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THE ACCOUNTING EQUATION INEQUALITY: A SET THEORY APPROACH

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ABSTRACT

The basics of financial accounting in the balance sheet and the accounting equation are revisited from the viewpoint of axiomatic set theory and predicate logic. The conceptual distinction between assets and claims on the assets are pointed out; next, it follows an application of the axioms of the theory. By a combination of axioms, this application leads to obtain two sets of capital units, which contains assets and claims (Liabilities plus Owners' Equity) on the assets, respectively. These sets are properly built, according to the use of the axioms; they contain all the lowest level items of the financial statements that still have financial meaning in the balance sheet. An analysis of the equality between these sets was applied to test the equality of the assets to the union of liabilities and equity. The analysis determined that these sets were not equal and as a conclusion assets are not equal to liabilities plus equity. This inequality is interpreted within the restrictions of the application of the set theory to financial data and algebraic sum. Nevertheless, the particular case where the accounting equation holds is described; however, this case has no financial meaning.

JEL: G3, M2, M4

KEYWORDS: Corporate Finances, Financial Accounting, Balance Sheet, Accounting Equation, Set Theory

INTRODUCTION

The balance sheet is based on the equality of assets to liabilities plus owners' equity. This equation is the fundamentals of the financial statements and analysts make significant efforts to classify items and fit the equation. However, it is well-known that different views in financial accounting analysis lead to different financial decision-making as it happens regarding conservatism accounting (Wang, 2013). According to Wang, conservatism involves the prudence principle and its application results in asymmetric different timeliness for recognizing earnings and losses. Therefore, it shows the influence of the figures significance; subjectivity is associated with numbers and changes the financial operations.

Accounting report analysis also contains different approaches. The semiotic linguistic theory analyzes the accounting reports as texts rather than from an economic viewpoint (Macintosh and Baker, 2002). Macintosh and Baker use an approach based on the notion of heteroglossic novel, where accounting has a representational nature; this approach uses the perspective of the literary theory in the analysis of accounting data. Accordingly, it calls for a conversational rather than a monologic process of accounting and, again, the diverse interpretations of accounting information are significant.

Chaos and complexity theories provide a different approach to financial statements and financial accounting. Lewin (1999) explored the implications of complexity in management studies, and Richardson (2008) pointed out that the metaphorical language is a characteristic of the sciences of complexity in management. According to the use of complexity theory in management and corporate finances, financial statements are not a fixed structure but a complex dynamic system. This system comprises many processes and various subsystems located at different levels, interacting with each other and resulting in a proper fit (Juárez, 2013). Juárez (2013) described how the use of belief logic (see belief logic in Smullyan, 1986) is implicit in the management discussion & analysis section and notes of the financial statements. The manner that the analyst communicates the findings in these sections is relevant; it is a communicative act where results depend on the type of reasoning used. In this line, belief and paraconsistent logics are tools that help in the analysis and conclusion of these sections (see Juárez, 2012, 2013, 2014).

Historically, it has been an interest in reviewing the foundations of financial accounting; in 1967, Sterling discussed the theoretical basis of accounting. In 1976, Wells provided an outstanding analysis of the developments and anomalies in accounting thought and, later on, Bauman (1996) explored the theoretical framework in fundamental analysis. Still later, Christensen (2010) discussed the conceptual frameworks of accounting from an information perspective.

Therefore, the basis of financial accounting change continuously and the quest for well-stablished foundations has no end. The possibilities of accounting equation have also been explored (see Lipkin, 1959, Nicol, 1968). However, the accounting equation is always assumed to be true and never questioned.

Usually, the bookkeeper operations and a set of rules guarantee that the accounting equation holds. While this is worthy, a rationale must exist that explain the veracity of the equation and allows reviewing it. Consequently, this research analyzes the justification for the accounting equation.

The remainder of the paper begins with a review of the relevant literature. Next, the paper provides a description of the data and methodology used. The following section presents analytical results. Finally, the paper closes with some concluding comments.

LITERATURE REVIEW

Few works introducing logic and axiomatic set theories into accounting have been done. No doubt, Mattessich (1957, 1964) is one of the first and great authors looking at the accounting system as a logical system; he delivered a consistent theory and formalization of the language of accounting. The developments made by Mattesich were impressive, and he redefined accounting with logical and theoretical research foundations.

Carlson and Lamb's system (1981) is complete and profound in their use of the axiomatic theory, giving a right direction to conduct research on the field. They point out the fact that Mattesich system uses set theory while they use predicate logic, but also that similarities exist between the two systems.

