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IS THREE A CROWD? CONSIDERING THE VALUE OF MANAGER DIVERSIFICATION FOR ADDING ALPHA

Lynda S. Livingston, University of Puget Sound

ABSTRACT

Creating a portfolio that consistently generates alpha—market-adjusted abnormal returns—is the holy grail of active management. Given that excess returns can come both from manager skill and from luck, some advocates of active management suggest that active funds should be combined into diversified portfolios, eliminating all but "pure" active risk and thereby optimizing the risk/return trade-off. In this paper, we present a simple model of such a diversified portfolio, and show that under certain conditions a portfolio manager actually would be better off by not diversifying.

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INTRODUCTION

Active fund managers are paid to deliver excess returns. However, even when they succeed, it is difficult for the portfolio manager who employs them to determine whether those returns were borne of luck or skill. It is obviously not the goal of an portfolio manager to pay extra for luck. Instead, her goal is to identify the active fund managers who earn their fees through skill, consistently delivering "pure alpha."

However, given that even skilled fund managers can be unlucky sometimes, is it not a good idea to create a diversified portfolio of these managers, reducing risk while protecting pure alpha? Should we not run portfolios of funds the same way we run portfolios of stocks, deriving the benefits of diversification?

If a portfolio manager chooses to concentrate, she is exposed to the full risk of one fund manager, eliminating the potential for risk reduction. However, if she diversifies, she must choose *many* managers who will outperform, not just one. Successful active management requires two levels of active skill: the first at the fund level (skillful active fund managers must exist), and the second at the portfolio level (the portfolio manager must be able to identify those skillful fund managers). Proponents of diversified active portfolios, such as Waring and Siegel (2003), suggest that if the portfolio manager is not skilled, she should index. However, as we will show, even if she is skilled, she may nonetheless be better off by choosing not to diversify her active portfolio.

In this paper, we present a model of the tradeoff between a diversified portfolio of active managers and a concentrated one. Using the information ratio (the ratio of alpha to active risk) as the evaluation criterion, we find that a portfolio manager's choice between a concentrated and a diversified portfolio depends upon the proportion of truly skillful managers in the active pool, the correlations among active managers' returns, and the number of fund managers chosen for the portfolio. If there are too few skilled fund managers or if correlations among the merely lucky are too high, a portfolio manager is better off by not diversifying her portfolio.

The paper proceeds as follows. In the next section, we briefly review related literature. We then discuss various expressions for alpha that are common in the institutional literature, and define the decision metric, the information ratio. In the fourth section, we present and discuss the model of the portfolio manager's decision, and present a numerical example. After a brief discussion of the model's implications in the fifth section, we summarize and conclude.

RELATED LITERATURE

Two strands of literature inform this study. The first addresses squarely the question at hand: how many funds should a portfolio manager employ? The answers provided tend to be either in the range of 20 to 30 funds (perhaps an extension of earlier work on the optimal number of stocks in a portfolio; see, for example, Statman, 1987) or, at the other end of the spectrum, no more than ten. Both answers come primarily from experience and simulation, rather than from theory. However, the more recent work, which moves beyond the traditional focus on mean and variance to incorporate higher moments of the return distribution, tends to suggest that smaller is better. We will survey this literature in this section. The second strand of related literature concerns the variables underlying our model, the most important of which are the correlations among funds. We will discuss this literature in section five, after we present the model.

The conventional wisdom is that diversification of funds is good. For example, Ross (2003), in a paper promoting an allocation to hedge funds in institutional portfolios, emphasizes the need for diversification within the added hedge fund portfolio. He stresses that "[t]he returns on a *diversified* portfolio of hedge funds are relatively uncorrelated with other assets" and that "a *diversified* portfolio of hedge funds offers stable returns that are significantly higher than those for bonds, exhibiting very low volatility" (emphasis added). Thus, hedge funds can offer superior risk/return characteristics to bonds, but only if their own unique risks are properly diversified. This requires not only diversification across hedge fund strategies, but also within each strategy: "*a well managed portfolio of hedge funds should have adequate diversification across managers as well as across strategies*" (page 7; emphasis original).

Park and Staum also advocate for significant diversification in funds of funds. They note that, while the average number of funds in portfolios is about five, "…with hundreds… of hedge funds in existence, it is hard for a fund of funds manager to claim that he can only find five good ones" (page 3). Asserting that under-diversifying means foregoing profits, they urge fund managers to "embrace diversification more fully." However, like Ross (2003), Park and Staum do not suggest a specific number of funds that would be required to effect this embrace.

