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ASSET PRICE VOLATILITY AND EFFICIENT DISCRIMINATION IN CREDIT MARKET EQUILIBRIUM

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ABSTRACT

Significant variation in the terms and volume of lending across classes of borrowers distinguished only by qualities independent of credit risk is often interpreted as evidence of inefficient or inequitable discrimination in credit markets. Increasing accuracy in the measure of credit risk renders common theories of lending discrimination and credit rationing based on lender preferences or asymmetric information increasingly implausible. We consider a traditional model of lending with complete markets, in which equilibria may exhibit disparate loan terms or access to credit across such classes of borrowers, despite common knowledge about the parameters describing the market. Rather than evidence of inefficient equilibria owing to discrimination, however, such equilibria can arise solely from the influence of asset price volatility on participants strategically exercising the options embedded in standard debt contracts. Extending substantially different loan terms or even rationing credit to different classes of borrowers can be a rational response by value-maximizing lenders when borrower classes are correlated with the degree of price volatility exhibited by the otherwise similar assets being financed by members of each of these classes. We discuss our results in the context of actual credit markets.

JEL: G13, G15, O16

KEYWORDS: Credit Rationing, Debt Contracts, Discrimination, Stochastic Differential Game

INTRODUCTION

Observations that some borrowers encounter significantly different loan terms than other Borrowers, from whom they appear indistinguishable with respect to credit risk, are common in many economies. Similar observations of periodic credit rationing to such borrowers are also frequently alleged. If true, these events pose a significant anomaly for the efficiency of credit markets. Two types of explanations for these observations are traditionally offered. The first, popular among policymakers and the public, asserts that lender preferences over classes of borrowers distinguished by observable qualities or traits which are independent of credit risk. Assuming lenders possess market power, these preferences could lead to systematic discrimination between these classes through the offer of different terms and magnitudes of credit. The second type of explanation involves an asymmetric dispersion of information across lenders and borrowers. Both adverse selection and moral hazard can produce different loan terms and credit rationing across distinct groups of borrowers who differ by unobservable probabilities of default. These models, however, cannot explain patterns of lending discrimination between borrowers based on observable qualities that are independent of credit risk.

We offer a novel explanation of discrimination and rationing in credit markets that is fundamentally different from these. We demonstrate that allocations of credit consistent with observations of discrimination can arise in efficient credit markets, in contrast to the reliance on exogenous sources of inefficiency, such as preferences and market power or asymmetric information. Our model features a representative credit market embedded in a continuous-time economy exhibiting complete markets and arbitrage-free valuation. We augment this classical environment through consideration of the strategies chosen by lenders and borrowers bargaining over the terms of a standard secured loan. These strategies include the respective choices of the parties to exercise the options embedded in such a loan. The strategic timing of their exercise depends, in part, on the properties of the asset serving as collateral. Optimality in the respective selection of strategies yields a unique non-cooperative equilibrium in which contrasting loan terms are offered to different classes of borrowers. Each such class is distinguished only by an observable trait displayed by its members while these members appear identical by standard measures of credit risk. Evidence of systematic discrimination, commonly interpreted as inefficient or inequitable, is entirely based on these two criteria. Under the condition that lenders perceive a correlation between a given class of borrowers and the properties of the representative asset securing their loans, however, we show that endogenous discrimination in the allocating credit across different classes of borrowers will be observed in market equilibrium. Unlike the inefficient equilibria exhibited in earlier approaches involving lender preferences or asymmetric information, however, our equilibrium arises in an economy with complete markets and is efficient in its allocation of credit risk. The paper is organized as follows. We review the relevant previous research on equilibrium credit discrimination in the next section. Our model is described in the succeeding section, followed by selected remarks. Concluding remarks appear in the final section.

LITERATURE REVIEW

The significance of this paper is in its resolution of the anomaly of discriminatory loan terms and credit rationing within a market satisfying the classical assumptions of finance theory. This section compares and contrasts our analysis to previous explanations of lending discrimination and credit rationing. A traditional approach to explaining idiosyncratic lending patterns across different borrowers was pioneered by Hodgman (1960) and Freimer and Gordon (1963) , and refined in subsequent models by Jaffee and Modigliani (1969), Barro (1976), Baltensperger (1976), and others. Differential access to credit arose from an exogenous inability of lenders to price default risk across individual borrowers. While able to predict lending patterns, which, for a suitable choice of parameters, could approximate the empirical patterns commonly, interpreted as demographic discrimination, these models could not generate endogenous credit market equilibria exhibiting discrimination based on borrower characteristics.