Carlson and Lamb (1981) state that predicate logic is well suited to accounting and the way accounting theory works. They introduce a well-formed system with axioms, inference rules, and theorems. Besides, their methodology includes a nontechnical language, easy to understand, to characterize the axiomatic system. In this system, references to the duality of accounting exist, recognizing the fact that someone exerts a claim on every potential asset inflow. The system also has mechanisms to preserve the accounting equation; i.e. an asset inflow is a debit to the asset account and a credit to a liability or equity account.

Moreover, Carlson and Lamb (1981 introduce some postulates to deduce accounting equation. The equation is defined as the equality of the valuation of the class of assets to the valuation of the class of liabilities plus the valuation of the class of owners' equity. Nevertheless, this rationale for the equation requires the assumption that every time an inflow in assets exists there is recognition in a liability or equity account. Therefore, by these operations, the equality is always maintained.

The application of formal logic to financial accounting leads to different financial statement analysis, with an impact on financial decision-making. The approach of chaos and complexity allows introducing propositional logic, belief logic, circumscription logic and dialogic in financial statements (Juárez, 2013). According to this perspective, transactions on the financial statements are what guarantee that the accounting equation holds; however, contradictions exist in this type of reasoning that gives support to the accounting equation.

As it was stated (see Juárez, 2012, 2013, 2014), several logics coexist in the financial statements analysis. These logics are the following: a) default reasoning; it allows allocating a capital to an item in financial statements unless a counter-example exists; b) abductive reasoning; it justifies assigning a capital to an item whenever a reason exists to do so; c) circumscription logic (see McCarthy, 1980); it lets to enter a new explanation that contributes to solve the problem; d) paraconsistent logic; it focuses on the existent contradiction when an asset valuation is also a debt or equity; and e) the circularity of dialogic (see Morin, 2007) that leads to circularly link consequences to causes. Within this landscape, the complexity of financial statements makes it necessary to review some of the accounting equation foundations.

Based on the above arguments, the aim of this research is not the formulation of a new axiomatic theory about the foundations of financial accounting because good descriptions have already ben provided in the literature. This investigation focuses on the logical truth of the accounting equation, which is the basis of the balance sheet and financial accounting.

DATA AND METHODOLOGY

The method used is objective, rationalistic, deductive and analytical; it uses predicate logic and axiomatic set theory to derive arguments and conclusions with the rationales of this logical and theoretical framework.

The axiomatic set theory of Zermelo-Fraenkel (Zermelo, 2004, original 1908), which is a widely accepted framework in this field, uses predicate logic to postulate its axioms. This theory comprises a set of well-defined axioms that determine the logical operations allowed applying to sets. The theory refers only to sets, so the elements of a set are always sets; it includes the axiom of extensionality, axiom of regularity, axiom of specification, axiom of pairing, axiom of union, axiom schema of replacement, axiom of infinity, axiom of power set, and axiom of choice.

Zermelo created this system because advances in set theory were not accompanied by a proper definition of set (Kragh, 2001). Some logical paradoxes resulted in an intolerable situation to the very notion of set, which is not simply a collection of objects, but objects according to an axiomatic system (Kragh, 2001). The system created by Zermelo remained as the most popular and accepted theory of sets. Fraenkel did some adjustments to the theory and introduced the replacement axiom, resulting in the Zermelo-Fraenkel set theory.

To test the equality of the accounting equation, the set union is assumed to be equivalent to the algebraic sum (Kragh, 2001). The fact that set union and algebraic sum have different properties does not affect the rationale provided.

RESULTS AND DISCUSSION

The accounting equation is

$$\text{Assets } (A) = \text{Liabilities } (L) + \text{Owners' Equity } (E) \tag{1}$$

In the equation, financial resources are allocated to into two groups; one of them is assets, and the other one is liability plus owners' equity.

Any capital unit $\{u_i\}$ (whatever it is) is an asset, liability or owners' equity, based on the financial definition of what an asset, liability or owners' equity is. Financial definitions can be of any type; they are of no particular interest here, and the only requirement is that they agree with the standards in the academic field; so no new definitions are needed. Besides, any pair of capital units are different, i.e. $\{u_i\}$ and $\{u_j\}$ are different, even having the same value. In set theory, it is necessary to make explicit this difference; in case these capital units were not different, a set having several capital units, would be the same as a set having just one.