Nesbitt, *et al.* (2003) do provide a number. Based on samples from historical returns (which we will discuss in more detail later), they conclude that "… an optimal portfolio of hedge funds includes between 20 and 30 individual funds. This provides substantial diversification without substantially reducing the potential for adding alpha" (reported in Bonafede, *et al.*, 2004, page 22). This conclusion is based on the behavior of portfolio mean and variance as n, the number of included funds, increases. However, expanding the set of relevant moments to include skewness and kurtosis, as in recent academic work, suggests that using even 20 funds may lead to portfolio overdiversification.

For example, Brands and Gallagher (2005) use simulations to study the effects of diversification on Australian funds of funds (FoFs). The authors find that return stays almost constant as the number of funds increases (in contrast to Park and Staum's foregone profits argument). Variance decreases at a decreasing rate, with most of the benefits accruing by n=6. However, the risk reduction is "slight," decreasing variance by at most 7% for a portfolio of 30 funds. The authors explain this result by noting "a diminishing increase in the number of unique securities added to the FoF as the number of funds in the FoF rises, given increasing levels of common stock holdings across funds" (page 190). (In fact, as the number of funds employed rises, portfolios run increasing risk of becoming closet index funds—forcing investors to pay active management fees for benchmark-like performance.) Combining reward and risk measures into the Sharpe ratio (a common performance metric, discussed more fully in the next section), Brands and Gallagher note that diversification can increase the ratio by 3.57% as n increases, but that "beyond a portfolio of 4 funds, it is not possible to significantly improve the fund's Sharpe Ratio" (page

193). Thus, these authors suggest that diversification benefits, while possible, are limited, and that they can be fully captured using only a very few funds.

On the other hand, they also note that diversification may hurt higher-level moments of portfolio performance. For example, investors are assumed to prefer positive skewness and lower kurtosis. However, as *n* increases, skewness in their funds of funds becomes increasingly negative—the chance of including poor funds increases as more funds are chosen. (This is exactly the effect that we will discuss below.) There is a comparable effect on kurtosis: as *n* rises to about ten, distributions become more peaked, and less attractive to investors. Amin and Kat (2002) find similar results: using data from 2,183 hedge funds and funds of funds from 1994 through 2001, they show that increasing the number of funds in a portfolio decreases skewness while increasing both kurtosis and correlation with the market. They summarize this as follows: "when adding more funds, the probability of a relatively large loss rises while the diversification potential within the context of a larger stock-bond portfolio drops," implying that, "with hedge funds, diversification is not necessarily a good thing" (page 5). Lhabitant and Learned (2004) concur; given their similar empirical results, they summarize their findings by saying that, "for some strategies, too much diversification results in undesirable side effects in the higher moments of the return distribution. Thus, while a fund of hedge funds may mitigate the negative effects of a hedge fund failure through diversification, too much diversification is also likely to result in deworsification" (page 2).

For fund portfolios, then, diversification is not a free lunch. Even where it seems to help—mean and variance—its benefits play out very quickly. Our model, which focuses on mean and variance, may help explain why. We will present the model after a brief review of the performance metrics we use.

ALPHA AND THE INFORMATION RATIO

Searching for alpha is often complicated by its multiple definitions—it is hard to find something you cannot describe. In this section, we briefly review some of the common definitions for alpha. We then justify using the ratio of alpha to active risk, the information ratio, as our portfolio manager's decision criterion.

Alpha is a portfolio's return above a benchmark. Benchmark specification therefore is central to the identification of superior returns. Also critical is the recognition that variations from the benchmark can occur both from luck and from manager skill. Thus, differences in the benchmarks chosen and varying acknowledgments of the effects of luck are the central points of contention among alpha definitions.

The earliest of the alpha measures was Jensen's (1967). Jensen defined this risk-adjusted return as the difference between a portfolio's average return and its predicted return using the Capital Asset Pricing Model (CAPM):

$$\alpha_p = \overline{R_p} - \{r_f + \beta_p * [\overline{R_M} - r_f]\}, \qquad (1)$$

where r_f is the risk-free rate, $\overline{R_p}$ is the average return on the portfolio, $\overline{R_M}$ is the average return on the "market" portfolio, and β_p is the portfolio's beta coefficient, defined as $\sigma_{pM}/\sigma_M{}^2$ (see Bodie, Kane, and Marcus, 1993, page 804).