Stiglitz and Weiss (1981) analyzed the endogenous decisions of lenders who, owing to the assumed presence of either adverse selection or moral hazard, could price risk to individual borrowers only on the average characteristics commonly exhibited by a group of borrowers, despite differences in credit risk among individual members of such groups. Second-best debt contracts incorporating screening resulted in lending terms, which differed across groups, as well as circumstances in which a group could experience the rationing of credit. More recent research, including Hart and Moore (1998), Gorton and Kahn (2000), Martin (2008), Saavedra (2015) and others, have examined the influence of monitoring costs and renegotiation in the design of contracts in credit markets with adverse selection or moral hazard. Models based on asymmetric information are unsuitable, however, in explaining discrimination owing to observable traits distinguishing borrowers of equal credit risk, since the equilibrium allocation of credit depends entirely on contracts screening borrowers only for unobservable characteristics directly germane to individual credit risk. The market equilibria in such models are necessarily inefficient in allocating risk. Our model and results are distinguished from earlier quantitative as well as anecdotal research through both endogenously deriving lending discrimination using only the traditional assumptions in finance and in generating equilibria on this basis, which are efficient in the allocation of credit risk.

METHODOLOGY

We model the negotiations of a loan contract by a representative borrower and lender with a stochastic differential game. The optimal strategies of each player are determined through the valuation functions of each party about their respective claims on the asset serving to secure the loan. Since the economy is assumed to have complete markets, we apply the standard arbitrage-free valuation method to derive a pair of linked partial differential equations and corresponding boundary conditions, which must be satisfied by the values of these contingent claims. Defining the state space of the game to be the support of all asset values and dates relevant to the decisions of the borrower and lender, we employ recursive methods to endogenously derive, through these differential equations, those subsets of this state space which represent the strategy spaces of each of the parties. A numerical solution for the game is obtained by finding those sequential paths in the state space which represent the "best-reply" strategies of the parties and which, in turn, determine market equilibrium. Each choice of parameter values for the market yields a distinct and unique equilibrium. The comparison of the properties of interest exhibited by alternative equilibria are compared and through this comparison we characterize lending discrimination in the form of loan terms and the volume of credit exchanged between lenders and different classes of borrowers.

Model

Consider a representative credit market in which participants trade, over dates t a single risky asset of value a. Participants also have access to a riskless asset with maturity T and return r. The market operates in a continuous time economy, which satisfies the assumptions of the classical asset valuation environment. In particular, this economy has a complete filtered probability space $[\Omega, \Im, P]$, where Ω represents the space of events in this economy, $\Im$ represents the corresponding filtration, a set of sequential sigma algebras $\{ \Im_t \}_{0\leq t \leq T}$ representing information available to traders at time t, and P is the actuarial probability measure over asset value and defined on Ω . All market participants observe each $\Im_t$ and base all decisions at each date t, $0 \leq t \leq T$, on this observation. All such observations are also assumed common knowledge among participants. All diffusions describing asset and contract values in the economy are also assumed to be well defined and adapted in this space. Asset markets are complete with respect to the source of risk arising in our representive credit market. It exhibits, consequently, arbitrage free valuation.

Debt contracts in this market correspond to a standard nonrecourse form under which a lender advances a unit of credit to a borrower at date 0 for which the borrower is obligated to remit both continuous coupon payments at a constant rate c until maturity T and a terminal balance $C(T)$, where $C(t)$ represents the unpaid loan balance at all dates $t \in [0,T]$. The rate premium γ paid by the borrower is determined by the constant coupon rate specified in the contract, relative to the riskless rate r. The asset being financed serves as the unique collateral securing the loan. Its value at date t, $a(t)$, evolves according to the diffusion

