wholesale price  $P_w$ , which is also the marginal cost each retailer faces,  $r_i$ 's best response, given  $\alpha$ , is

$$q_i = \frac{\alpha - q_{-i} - P_w}{2}, \tag{1}$$

where  $q_{-i}$  is the amount sold by the retailer other than  $r_i$ . Under LSQBs, the best response function in Equation (1) will no longer hold at all times. A jump will occur in the best response function because the reward to retailers is discontinuous due to LSQBs, and because retailers are encouraged to sharply increase their sales when sales are close enough to the threshold  $\bar{q}$ .

Figure 1:  $r_i$ 's Generic Response Function under LSQBs



A jump will occur in the best response function because the reward to retailers is discontinuous due to LSQBs.

Lemma 1 Under LSQBs,  $r_i$ 's best response function jumps as  $q_{-i} = \alpha - P_w - 2\bar{q} + 2\sqrt{\bar{q}}$ . After the jump,  $r_i$  continues to choose  $q_i = \bar{q}$  as  $q_{-i}$  decreases.  $q_i$  does not change until  $q_{-i} < \alpha - P_w - 2\bar{q}$ .

Lemma 1 shows that how the discontinuous reward scheme of LSQBs creates a jump in  $r_i$ 's best response function. Except for the jump  $r_i$ 's best response function is the same as indicated by Equation (1). The generic best response function  $r_i$  is depicted in Figure 1 based on the formula.

$$q_i = \begin{cases} 0 & q_{-i} \geq \alpha - P_w & \text{Part I} \\ \frac{\alpha - q_{-i} - P_w}{2} & \alpha - P_w > q_{-i} \geq \alpha - P_w - 2\bar{q} + 2\sqrt{\bar{R}} & \text{Part II} \\ \bar{q} & \alpha - P_w - 2\bar{q} + 2\sqrt{\bar{R}} > q_{-i} \geq \alpha - P_w - 2\bar{q} & \text{Part III} \\ \frac{\alpha - q_{-i} - P_w}{2} & \alpha - P_w - 2\bar{q} > q_{-i} \geq 0 & \text{Part IV} \end{cases} \quad (2)$$

Note that when  $\alpha - P_w - 2\bar{q} + 2\sqrt{\bar{R}} \geq \alpha - P_w$  or  $\alpha - P_w - 2\bar{q} < 0$ , Part II or Part IV of  $r_i$ 's best response function disappears, respectively.

The necessary conditions of the equilibrium in the retail market can be induced by the best response function. For example, if each retailer sells  $\bar{q}$  units in the retail market, Inequation (3) must be satisfied.

$$\alpha - P_w - 2\bar{q} \leq \bar{q} < \alpha - P_w - 2\bar{q} + 2\sqrt{\bar{R}}. \quad (3)$$

Inequation (3) ensures that one retailer's best response function crosses the other's at Part III. Similarly, if each retailer sells  $q = \frac{\alpha - P_w}{3} < \bar{q}$  units in the retail market, Inequation (4) must be satisfied.

$$\alpha - P_w - 2\bar{q} + 2\sqrt{\bar{R}} \leq \frac{\alpha - P_w}{3} < \alpha - P_w. \tag{4}$$

And Inequation (4) ensures that one retailer's best response function crosses the other's at Part II. Note that, letting  $q_i = q_{-i}$ , the amount sold by each retailer can be easily solved from Equation (1).

The Self-Interested Manufacturer

We assume that the manufacturer simply maximizes its profits by using different types of vertical restraints in this section. Unfortunately, due to the nonlinearity of the reaction functions, the case of LSQBs becomes unnecessarily complicated. Again, the main purpose of our analysis is to prove that LSQBs can be superior to TT and RPM in resolving the DMP. Instead of solving all possible cases, we simply consider symmetric equilibrium in the retail market for simplicity. In some asymmetric cases, one potential retailer might quit the retail market. However, the result of our analysis will not be altered.