The specification axiom allows creating subsets by a formula ϕ; this axiom creates a subset with all its elements (sets) having the property defined by ϕ. The formula ϕ allows for identifying a subset y, such as it contains every element x of the set z with the property described in the formula ϕ. This formula could be

$$\phi_A\text{: } u_A \text{ is a company asset} \tag{2}$$

where u_A is a capital unit.

The formula is also applied to liabilities and owners' equity, creating the subsets A, L and E. They are subsets because are part of a financial system, and the financial statements of a company or other entity are subsets in this system. As far as all the financial statements have the same theoretical structure in the financial system, the specific company has no interest in the analysis.

The application of these definitions and axiom result in the sets A, L and E. This classification is what the analyst uses when assigning capital transactions to the financial statements. These sets comprise all the financial resources of the organization. The formula ϕ applies to all the sets in the financial statements, resulting in all the subsets or items in the financial statements.

The assets of the organization have the same value than the claims on the assets. Nevertheless, the conceptual distinction between assets and claims, along with the fact that financial statements are classified, make it necessary to check whether the equality proposed in the accounting equation is real. This conceptual distinction is what make it possible to define the accounting equation; in the absence of this distinction, it would not be necessary to verify that assets are equal to liabilities plus owners' equity.

The structure of financial statements needs defining it as a structure of sets and subsets. The Zermelo-Fraenkel set theory allows defining a subset as that whose elements are also elements of another set. Accordingly if a set z is a subset of x and x is a subset of a set y then z is a subset of y. Therefore, and using a simplified structure, the set A comprises the subsets current assets A_c and non-current assets A_{nc}. Current assets A_c comprise cash A_{cc} and accounts receivable A_{car}, while non-current assets A_{nc} comprise long-term investments A_{nclti}, property, plant and equipment A_{ncppe} and intangible assets A_{ncia}.

In the same manner, the set L contains different subsets, such as current liabilities L_c and non-current liabilities L_{nc}. Current liabilities L_c includes, in turn, other sets such as accounts payable L_{cap} and unearned revenues L_{cur}, while non-current liabilities L_{nc} include the set mortgage payable L_{ncmp} and notes payable L_{ncnp}. The set owners' equity E contains issued capital E_{ic}, common stocks E_{cs} and retained earnings E_{re}.

Another application of the specification axiom with the formula ϕ_C is

$$\phi_C\text{: } u_C \text{ is a claim on a company asset} \tag{3}$$

It results in another set C containing all the claims made on the assets of the company. This set C contains the subset L and E, and the elements of L and E are elements of C.

The axiom of union states that the union of sets is a set that contains the elements of the elements of another set. Therefore, if a set X contains several subsets z and these elements contain several subsets w, then the union of the elements w of the subsets z of the set X, is another set Y.

According to the axiom of union, the set L_u consists in the union of all of the liabilities individual elements or subsets. In this way, the set L_u comprises all the liabilities of the organization. By the axiom of specification, this set is included in C.

Then, a new set can be defined, the set C_u, which is the union of L_u and E; this new set comprise all the elements that are claims on the assets. These elements contain capital units grouped in categories with financial meaning; these categories are the items in the financial statements. Applying the same procedure to the assets, the set A_u is the union of all of the elements of A_c and A_{nc}. The elements of A_u are those elements that still keep financial meaning. These elements contain just capital units.

As a result of this previous operations on sets, there are two sets A_u and C_u, and these sets contain all the lowest level items in the financial statements that still have financial meaning. It is so because these items (or elements of A_u and C_u) have a proper item name in the financial statements. The elements of these sets (items) are the single capital units $\{u_i\}$. Accordingly, the sets A_u and C_u contain subsets that comprise sets of capital units; nevertheless, the subsets are not the same in A_u and C_u, because the classified financial statements do not have the same items in assets and liabilities or owners' equity.

The truth of the accounting equation is analyzed by comparing the sets A_u and C_u. Thus the quality to test is

$$A = L \cup E \tag{4}$$

Successive applications of the set theory axioms, the union $L \cup E$, and the union of the subsets A_i of A, resulted in two sets A_u and C_u; each of them has the classified capital units of the assets and claims on the assets, respectively. Accordingly, the equation is interpreted as the equality of these sets.

In Zermelo-Fraenkel set theory, the equality of two sets is defined by the axiom of extensionality. According to this axiom, for two sets to be equal they need to have the same elements or subsets. Therefore, the set x would be equal to the set y if for all the subsets z, whenever z is a subset of x then z is a subset of y, and whenever z is a subset of y then z is a subset of x. Only under these conditions, a set x is equal to y.