$$a(t) = \alpha(a,t)dt + \sigma a(t)dz(t) \tag{1}$$

where $z(t)$ is a standard Brownian motion, $\alpha(a,t)$ is the expected drift at all t and σ is a constant volatility parameter. Since these features are exogenous and cannot be influenced by either party to the loan, neither moral hazard nor adverse selection are present in this market. The borrower receives a continuous flow of value $\pi(a,t)$, representing profits net of depreciation costs, while he services the loan. Foreclosure, following a failure by the borrower to pay c at any date t, results in sale of the asset. The lender receives $max[\, a(t) - b(a,t), 0]$ from this sale, where $b(a,t)$ is the liquidation cost incurred by the lender, and the borrower receives any residual funds exceeding the unpaid balance $C(t)$. Bargaining over the terms of the loan contract is represented by the choice of intertemporal strategies by borrower and lender. Their respective strategy spaces include choice of the timing of any exercise of the range of options normally embedded in the loan covenants. The principal element of the space of the lender is, in the current paper, his specification of the initial amount of credit advanced, *C(0)*, and, contingent on default, the date of foreclosure. This space could, in a more complex version of the model, be augmented to include several additional options, including the right to call the loan. The principal strategic elements chosen by the borrower are the timing of both his exercise of the option to default or to prepay the existing loan balance. The strategy of the lender and of the borrower are each selected to maximize, subject to the strategy of his counteparty, the value of his contingent claim on the asset collateralizing the loan.

Denote by *L(a,t)* the values to the lender of payments received from the borrower and from his option to foreclose and denote by *B(a,t)* the value of the borrower's position in the contract, also inclusive of his options to default or prepay. Consistent with the traditional features of our economy, application of standard arbitrage pricing methods yield the respective functions *L(a,t)* and *B(a,t),* generating these values for all possible combinations *(a,t).* Solutions for these value functions, under each choice of parameters, represent the respective values of the debt and equity claims on the asset in a perfect Markovian equilibrium. Solutions to these functions satisfy a pair of partial differential equations, linked by the best-reply strategies selected by each party and by the respective boundary conditions for each equation. This pair of equations is

$$rL = \left(\frac{1}{2}\right)(a\sigma)^2 L_{aa} + (ra - \pi)\mathrm{L_a} + c + L_{\mathrm{t}}, \tag{2}$$

$$rB = \left(\frac{1}{2}\right)(a\sigma)^2 (a\sigma)^2 B_{aa} + (ra - \pi)B_a + c + B_t \Big), \tag{3}$$

with the corresponding boundary conditions

$$L(\breve{a}, t) = max\{0, \breve{a} - \eta(\breve{a}, t)\}, \tag{4}$$

$$\pi(\breve{a}, t) - c + E_t B^*(\breve{a}, t) = 0, \tag{5}$$

and

$$L^*(\hat{a}, t) = C(t), \tag{6}$$

$$\pi(\hat{a}, t) - c + E_t B^*(\hat{a}, t) = 0, \tag{7}$$

The term $E_t(\bullet)$ is the expectations operator under the unique equivalent martingale measure induced by our assumption of complete markets and $B^*(a,t) = (e^r dt)B(a+da, t+dt)$ and $L^*(a,t) = (e^r dt)L(a+da, t+dt)$ are the respective risk-adjusted values of the claims of the borrower and lender, discounted at the riskless interest rate. Denoting by $\bar{\bar{B}}(a,t)$ and $\bar{\bar{L}}(a,t)$ the respective values of the parties' claims if the loan terminates through the exercise of an option by either party, the terms $\breve{a}$ and $\hat{a}$ are the respective asset values triggering default and prepayment b the borrower. At these values the functions *B(a,t)* and *L(a,t)* satisfy the value-matching and smooth-pasting criteria. The value-matching condition requires the borrower's value function to be continuous at the respective asset value inducing him to default at date *t*, $\breve{a}$, or to prepay at date *t*, $\hat{a}$, as defined respectively by

$$B(\breve{a}, t) = \bar{\bar{B}}(\breve{a}, t), \tag{8}$$

$$B(\hat{a}, t) = \bar{\bar{B}}(\hat{a}, t), \tag{9}$$

while the smooth-pasting condition requires the first derivatives of $B(\bullet)$ and $\bar{\bar{B}}(\bullet)$ to be continuous at these same points. The lender's value function is required to satisfy analogous criteria at those distinct points where he would exercise his option to foreclose or, if the game allows this option, to call.

Numerical Solution Procedure

Since the finite maturity of the loan precludes an analytical solution, we characterize market equilibrium through numerical solutions for the valuation equations and boundary conditions (2)−(7). We use a recursive finite difference procedure to obtain these solutions. This requires representation of the respective strategies of the lender and borrower in terms of subsets of the underlying state space of our game. This state space is defined by all specific pairs of exogenous values *a* of the asset and corresponding dates *t* relevant to the respective strategy choices by the lender and borrower. If the set of all asset values is denoted by $\mathcal{A}$ and the compact set of all dates relevant to the loan contract by $\mathcal{T}$, then the state space of the game is $\mathcal{A} \times \mathcal{T}$, which is the support of the continuum of all possible states *(a,t)*. It is within $\mathcal{A} \times \mathcal{T}$ that the strategies chosen by the lender and borrower are nested.