Two-Part Tariffs

Under TTs, the manufacturer needs to choose the wholesale price  $P_w$  and the franchise fee  $F$  in the retail contract. It is assumed that retailers realize the actual market size after agreeing to the retail market. Hence, retailers have to pay  $F$  before  $\alpha$  is realized. Consequently, the manufacturer simply determined the wholesale price that will maximize the expected channel profits and capture all of the expected channel profits through the franchise fee. The manufacturer faces a dilemma. A high wholesale price can increase profits when the market is large but may cause the manufacturer to totally lose the market when the market is small. If the manufacturer charges retailers by the wholesale price according to the average monopoly price, the wholesale price will be too low (high) when the market size is large (small).

*Proposition 1 Under TTs, the DMP cannot be resolved when the manufacturer maximizes its profits.*

Indeed, the manufacturer who adopts TTs in the retail contract is already taking channel profits into account. The DMP still cannot always be completely resolved due to the lack of tools. Recall that  $P_w$  is the only manufacturer's tool to affect a retailer's marginal condition. Unfortunately, one tool cannot account for two states simultaneously.

Resale Price Maintenance

Under RPM, the manufacturer needs to set the market price ceiling  $\bar{P}$  along with  $P_w$  in the retail contract. The market price ceiling often implicitly appears in the format of the manufacturer's suggested retail price (MSRP). Almost no retailers set a price strictly higher than MSRP. An appropriate  $\bar{P}$  should be binding in at least one state. Recall that without  $\bar{P}$  and given  $\alpha$ , the market quantity and the market price are  $\frac{2(\alpha - P_w)}{3}$  and  $\frac{\alpha + 2P_w}{3}$  respectively. Hence,  $\bar{P}$  becomes binding only when

$$\bar{P} \leq \frac{\alpha + 2P_w}{3}, \text{ or } \alpha > 3\bar{P} - 2P_w. \tag{5}$$

Under RPM, although  $\bar{P}$  cannot directly alter the retailers' marginal condition, it can still serve as a tool for regulating market price. Nevertheless, similar to the case under TTs, the manufacturer has insufficient tools to regulate the market prices in different states. Proposition 2 yields undesirable results.

Proposition 2 *Under RPM, the DMP cannot be resolved when the manufacturer maximizes its profits.*

Proof:

Please see Appendix.

Apparently the manufacturer can freely choose  $P_w$  and  $\bar{P}$  and can induce the market prices in both states to the monopoly level. One reasonable approach that may come to our mind immediately is to set  $\bar{P} = \frac{\alpha_H}{2}$  and  $P_w = \frac{\alpha_L}{4}$ , in which market price may be directly regulated by  $\bar{P}$  when  $\alpha = \alpha_H$  and the monopoly price can be induced by  $P_w$  when  $\alpha = \alpha_L$ . The DMP may be able to be resolved completely then. Unfortunately, this approach cannot be the manufacturer's strategy for maximizing its profits. Indeed, this strategy is not sustainable even when the manufacturer maximizes channel profits. Please see the related discussion in the next section.  $\bar{P}$  has influence only in the case of the large market size, under which the manufacturer's profits are  $(\alpha_H - \bar{P}) \cdot P_w$ . Clearly, given  $P_w$ , the manufacturer must choose  $\bar{P}$  as small as it can to enhance the profits. To prevent  $\bar{P}$  from being binding when  $\alpha = \alpha_L$ , the constraint  $\bar{P} \geq \frac{\alpha_L + 2P_w}{3}$  must be satisfied, and the manufacturer must choose  $\bar{P}$  as  $\frac{\alpha_L + 2P_w}{3}$ , which is also the market price when  $\alpha = \alpha_L$  (details can be found in the proof of Proposition 2).<sup>3</sup> Clearly, RPM empowers the manufacturer to choose different market prices in different states, but the manufacturer will not do so.