Translating that to the formulation of the accounting equation in the set theory, it would mean that the subsets of C_u are equal to the subsets of A_u, and they would have the same elements. Applying the axiom of extensionality to the subsets A_i of A_u and the subsets C_i of C_u, the subsets C_i must be equal to the subsets A_i, and all of the capital units $\{u_i\}$ in a single set C_i are in a unique set A_i. In other words, there must a subset A_i of A_u identical to a subset C_i of C_u and all of the subsets C_i must have a corresponding set A_i.

However, the final structure of the financial statements, obtained by the set operations, still keeps financial meaning, and the capital units are classified into the original items of the financial statements. The applications of the Zermelo-Fraenkel set theory axioms resulted in a lowest level item categories, but still they are the original items by the financial classification. The financial operations of a company do not take into account that every capital unit $\{u_i\}$ in an item C_i must be located in particular A_i. The capital of the company moves to different items A_i depending on the organization needs.

Therefore, the capital units of each subset in C_u are spread over different subsets in A_u,. The organization does not allocate the capital of a subset of C_u to a subset of A_u, but it distributes it among all possible types of assets and, at least, it exists a subset C_i of C_u, such as its elements spreads over several subsets of A_u. In the same sense, the specification axiom allows identifying capital units in the subsets A_i of A_u included in a C_i of C_u. By the application of this axiom, it is found that the subsets of A_u does not agree with the subsets of C_u. The axiom can use the formula

$$\phi = \text{elements } \{u_i\} \text{ of a set } A_i \text{ that are also members of a particular } C_i \tag{5}$$

The application of this axiom with this formula will restrict the elements $\{u_i\}$ of a new set called A_{ie} to those $\{u_i\}$ of A_i that are members of a given C_i. Then, by the axiom of extensionality, in case of $A_u = C_u$, it should be that $A_{ie} = A_i$;. It means that all of the elements of the set A_i are in A_{ie} because all of them are in a unique C_i. However, this is not the case and $A_i \neq A_{ie}$.

According to these rationales,

$$A_u \neq C_u \tag{6}$$

and

$$A_u \neq L_u \cup E \tag{7}$$

In a more general formulation

$$A \neq L \cup E \tag{8}$$

Alternatively, assuming the equivalence between the set union and algebraic sum

$$A \neq L + E \tag{9}$$

However, a special case exists that makes the equality of the equation true.

A new application of the axiom of the union to the sets A_u and C_u would result in sets A_{uu} and C_{uu}. The set A_{uu} has the elements of the subsets of A_u, i.e all the single capital unit sets $\{u_i\}$ of every A_i. In the same manner the set C_{uu} has the elements of the subsets of C_u,, i.e. all the single capital unit sets $\{u_i\}$ of every C_i. Applying the axiom of extensionality to the sets A_{uu} and C_{uu}, it is easy to see that these sets are equal because all the capital units are in both of them. Accordingly

$$A_{uu} = C_{uu} \tag{10}$$

This result would mean that assets A are equal to claims (L plus E) on the assets.

$$A = C \tag{11}$$

In general, it could be said

$$A = L + E \tag{12}$$

However, this solution is trivial and has no financial meaning as it is a comparison of just amounts of capital that are equal because they are the same capital. Finally, it must be noted that the inequality of the accounting equation was determined by breaking the structure of the financial statements into its smallest parts with financial meaning. Nevertheless, a rebuilding of this structure to higher levels would lead to the same result; in no other levels it is found a set equality.

CONCLUDING COMMENTS

According to the result, when using items with financial meaning, the accounting equation is an inequality. The result needs to be understood within the restriction of the set theory when applied to algebraic operations.

It has been pointed out that the union of two sets is equal to the sum (Kragh, 2001). However, the properties of the union of sets are not equal to those of the algebraic sum. Nevertheless, although the algebraic sum also has a formulation in the set theory, it was not introduced here. The analysis focused on the set union, which can be considered similar to the algebraic sum, in some way. The application of the axiomatic set theory along with predicate logic led to the conclusion that the accounting equation is an inequality. Even with the restrictions of the method, this finding has some implications for the foundations of the balance sheet and financial accounting.

This result requires more research to be conclusive. However, it is part of a research project in corporate finances, where the analysis of the foundations of financial accounting and, specifically, the accounting equation, has several approaches. These methods include the application of the axiomatic set theory, predicate logic and other types of logics, along with different forms of mathematical analysis. All of them led to similar results.

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