Any strategy chosen by either of these parties, consequently, can, for the purposes of a numerical solution for equilibrium, be represented as regions (subsets) of $\mathcal{A} \times \mathcal{T}$. These sets comprising each feasible strategy of the lender and borrower are defined by the property that any realization of the state *(a,t)* within them triggers the exercise of an option by one of these respective parties. One element of every strategy of the lender, for example, consists of his choice of the initial balance $C(0) \in \mathcal{A}$ at the origination of the loan. A second element is his choice of a date, contingent on default by the borrower, at which he chooses to foreclose. Since, in the current model, the lender will always choose to foreclose immediately at default, the lender's strategy space can be entirely represented by the closed set $\mathcal{A} \times \{0\}$. Similarly, the strategy of the borrower can be represented in terms of two closed regions of $\mathcal{A} \times \mathcal{T}$. The first is $\mathcal{D}$, which is that subset of $\mathcal{A} \times \mathcal{T}$ containing all states at which the borrower chooses to default by ceasing the coupon payment. The analogous region in which the borrower will prepay the outstanding loan balance, should his asset be sufficiently valuable, can be denoted by $\mathcal{P} \subset \mathcal{A} \times \mathcal{T}$. The strategy selected by the borrower is the union of these respective regions, $\mathcal{D} \cup \mathcal{P}$.

The endogenous derivation of numerical solutions for *L(a,t)* and *B(a,t)* proceeds by using the values of the claims of each party. Representing $\mathcal{A} \times \mathcal{T}$ as a discrete rectangular grid of *a* and *t* values, a solution for equilibrium of our game proceeds by calculating the respective values for $L(\bullet)$ and $B(\bullet)$ at each grid point (*a,t*). These values are determined using the Crank-Nicholson method of calculating the discrete approximation for the partial derivatives in equations (2) and (3). The regions comprising the strategy of each of the parties are then found by calculating, from every point at maturity $\mathcal{T}$, the respective values of the loan to each party. This calculation is repeated at $\mathcal{T}$-1 and, in general, at each prior grid point $t - 1$ accessible from each of the analogous points at *t*. These values indicate whether, at each such grid point, either party could do better by exercising his options and terminating the loan. A numerical solution for the unique equilibrium of the game, given the choice of parameter values describing the exogenous features of the market, is then obtained by using these values to find the unique 'path,' for each node at $\mathcal{T}$, of *(a, t)* values through our discrete approximation of the state space $\mathcal{A} \times \mathcal{T}$ for which the subsets we derived constitute best-reply strategies for the parties. The characterization of discrimination in the market is then obtained by comparing the terms and volume of credit for the loan contract in the equilibria generated by each of our choices of alternative parameter values.

RESULTS AND DISCUSSION

Selection of alternative sets of values of the parameters determining the stochastic evolution of the asset and institutional features of the market allow us to compare the properties of the strategies and debt and equity values in the equilibrium arising for each such set. Such comparisons allow us to measure the relative difference between loan terms and the volume of credit exchanged in each equilibrium. We can, therefore, numerically assess the existence and magnitude of systematic lending discrimination in these comparisons. Since the features most frequently cited in empirical evidence of lending discrimination, such as in analyses of HMDA data in the United States (Avery, Brevoort and Canner (2007)), are comparisons of the amount of credit obtained at loan origination and selected loan terms, we focus the presentation of our results on these two properties of credit market equilibrium. We use the initial loan balance, which is also the current value of the loan to the lender $C(0)$, to measure the amount of credit and the rate premium π to represent the terms of the loan and compare these values across the equilibria corresponding to alternative sets of parameters describing the exogenous features of the credit market. The existence of a perceived correlation between classes of loan borrowers distinguished by traits unrelated to credit risk and the parametric characteristics of the diffusion process describing the value of the representative asset securing the equilibrium loan contract for each class yields different loan terms and balances offered to the members of each such class.