### Lump-Sum Quota Bonuses

We only consider the symmetric equilibrium in the retail market. Under an appropriate  $\bar{q}$  is binding in at least one state. Further, if  $\bar{q}$  is binding when  $\alpha = \alpha_L$ , it must be binding when  $\alpha = \alpha_H$ . Only two cases in the retail market should be taken into account: 1.  $\bar{q}$  is binding when  $\alpha = \alpha_L$ ; 2.  $\bar{q}$  is binding only when  $\alpha = \alpha_H$ . However, the second case can be easily ruled out while considering whether the DMP can be resolved. In Case 2, if the DMP can be completely resolved,  $\bar{q}$  and  $P_w$  must be  $\frac{\alpha_H}{4}$  and  $\frac{\alpha_L}{4}$  respectively. Otherwise, the monopoly quantity and the monopoly price cannot be induced in both states. However, the solution  $\bar{q} = \frac{\alpha_H}{4}$  and  $P_w = \frac{\alpha_L}{4}$  cannot be sustained in Case 2. When  $\alpha = \alpha_H$  and each retailer sells  $\bar{q} = \frac{\alpha_H}{4}$  units in the retail market, Inequation (3) cannot be satisfied. More precisely, the left inequality of Inequation (3) is violated. Hence, only Case 1 will be considered, in which each retailer sells exactly  $\bar{q}$  units when  $\alpha = \alpha_L$  and each retailer sells an amount higher than  $\bar{q}$  when  $\alpha = \alpha_H$ . It is also the case that can always resolve the DMP, as indicated in the next section.

In contrast to RPM,  $\bar{q}$  in LSBQs provides the manufacturer with more power to choose the market quantity. In addition,  $\bar{R}$  in LSBQs provide some flexibility to choose  $P_w$ . With the compensation of  $\bar{R}$ , retailers are willing to accept a higher wholesale price. It seems that LSBQs are more likely to resolve the DMP. However, the instinctive conflict of choices associated with  $\bar{q}$  and  $\bar{R}$  when LSQB are used yields the undesirable results of Proposition 3.

*Proposition 3 Under LSBQs, the DMP cannot be resolved when the manufacturer maximizes its profits.*

Similarly, to make market quantity reach monopoly levels in two states, one may use the following strategy:  $\bar{q} = \frac{\alpha_L}{4}$  and  $P_w = \frac{\alpha_H}{4}$ . Again, this strategy cannot be the manufacturer's strategy for maximizing its profits. In this class of solutions, the manufacturer's expected profits are

$$2(1 - p)(\bar{q} \cdot P_w - \bar{R}) + 2p \left( \frac{\alpha_H - P_w}{3} \cdot P_w - \bar{R} \right).$$

Suppose that the manufacturer chooses  $\bar{q} = \frac{\alpha_L}{4}$  and  $P_w = \frac{\alpha_H}{4}$ . When market size is small, a higher wholesale price may not reduce the quantity sold because the binding quota. The manufacturer can raise

$P_w$  to increase its profits then. Because  $P_w$  is less than  $\frac{\alpha_H}{2}$ , the optimal level of  $P_w$  when market size is large. Because the manufacturer can increase its profits in both states by raising  $P_w$ , we can conclude now that  $\bar{q} = \frac{\alpha_L}{4}$  and  $P_w = \frac{\alpha_H}{4}$  cannot be the manufacturer's optimal strategy.

Resolving the Double Marginalization Problem

As we know, if the DMP can be ex post completely resolved, then both the market quantity and the market price must be  $\frac{\alpha}{2}$  no matter whether the actual  $\alpha$  is  $\alpha_H$  or  $\alpha_L$ . Hence, the whole channel, in each state, can earn monopoly profits  $\frac{\alpha^2}{4}$  and the expected channel profits occur at the highest level