Since the CAPM is an ex ante model, composed of market and risk-free benchmarks that exist only in theory, making Jensen's alpha operational requires some heroic assumptions. We need a proxy for the market portfolio (a perfectly diversified portfolio that lives on the efficient set of risky assets), a proxy for the risk-free rate, and a way to measure beta (specifying return measurement intervals, length of measurement period, and any accommodation for potential mismeasurement).

For example, Morningstar suggests that investors use the S&P500 for the market and the 90-day Treasury bill for the risk-free rate. Thus, we are to substitute a 500-stock, domestic large-cap equity index for the ex ante efficient portfolio of all existing risky assets, and an inflation-vulnerable, domestic money-market rate for the riskless benchmark. Even if we accept that these proxies are reasonable, we have no guidance as to how specifically to use them to calculate an asset's predicted return, since we are not told specifically how to measure beta. Since our identification of excess return is dependent upon our calculation of the predicted return, these measurement problems complicate our application of Jensen's alpha.

Academic theoretical and empirical work on alpha has addressed these sorts of issues by increasing the complexity of the models used to define alpha and the econometric techniques used to identify it. For example, Avramov and Chordia (2006) model a K-factor specification for stock returns, utilizing a set of macroeconomic variables that may help predict them. In their empirical model, alpha may vary over time. Similarly, Kosowski, Naik, and Teo (2007) use a seven-factor return model to define alpha. They then use a seemingly unrelated regression approach to create Bayesian posterior alphas. These Bayesian alphas measure performance more accurately and are better at predicting future superior performance. Ordinary least squares (OLS) alphas, on the other hand, tend to overstate excess return while simultaneously prohibiting that excess performance from easily being distinguished from luck. Lo (2007), criticizing current performance metrics for being static measures of dynamic processes, proposes an "active/passive" approach that defines active risk using the covariance between portfolio weights and returns. He breaks the active component of returns into two parts: that coming from security selection (related to the intercept of a K-factor model, alpha, "identifying untapped sources of expected return" [page 14]) and that coming from factor timing (related to exposure to the model's factors). Nielsen and Vassalou (2004) also extend traditional performance measures from discrete to dynamic processes. They develop both the Sharpe ratio (discussed below) and Jensen's alpha in continuous time, under conditions that allow a fund manager to incorporate active strategies, including the use of options. They show that alpha in this case equals the discrete version, (1), minus half of the variance of the portfolio, plus half of its covariance with the market benchmark. Managers identifying a fund with positive alpha can improve their performance by adding some of the fund to their core portfolio.

Although academic characterizations of excess return have become increasingly sophisticated, these models have yet to be fully incorporated into practice. Instead, institutional work often relies on problematic conceptions of alpha. For example, statements that merely refer to alpha in passing with phrases such as "returns above and beyond those of the market" (Bonafede, *et al*., 2004, page 2) may lull us into thinking that alpha is simply ($\overline{R_p}$ - $\overline{R_M}$), increasing the risk that we will ignore beta effects when evaluating managers' above-market returns. Lo (2007) puts it this way: "active management is not simply adding value in excess of a passive benchmark, which can be done passively by taking on non-benchmark factor exposures in a multi-factor world" (page 9). (Thus, there is a third reason that a manager's returns can vary from a benchmark: not just luck, not just skill, but also exposures to non-market [priced] risk factors.)

More serious than this type of obviously simplistic, throwaway reference are the models that attribute all ex-market returns to alpha, ignoring error. For example, Bonafede *et al*. (2004) present the following single-factor model:

$$R_p = r_f + \beta^*(R_M - r_f) + \alpha. \qquad (2)$$

Viewed as an ex ante measure, or as an ex post relationship among averages, this correlates nicely with Jensen's measure, (1). However, (2) is not a fully specified empirical model. This distinction is clouded when the authors state that "[i]nvestment returns can be broken into three distinct parts," as shown in (2), and that this tripartite description implies that portfolio risk must therefore be:

$$\sigma^2_p = \beta^2_p * \sigma^2_M + \sigma_e^2, \tag{3}$$

where σ_e "is the variance [standard deviation] of the alpha component of return" (Bonafede, *et al*., 2004, page 4). σ_e^2 is actually active risk, and is composed not only of variation attributable to "pure" active risk, but also of random error. Waring and Siegel's (2003) work is similar, as is Foresti's (2005) and Foresti and Toth's (2006). For example, although Waring and Siegel explicitly recognize the role of luck, using the term "pure active risk" to distinguish "the portion of the return that is not explained by market or beta risk exposures of the portfolio" (page 2), they also present versions of equations (1) and (2). They also echo Bonafede *et al*.'s dichotomy when they state that risk can be broken into systematic and "pure active risk," and when they describe alpha as the residual from a return/benchmark regression. Where is luck here? Attributing all nonmarket risk to the variance of alpha ignores error. The whole point of performance measurement is to be able to identify those managers who beat the benchmark consistently—because of skill. Just beating the benchmark is insufficient. Expressions such as (2), when considered as an empirical model, ignore the distinction between luck and skill, missing the whole point of the quest for alpha.

While common institutional definitions of alpha may obscure the distinction between skill and luck, these types of definitions are unlikely to be the measure of an active manager's performance. Instead, managers are usually evaluated using reward-to-risk measures. Two commonly reported by portfolio consultants are the Sharpe measure and the information ratio (see, for example, Bonafede, *et al*., 2004, Appendix B). The Sharpe measure is defined as:

$$S_p = (\overline{R_p} - \overline{r_f})/\sigma_p, \tag{4}$$

(where expressing the risk-free rate as an average, $\overline{r_f}$, recognizes that this rate may vary over the sample period). This ex post measure relates excess return to total risk; the higher the ratio, the better the performance. The Sharpe ratio is the appropriate measure of performance when plunging into a single portfolio (see Bodie, Kane, and Marcus, 1993). This case is similar to identifying the Capital Market Line in (expected return, standard deviation) space: the goal is to choose the efficient risky asset that, when combined with the risk-free asset, produces the steepest opportunity set. In the theoretical derivation of the CAPM, this linear opportunity set defines the "market" portfolio, which is chosen by all market participants. In the manager-evaluation application, the steepest line identifies the best single risky portfolio to choose.

However, when an active portfolio is to be added to a diversified core portfolio, the total risk of the addition is not the appropriate risk measure. In this case, we use the information ratio (IR):

$$IR_p = \alpha_p / \sigma_{ep}. \tag{5}$$

The IR compares a portfolio's active excess return to its total nonsystematic risk. Bodie, Kane, and Marcus (1993) note that the square of the Sharpe measure for overall portfolio—the combination of the core and the active "satellite"—equals the square of the market's Sharpe ratio plus the square of the added portfolio's information ratio. (See also Nielsen and Vassalou, 2004.) The IR therefore measures the contribution of the addition to the performance of the complete portfolio.

Kosowski, Naik, and Teo (2007) use a measure similar to the IR, the t-statistic of alpha, to evaluate the performance of hedge fund managers. (This statistic is approximated by the IR multiplied by the square root of the number of observations, *N*: [IR* $\sqrt{N}$]. Note that the standard error of the alpha estimate is $\sigma_e*[\Sigma X^2/(n*S_{xx})]^{1/2}$, where S_{xx} equals $[\Sigma X^2 - (\Sigma X)^2/n]$; see Ott, 1984, page 284.) They find that this measure, compared to alpha alone, allows them to better distinguish skillful from lucky performance. They report that these results "empirically validate the industry practice of ranking fund managers on their information ratios as opposed to their Jensen's alpha." We will follow that practice.

In this paper, we use the IR to consider a combined core/satellite portfolio. This strategy is consistent with Wilshire Associates' suggested use of hedge funds in institutional portfolios (see Bonafede *et al.*, 2004), and with the exposition in Waring and Siegel (2003). Specifically, we consider the contention that maximizing the IR of an added portfolio of active managers is best achieved by diversifying across many active managers: by treating the satellite as we would any other portfolio, perhaps we can decrease active risk, σ_e^2, by diversifying across multiple alpha sources. If we can, the lowering of active risk through diversification would translate directly into higher information ratios, and therefore into superior performance.

However, the problem with diversifying across active managers is that you may actually be diversifying away from skill and toward luck. Not all managers are equally skillful. In fact, some are just lucky. As Waring and Siegel (2003) note, a portfolio manager who wants to hire active managers "must believe that he or she can identify which ones will be the winners" (page 20). If she cannot, she may be better off by not diversifying.