The exogenous features of the market we consider include the instantaneous mean $\alpha(a,t)$and proportional volatility σ exhibited over time by the collateral asset; the net flow of value $\pi(a,t)$ accruing to equity in the asset; the cost $b(a,t)$ incurred by the lender in liquidating the asset in the event of foreclosure; and any cost $f(C(t))$ incurred by the borrower should he prepay the loan. We assume, for simplicity in the presentation and discussion of the parametric cases below, that the net revenue flow from the asset $\pi(a, t)$, liquidation costs $b(a, t)$ and prepayment costs $f(C(t))$ are all independent of time and homogeneous in their arguments. These allow us to represent conveniently the initial loan balance as a percentage of the initial value of the asset. We interpret as annual the per-period values for the riskless rate and rate premium, the asset volatility and the maturity of the loan as a convenience for the reader.

We present our results as equilibrium values of the initial loan balance $C(0)$, per corresponding rate premium γ, for alternative values of a chosen exogenous variable, holding constant all other parameters at benchmark values. The benchmark values underlying the results presented below are, respectively, a riskless annualized interest rate of $r = .03$, an annualized proportional volatility (standard deviation) in the value of the collateral asset of $\sigma = .2$; an instantaneous flow of value to equity π of ten basis points, a maturity T of five periods, a one basis point flow of coupon payments c; and both liquidation $b(a,t)$ and refinancing $f(C(t))$ costs of zero. We also specify six gridpoints, or equivalently five 'periods,' over the state space $\mathcal{A} \times \mathcal{T}$ for our calculations of the numerical solutions to equations (2) – (7).

We first consider the influence of liquidation costs on the equilibrium amounts and terms of credit and show that, above a certain threshold rate premium, the lender will rationally ration credit. Table 1 illustrates this by depicting how loan balances $C(0)$ vary with successive two percentage point increases in the corresponding equilibrium rate premium γ , as the costs b of asset liquidation in foreclosure increase from a 'low value' (10%) to a 'high' value (30%). Higher balances correspond to higher rates in each case, but, as expected, higher liquidation costs reduce the initial loan balance at each rate premium. Averaged over the 12-percentage point range of rates, initial balances are 86.34% and 78.37% at the respective low and high liquidation costs, which is approximately an eight-percentage point difference. Balances also increase, at a decreasing rate, as rates rise from 2% to 8%, but with a lower average increase as liquidation costs increase. Note, in particular, that, as liquidation costs become sufficiently high, rate increases beyond a threshold point (8% in this example) elicit virtually no increases in initial balances. Credit, after this threshold, is strictly rationed.

Table 1: Loan Terms: Effects of Liquidation Costs

Rate Premium	Case One (*B* = 10%)	Case Two (*B* = 30%)
□	*C (0)*	*C (0)*
0.02	71.70	57.60
0.04	82.70	65.00
0.06	89.00	68.00
0.08	93.00	70.70
0.10	95.50	71.80
0.12	97.20	71.90
0.14	98.30	71.90

Table1 illustrates the influence of liquidation costs on equilibrium lending terms. Case One shows that, for relatively low costs (b=10%) of liquidating the asset at default, the initial loan balance is 71.7% at a rate premium of 2%. This balance increases monotonically with this rate until it reaches 98.3% of the asset value at a 14% rate. Case Two illustrates, however, that higher liquidation costs reduce both the amount of credit available at each rate premium as well as rate at which that amount increases at each successively higher rate. Liquidation costs of 30% reduce the initial loan balance to only 57.6% at a two percent premium while the increases in the balance for equal rate increases steadily decline until, at a rate premium of 10%, the balance reaches only 71.90%. True credit rationing occurs after this point, with subsequent rate increases producing no increase in credit. Higher rates provide no increase in value for the lender because liquidation costs imply that the increased risk of default they induce outweighs any increase in the value of higher coupon payments.

Table 2 illustrates the influence of price volatility on credit available at each rate. When volatility is relatively 'low' (σ =15%), for each increase in rates balances increase, but at a rate that decreases sharply from 14% at a 2% premium to 1% at a 12% premium. Doubling the volatility, as in Case Two, reduces the amount of credit available at each corresponding rate at an average decline of 10.2 percentage points relative to the lower volatility. Successive rate increases accompany balances increasing, again at a decreasing rate. Note, however, that in contrast to the analogous comparison involving liquidation costs, the average successive increase in balances is greater when volatility is high than when it is low.