$$p \frac{\alpha_H^2}{4} + (1-p) \frac{\alpha_L^2}{4}.$$

Due to the incompatibility between the manufacturer's interests and the channel interests, the manufacturer cannot maximize its profits and the channel profits at the same time by using RPM or LSQBs in the retail contract. The manufacturer simultaneously maximizes its profits and the channel profits when it uses TTs in the retail contract. Due to the lack of tools available, the manufacturer can resolve the DMP in one state at most. If the manufacturer is instead allowed to use *ex ante* franchise fees to exploit retailers along with RPM or LSQBs in the retail contract, the manufacturer must be willing to maximize channel profits when designing its retail contracts. However, it emerges below that only LSQBs can always resolve the DMP. First, we consider RPM. Recall that the manufacturer does not have complete freedom to choose  $P_w$  and  $\bar{P}$ . In general, a lower wholesale price must be associated to a lower effective price ceiling, and *vice versa*. In this case, when  $P_w$  is as low as  $\frac{\alpha_L}{4}$ , the equilibrium market price when the market is large (which is also the lower bound of the effective price ceiling) is  $\frac{2\alpha_H + \alpha_L}{6}$  less than the monopoly price  $\frac{\alpha_H}{2}$ . Hence, there is no way of resolving the DMP completely by using RPM in the retail contract.

*Proposition 4: Under RPM, the DMP cannot be resolved when the manufacturer maximizes the channel profits.*

We now turn to LSQBs. Ironically, the failure of RPM sheds light on when the DMP can be resolved. To ensure that the DMP is resolved in both states, the manufacturer must charge a high wholesale price which can cause retailers' losses from sales, especially when the market is small. However, as long as the retailers receive adequate compensation, including through the bonuses under the LSQB scheme, they can be convinced to accept a high wholesale price. As mentioned in the last section, the LSQB scheme does provide the manufacturer with option of charging a high wholesale price. Consider the special class analyzed in the last section.

*Proposition 5 Under LSQBs, when the manufacturer maximizes channel profits, the retail contract with  $\bar{q} = \frac{\alpha_L}{4}, P_w = \frac{\alpha_H}{4}$ , and  $\bar{R} \geq \frac{\alpha_H}{4}$  can always be used to resolve the DMP.*

In the proof of Proposition 5, using first order conditions indicates that the retail contract that can maximize the expected channel profits is exactly the one in Proposition 5. The validness of the strategy is also checked in the following way. The manufacturer's goal is clear.  $\bar{q}$  and  $P_w$  must be  $\frac{\alpha_L}{4}$  and  $\frac{\alpha_H}{4}$  respectively. As long as we can show that there exists a  $\bar{R}$  that ensures that the retail contract is sustainable for the class of solutions considered, the proof is complete. In general, if we can find a set of  $\bar{q}$ ,  $P_w$  and  $\bar{R}$  that can resolve the DMP for a class of solutions, then we can prove that the DMP can be resolved by using LSQBs. In this way, it is easy to show the importance of the flexibility provided by  $\bar{R}$ . Clearly, the superiority of LSQBs is resulted by more instruments to manipulate retailers' behavior than RPM and TTs.

## CONCLUDING COMMENTS

In the modern economy, manufacturers are usually not directly involved in the retail business. Coordination failure among agents in the channel, referred as the double marginalization problem (DMP), results in lower channel profits. To coordinate agents in that channel, different types of vertical restraints are used. This paper focuses on the performance of vertical restraints when the market size is uncertain. Hence, we analyze lump-sum quota bonuses (LSQBs), two-part tariffs (TTs), and resale price maintenance (RPM) in this paper.

The quantity-fixing effect helps to distinguish LSQBs from other vertical restraints in cases in which the market size is uncertain. However, if the quantity-fixing effect is all the manufacturer needs, then why should the manufacturer bother to use LSQBs in the retail contract rather than using the following pricing scheme: the wholesale price for the amount less than  $\bar{q} = \frac{\alpha}{4}$  is  $P_w = \frac{\alpha}{2}$  and an extremely high price for the amount exceeding  $\bar{q}$ . The proposed scheme is highly similar to the tariff quota measure used in the agricultural trade. The LSQB scheme can actually make retailers willing to accept a wholesale price even higher than the market price because bonuses will be awarded. This strategy thus gives the manufacturer more flexibility to choose the wholesale prices, and this flexibility is important to resolving the DMP when the market size is uncertain.