Table 2: Loan Terms: Effects of Asset Price Volatility

Rate Premium	Case One (σ = 15%)	Case Two (σ = 30%)
π	*C (0)*	*C (0)*
0.02	78.10	58.10
0.04	88.70	73.70
0.06	93.60	82.10
0.08	96.20	87.50
0.10	97.70	91.10
0.12	98.70	93.70
0.14	99.20	94.40

Analogously, Table2 illustrates the influence of annual volatility in asset value on equilibrium lending terms. Case One shows that, for relatively low volatility (σ =10%), the initial loan balance is 78.10% at a rate premium of 2%. Credit available to the borrower increases monotonically with this premium until it reaches 99.2% of the asset value at a 14% rate. This increase in credit, however, exhibits a sharply decreasing rate of increase, from approximately a 14% growth in initial balance as the premium rises from a value of 2% to 4%, to only 1% growth as the premium rises from a value of 12% to 14%. Higher volatility (σ = 30%), as expected, reduces the amount of credit available, at each corresponding value of the rate premium, to an average balance of 83% relative to an average of 93.4% at the lower volatility. ,

We now illustrate, in the last set of our selected results, the effects on the availability of credit from simultaneous variation in the volatility of asset value and costs of liquidating that asset at foreclosure. Table 3 measures these effects through four parametric combinations corresponding to those in Tables 1 and 2. In the first case, loan terms are shown for 'low' (b=10%) and 'high' (b=30%) liquidation costs, conditional on the 'low' value (15%) of price volatility. The second case repeats this same increase in liquidation costs, but now at the 'high' volatility value (30%).

Table 3: Loan Terms: Combined Effects of Liquidation Costs and Asset Price Volatility

Volatility (15%)			Volatility (30%)		
	b = .10	b = .30		b = .10	b = .30
γ	C(0)	C(0)	γ	C(0)	C(0)
0.0200	0.7690	0.7050	0.0200	0.5230	0.4990
0.0400	0.8400	0.7660	0.0400	0.6610	0.5600
0.0600	0.8700	0.7910	0.0600	0.7400	0.6030
0.0800	0.8840	0.7980	0.0800	0.7890	0.6340
0.1000	0.8900	0.8040	0.1000	0.8220	0.6540
0.1200	0.8940	0.8090	0.1200	0.8440	0.6480
0.1400	0.8970	0.8130	0.1400	0.8590	0.6420

The effects on equilibrium loan terms of simultaneous variation in the volatility of asset value and costs of liquidating that asset at foreclosure are illustrated in Table 3. Loan terms, again represented by the rate premium and corresponding initial loan balance, are shown for four combinations of parameter values. In the first case, loan terms are shown for the same 'low' (b=10%) and 'high' (b=30%) liquidations costs as considered in Table 1, conditional on the annual price volatility of 15% used for Case One in Table 2. The second case shows these same loan terms for the same two values of liquidation cost, but now conditional on the annual price volatility of 30% used for Case Two in Table 2.

Three aspects of our results are particularly notable. First, the data in Tables 1-2 clearly illustrate, as expected, that loan balances at any given rate are considerably lower for borrowers when lenders incur higher liquidation costs at constant price volatility or when price volatility increases at constant costs of liquidation. Table 3 illustrates, in addition, that the adverse impact on credit caused by a given increase in liquidation costs or volatility is significantly worsened by a respective increase in volatility or liquidation costs. Consider, for example, that the approximately 6 percentage point decline in average balances caused by a given rise in liquidation costs from 10% to 30% at an annual volatility of 15% increases to an approximately 22 percentage point decline in average balances for the same increase in liquidation costs at an annual volatility of 30%, a difference of 16 percentage points. A similar difference in average balance decline caused by a doubling of volatility occurs when the costs of liquidation increase.

Second, the response of balances to small variations in loan rates also differs in these same situations. Our results in Table 3 suggest that a given increase in liquidation costs will have a significantly smaller effect on the rate balances grow, per unit increase in loan rates, when price volatility is relatively low than when it is high. When annual volatility is 15%, for example, an increase in liquidation costs from 10% to 30% corresponds to a 20 basis point decrease in the average growth of balances per unit rate increase, but the same increase in liquidation costs at an annual volatility of 30% corresponds to a 454 basis point decrease. The opposite is true, however, for the case of a given doubling of price volatility at low, relative to high, liquidation costs.