With the quantity-fixing effect and the flexibility to choose wholesale prices, only LSQBs can always resolve the DMP in our two-state case. In contrast, TTs or RPM can only resolve the DMP in one of two possible states at most. Although the LSQBs may not always resolve the problem when there are more than two states, it still reasonable to believe that using LSQBs may create higher channel profits than do the other two vertical restraints due to the quantity-fixing effect and the flexibility to choose wholesale prices. Further, the vertical restraints with more instruments, such as the all-units discounts in Kolay et al. (2004), should be more powerful to resolve the DMP in the case of uncertain market demand.

We do not address welfare analysis in this paper because welfare implication is intuitive in our simple model. Once the DMP is resolved, the lower market price and the higher channel profits guarantee a higher social welfare. However, it is inappropriate to conclude that antitrust authority should not regulate the measure of LSBQs without deeper analysis. TTs and RPM only regulate the aggregate figures in the market, including market price and market quantity. Instead, through the setup of quota and bonus, under LSBQs, the manufacturer has stronger influence on individual retailer's behavior. For example, if the asymmetric equilibrium in the retail market is allowed, the manufacturer can expel one retailer by raising quota and bonus. Clearly, the manufacturer gains more market power under LSBQs. Hence, the impact of LSBQs on social welfare could be controversial. LSBQs can help abating the DMP in the short run, but an upstream firm with strong market power may jeopardize market competition in the long run.

## APPENDIX

### The Proof of Lemma 1

Assume that given  $P_w, \bar{q}$  and  $\bar{R}$ , the best response for  $r_i$  jumps to  $\bar{q}$  as  $q_{-i} = x$  because of the extra bonuses  $\bar{R}$ . Because  $r_i$ 's best response function jumps as  $q_{-i} = x$ ,  $r_i$  should enjoy the same profits regardless of whether  $q_i$  equals  $\bar{q}$  or  $\frac{\alpha - x - P_w}{2}$ . Hence,  $x$  can be solved from

$$(\alpha - (\bar{q} + x) - P_w)\bar{q} + \bar{R} = \left( \alpha - \left( \frac{\alpha - x - P_w}{2} + x \right) - P_w \right) \frac{\alpha - x - P_w}{2}. \quad (6)$$

Accordingly,



$$x = \alpha - P_w - 2\bar{q} \pm 2\sqrt{\bar{R}}, \tag{7}$$

When  $q_{-i}$  is equal to  $\alpha - P_w - 2\bar{q} + 2\sqrt{\bar{R}}$ ,  $r_i$  may respond with a quantity of either  $\bar{q} - \sqrt{\bar{R}}$  or  $\bar{q}$ . Although  $\bar{q}$  yields lower profits from sales,  $r_i$  can earn  $\bar{R}$ . In contrast, when  $q_{-i}$  is equal to  $\alpha - P_w - 2\bar{q} - 2\sqrt{\bar{R}}$ ,  $r_i$  must respond with  $\bar{q} + \sqrt{\bar{R}}$  rather than  $\bar{q}$ . Note that  $r_i$  can still earn  $\bar{R}$  by choosing  $\bar{q} + \sqrt{\bar{R}}$ . Hence, the jump in  $r_i$ 's best response function must occur at  $x = \alpha - P_w - 2\bar{q} + 2\sqrt{\bar{R}}$ .