The third, and most striking, feature of our results is that not only can the 'rationing' of credit occur to different borrowers, based on the characteristics of the assets used to collateralize their respective loans, but, for possible, if extreme, parameter values in the market, increases in the rate premiums that rationed borrowers are willing to pay can actually <u>reduce</u> the loan balance they receive. A comparison of Tables 1 – 3 illustrates that, as rate premiums rise, the successive increases in balances decline at any combination of liquidation costs and price volatility. This declining responsiveness of balances can appear as incipient or actual credit rationing. However, when liquidation costs and price volatility are both high, Table 3 demonstrates that, above a threshold rate of 10%, each successive increase of two percentage points in rates actually causes credit to decline by 60 basis points.

Each of the situations respectively illustrated in Tables 1 – 3 is, arguably, empirically plausible. In actual credit markets, capital investments will be made by borrowers exhibiting observable traits that are commonly regarded as being independent of credit risk but which are correlated with the qualities of the assets borrowers, who have a common observable trait, are financing. Such investments include residential or commercial property, various types of physical capital used by entrepreneurs involved in various kinds of small business, and proprietary designs or patents associated with highly innovative and technologically advanced sectors of the economy.

The most extensively documented example of this occurs in the American market for residential mortgage loans (Ross (1996), Nickerson, Nebhut and Courchane (2000), Li, Hossain and Ross (2010) and many others.) Consider a situation in which two borrowers possessing observable differences in demographic or other traits seek financing in order to purchase a house. Assume that, on the basis of such conventional underwriting measures as an individual's financial capacity, credit history and employment status, these borrowers appear to pose similar degrees of credit risk to a lender. Borrowers with these contrasting traits, however, each tend to reside in geographically distinct neighborhoods. The average property in the neighborhood of a borrower with trait A is known, for a variety of reasons, to have higher liquidation costs per dollar, or greater price volatility or poorer information about its selling price over time, than the average property in the neighborhood of a borrower with trait B. Correlation between the demographic traits of these two types of borrowers and the respective characteristics of the properties they wish to finance will inevitably lead to a consistent difference in loan terms obtained from a given lender and each of these two types of borrowers. Observations of statistically significant differences in loan terms offered to each type would, based on public opinion as well financial regulations in a variety of countries, be interpreted as discrimination on a basis independent of credit risk. Contrary to this common interpretation, however, our model and its results prove that such systematic differences can occur in markets embodying the traditional assumptions of financial theory and be consistent with an efficient allocation of credit.

CONCLUDING REMARKS

Empirical evidence exists of discrimination in the credit markets of many economies, including those of the U.S. and E.U. This evidence largely consists of observations of significant differences in lending terms among different borrowers who, by conventional measures, appear identical in the credit risk they pose to lenders. These observations are most often interpreted as discrimination on the basis that borrowers are distinguished by contrasting demographic characteristics rather than on any objective economic basis. If this interpretation is correct, this evidence poses a serious anomaly for the traditional presumption by financial economists that the market allocation of credit is efficient.

Previous explanations of this evidence share the presumption that some form of market failure pervades lending and that, consequently, credit markets necessarily exhibit allocational inefficiencies. In stark contrast, this paper offers an explanation based on complete markets, arbitrage-free valuation and other features standard in traditional valuation models. By taking into account the strategic interaction between lenders and borrowers in negotiating loan terms, we demonstrate that the observable properties of non-cooperative equilibria in credit markets depend heavily upon price volatility and other exogenous characteristics of the asset securing a loan. Although absent from standard statistical measures of credit risk, differences in these characteristics induce both lenders and borrowers to change the conditions under which they strategically choose to exercise the options available to them in a standard loan contract. These changes, in turn, produce equilibria with significant differences in loan terms and available credit.

If lenders perceive a correlation, unaccounted for in existing credit discrimination data, between an observable but intrinsically irrelevant trait distinguishing borrowers and the properties of the representative asset securing the borrower's loan, market equilibrium will display differences in the terms and amount of credit obtained by these different borrowers. These differences will, for plausible parameter values, be observationally equivalent to those in the empirical evidence of discrimination in credit markets, but will be consistent with an efficient allocation of credit among all borrowers. Our paper also demonstrates the feasibility of applying the contingent claims method of valuation to the analysis and explanation of differences in credit available to borrowers based on factors other than credit risk. Such applications could significantly widen our consideration of factors in future research on markets for credit, and, of equal importance, offer a new basis for the design and assessment of financial regulations.

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