When  $r_i$  follows the original best response (Equation (1)),  $r_i$ 's profits are

$$\pi_i^{BR} = \frac{(\alpha - P_w - q_{-i})^2}{4}. \tag{8}$$

When  $q_i = \bar{q}$ ,  $r_i$ 's profits are

$$\pi_i^{\bar{q}} = [\alpha - (\bar{q} + q_{-i}) - P_w]\bar{q} + \bar{R}. \tag{9}$$

We differentiate  $\pi_i^{BR}$  and  $\pi_i^{\bar{q}}$  with respect to  $q_{-i}$ ; this yields

$$\frac{\partial \pi_i^{BR}}{\partial q_{-i}} = -\frac{\alpha - P_w - q_{-i}}{2} < 0. \tag{10}$$

$$\frac{\partial \pi_i^{\bar{q}}}{\partial q_{-i}} = -\bar{q} < 0. \tag{11}$$

Comparing Equation (10) and Equation (11); we see that  $\frac{\partial \pi_i^{\bar{q}}}{\partial q_{-i}} < \frac{\partial \pi_i^{BR}}{\partial q_{-i}}$  as  $\bar{q} > \frac{\alpha - q_{-i} - P_w}{2}$ . Recall that  $q_i$  equal to  $\bar{q} - \sqrt{\bar{R}}$  or  $\bar{q}$  yields the same profit as  $q_{-i} = \alpha - 2\bar{q} + 2\sqrt{\bar{R}}$ . Furthermore,  $\bar{q} - \sqrt{\bar{R}} < \bar{q}$ . As  $q_{-i}$  decreases from  $\alpha - 2\bar{q} + 2\sqrt{\bar{R}}$ ,  $\pi_i^{BR}$  is less than  $\pi_i^{\bar{q}}$  until  $\bar{q} < \frac{\alpha - q_{-i} - P_w}{2}$  or  $q_{-i} < \alpha - P_w - 2\bar{q}$ . Hence, we know that  $r_i$  chooses  $q_i = \bar{q}$  when  $q_{-i}$  is between  $\alpha - P_w - 2\bar{q}$  and  $\alpha - P_w - 2\bar{q} + 2\sqrt{\bar{R}}$ .

The Proof of Proposition 1

Note that the relative relationship among  $\alpha_L, \alpha_H$ , and  $P_w$  determines the way of calculating the expected channel profits. Because a  $P_w$  higher than  $\alpha_H$  guarantees no profits, there are only two possible cases below.

Case 1:  $P_w \leq \alpha_L \leq \alpha_H$

No matter what  $\alpha$  is, each retailer sells  $\frac{\alpha - P_w}{3}$  and the market price is  $\frac{\alpha + 2P_w}{3}$ . Accordingly, the expected channel profits are

$$E\left(\frac{\alpha + 2P_w}{3} \cdot \frac{2(\alpha - P_w)}{3}\right) = \frac{2}{9}[E(\alpha^2) + P_w E(\alpha) - 2P_w^2]. \tag{12}$$

Based on the first order condition, we have  $P_w = \frac{E(\alpha)}{4}$ , and the expected channel profits are  $\frac{2}{9}[\alpha + \frac{1}{8}(E(\alpha))^2]$ . Accordingly, the market price and the market quantity are  $\frac{(2+p)\alpha_H+(1-p)\alpha_L}{6}$  and  $\frac{(4-p)\alpha_H-(1-p)\alpha_L}{6}$  respectively when  $\alpha = \alpha_H$ , and the market price and the market quantity are  $\frac{p\alpha_H+(3-p)\alpha_L}{6}$  and  $\frac{(3+p)\alpha_L-p\alpha_L}{6}$  respectively when  $\alpha = \alpha_L$ . Note that the necessary condition for this solution is  $P_w \leq \alpha_L$ . That is,  $\frac{\alpha_H \leq 3+p}{\alpha_L \leq p}$ .

Case 2:  $\alpha_L \leq P_w \leq \alpha_H$

Only if  $\alpha = \alpha_H$ , then each retailer sells  $\frac{\alpha_H - P_w}{3}$ , and then the market price is  $\frac{\alpha_H + 2P_w}{3}$ . Otherwise, the retailers sell nothing. Accordingly, the expected channel profits are

$$p \cdot \frac{\alpha_H + 2P_w}{3} \cdot \frac{\alpha_H - P_w}{3}.$$

Based on the first order condition, we have  $P_w = \frac{\alpha_H}{4}$ , and the expected channel profits are  $p \cdot \frac{\alpha_H^2}{4}$ . Accordingly, the market price and the market quantity are both at the monopoly levels when  $\alpha = \alpha_H$ , but there are no sales at all when  $\alpha = \alpha_L$ . Note that the necessary condition is simply  $\alpha_L \leq P_w$  because the optimal  $P_w, \frac{\alpha_H}{4}$ , is surely less than  $\alpha_H$ . That is,  $\frac{\alpha_H \geq 4}{\alpha_L}$ .

Depending on the value of  $\alpha_L, \alpha_H$ , and  $p$ , Case 1, Case 2, or both cases are sustainable. However, neither the monopoly quantity nor the monopoly price is induced in either state. Hence, the DMP cannot be resolved.

The Proof of Proposition 2

An appropriate  $\bar{P}$  should be binding in at least one state. Hence, there exist three types of possible solutions when the manufacturer adopts RPM in retail contracts.

- Case 1:  $P_w \leq 3\bar{P} - 2P_w \leq \alpha_L \leq \alpha_H$
- Case 2:  $P_w \leq \alpha_L \leq 3\bar{P} - 2P_w \leq \alpha_H$
- Case 3:  $\alpha_L \leq P_w \leq 3\bar{P} - 2P_w \leq \alpha_H$

Because the complete resolution of the DMP requires two different market prices in two different states,  $\bar{P}$  cannot be binding all the time, and no markets can be entirely surrendered. Hence, we only consider Case 2 below.

Although the manufacturer and retailers sell goods in both states,  $\bar{P}$  is only binding when  $\alpha = \alpha_H$ . The manufacturer's expected profits are

$$(1 - p) \cdot \frac{2(\alpha_L - P_w)}{3} \cdot P_w + p \cdot (\alpha_H - \bar{P}) \cdot P_w. \tag{13}$$

If we differentiate Equation (13) with  $\bar{P}$  and  $P_w$ , we have

$$\bar{P}: -p \cdot P_w \leq 0, \tag{14}$$

$$P_w: \frac{2(1 - p)}{3}(\alpha_L - 2P_w) + p(\alpha_H - \bar{P}) = 0. \tag{15}$$

Similarly, based on Inequation (14),  $\bar{P} = \frac{\alpha_L + 2P_w}{3}$ . However, the internal solution suggested by Equation (15) is not necessarily sustainable. After we substitute  $\bar{P}$ , the internal solution becomes

$$P_w = \frac{2\alpha_L + 3p(\alpha_H - \alpha_L)}{4 - 2p}, \tag{16}$$

and the internal solution must satisfy the constraints below.

$$\frac{2\alpha_L + 3p(\alpha_H - \alpha_L)}{4 - 2p} \leq \alpha_L, \text{ or, } \frac{\alpha_H}{\alpha_L} \leq \frac{2 + p}{3p}. \tag{17}$$

If  $\frac{\alpha_H}{\alpha_L} \leq \frac{2+p}{3p}$ , then

$$P_w = \frac{2\alpha_L + 3p(\alpha_H - \alpha_L)}{4 - 2p}, \bar{P} = \frac{4(1 - p)\alpha_L + 3p\alpha_H}{3(2 - p)}. \tag{18}$$

Accordingly, the market price and the market quantity are  $\frac{2\alpha_L + 3p(\alpha_H - \alpha_L)}{4 - 2p}$  and  $\frac{4(1 - p)\alpha_L + 3p\alpha_H}{3(2 - p)}$  respectively

when  $\alpha = \alpha_L$ , and the market price and the market quantity are  $\bar{P}$  and  $\alpha_H - \bar{P}$  respectively when  $\alpha = \alpha_H$ . Otherwise,  $P_w = \bar{P} = \alpha_L$ . In both states, the market price is always  $\alpha_L$ , and the market quantity is  $\alpha - \alpha_L$  when the market size is  $\alpha$ .

The Proof of Proposition 3

In this class of solution, the manufacture earns

$$2(1 - p)(P_w \cdot \bar{q} - \bar{R}) + 2p \left( \frac{\alpha_H - P_w}{3} \cdot P_w - \bar{R} \right), \text{ and the first derivatives with respect to } \sqrt{\bar{R}}, \bar{q}, \text{ and } P_w$$

are

$$\sqrt{\bar{R}}: (-4(1 - p) - 4p)\sqrt{\bar{R}} \leq 0, \tag{19}$$

$$\bar{q}: 2(1 - p)P_w \geq 0, \tag{20}$$

$$P_w: 2(1-p)\bar{q} + \frac{2}{3}p(\alpha_H - 2P_w) = 0. \tag{21}$$

Given any  $P_w$ , the manufacturer should raise  $\bar{q}$  and lower  $\bar{R}$  as possible as it can though the exact choices are still regulated by the necessary conditions.

However, no matter whether the necessary conditions are satisfied or not, the DMP cannot be resolved while the manufacturer maximizes its own profits. The DMP can be resolved only when  $\bar{q} = \frac{\alpha_L}{4}$  and  $P_w = \frac{\alpha_H}{4}$ . It is clear that the combination of  $\bar{q}$  and  $P_w$  cannot satisfy Equation (21).

The Proof of Proposition 4

As seen in the proof of Proposition 2, we only analyze the case in which  $\bar{P}$  is only binding when  $\alpha = \alpha_H$ . The expected channel profits are

$$(1-p) \cdot \frac{2(\alpha_L - P_w)}{3} \cdot \frac{\alpha_L + 2P_w}{3} + p \cdot (\alpha_H - \bar{P}) \cdot \bar{P},$$

The first order conditions are

$$\alpha_L - 4P_w = 0, \tag{22}$$

$$\alpha_H - 2\bar{P} = 0. \tag{23}$$

Accordingly,  $P_w = \frac{\alpha_L}{4}$  and  $\bar{P} = \frac{\alpha_H}{2}$ . Unfortunately, this internal solution, which is the only candidate for resolving the DMP under RPM, does not satisfy the necessary condition in such a case. When  $P_w = \frac{\alpha_L}{4}$ , the equilibrium market price is  $\frac{2\alpha_H + \alpha_L}{6} \leq \frac{\alpha_H}{2}$  when  $\alpha = \alpha_H$ . Indeed  $\bar{P}$  cannot be binding in either state.

The Proof of Proposition 5

Only one case should be taken into account, in which  $\bar{q}$  are binding in both states and each retailer sells exactly  $\bar{q}$  units when  $\alpha = \alpha_L$  (please refer to the discussion in the main text). The expected channel profits are

$$(1-p) \cdot (\alpha_L - 2\bar{q}) \cdot 2\bar{q} + p \cdot \frac{2(\alpha_H - P_w)}{3} \cdot \frac{\alpha_H + 2P_w}{3}.$$

The first order conditions are

$$\alpha_L - 4\bar{q} = 0, \tag{24}$$

$$\alpha_H - 4P_w = 0. \tag{25}$$

Accordingly,  $\bar{q} = \frac{\alpha_L}{4}, P_w = \frac{\alpha_H}{4}$ , which can induce the monopoly quantity and the monopoly price in both states. We need to check whether the below necessary conditions are satisfied.

$$\alpha_L - P_w - 2\bar{q} \leq \bar{q} \leq \alpha_L - P_w - 2\bar{q} + 2\sqrt{\bar{R}}, \quad (26)$$

$$0 \leq \frac{\alpha_H - P_w}{3} \leq \alpha_H - P_w - 2\bar{q}. \quad (27)$$

As long as the manufacturer sets  $\sqrt{\bar{R}} \geq \frac{\alpha_H}{8}$  the necessary conditions must be satisfied.

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