THE MODEL

Assume that there are three types of fund managers: passive managers (P), active managers who have no special skill (U), and active managers who are skilled (S). Thus, denoting the type of fund manager by T, we have T ∈ (U, P, S). The proportions of the three types of fund managers in the market, w_T, must sum to one. We will assume that, among active managers, there are fewer who are skilled, so that $w_U > w_S$.

Regardless of the return model we adopt, the average risk-adjusted abnormal performance, $\bar{\alpha}$, must equal zero across the full market. Thus, we must have:

$$\bar{\alpha} = w_S*\bar{\alpha}_S + w_U*\bar{\alpha}_U + w_P*\bar{\alpha}_P = 0 \quad (6)$$

If we assume that the average excess return to the passive managers, $\bar{\alpha}_P$, itself equals zero, then (6) implies that:

$$\bar{\alpha}_U = -\bar{\alpha}_S*(w_S/w_U). \quad (7)$$

Since we assume that the skilled managers deliver a positive alpha, (7) says that $\bar{\alpha}_U$ will be negative. In the zero-sum game of delivering excess performance, the skilled managers are taking their excess from the unskilled.

To determine the variance of any portfolio of fund managers that a portfolio manager chooses, we must describe the covariance matrix for the returns for the various fund manager types. First, we will assume that all active managers' excess returns are distributed normally with a common variance σ_e^2, so that excess returns are distributed $N(\bar{\alpha}_S, \sigma_e)$ for skilled managers and $N(-\bar{\alpha}_S*(w_S/w_U), \sigma_e)$ for unskilled.

(Throughout the remaining exposition of the model, we will drop the "e" subscript from σ_e for notational simplicity. Also, we will refer to managers' σ_e^2 simply as the "variance.")

These assumptions about the diagonal elements of the covariance matrix warrant some comment. Normality is a simplifying assumption: as Kosowski, Naik, and Teo (2007) note, hedge fund alphas will probably not be normally distributed, given the complex and often option-like strategies they employ. However, we are being consistent with institutional work, which also assumes normality. On the other hand, the institutions may not assume that the variances for the skilled and unskilled managers would be equal. For example, Waring and Sigel (2003) suggest that we should "prefer skillful lower active risk managers to higher risk, concentrated managers," implying that active managers' variances will depend upon their strategies. However, they do note that the "luck component" of risk is comparable for skillful and unskillful managers (page 6). This comparability is sufficient to motivate our simplifying assumption: we do not distinguish among strategies, and allowing for unequal variances would unnecessarily complicate the model.

For the off-diagonal elements of the covariance matrix, we will assume that skilled managers' returns are uncorrelated with each other and with those of the other two types of managers, so that $\rho_{SS} = \rho_{SU} = \rho_{SP} = 0$. This models the unique contribution of a manager who is truly skilled. For simplicity, we will also assume that unskilled active managers' returns are uncorrelated with the passive, $\rho_{PU} = 0$. However, they are positively correlated with each other: $\rho_{UU} > 0$. This models the observed behavior of active managers' following similar strategies (we discuss this point more fully below). Given our assumption that only ρ_{UU} is nonzero, we will refer to this correlation from now on simply as ρ.

We want to compare the active risk/return tradeoff between a concentrated, skilled portfolio and a diversified portfolio of active managers. For the skilled portfolio, we will use only one manager, denying us the diversification potential that would undeniably come from a portfolio of skilled, uncorrelated managers. (If it is difficult to distinguish luck from skill, and if skilled managers are scarce, we would be fortunate to identify our one skilled manager.) The variance of this skilled portfolio will therefore be σ^2, and we expect that the delivered information ratio will be $\bar{\alpha}_S/\sigma$.

For the diversified portfolio, a portfolio manager's expected alpha and variance will depend upon the proportion of unskilled managers she chooses. Calling this proportion w, and calling the total number of managers used n, we can describe the portfolio characteristics as:

$$\alpha_p = w*(-w_S/w_U)*\bar{\alpha}_S + (1-w)*\bar{\alpha}_S \quad (8)$$

$$\sigma_p^2 = n*(1/n)^2*\sigma^2 + n*w*(n*w-1)*(1/n)^2*\sigma^2*\rho_{UU} \quad (9)$$

The first term in the variance expression (9) represents the contributions to variance from the n individual managers. The second term represents the contributions from the covariances among the returns of the $n*w$ unskilled managers. All of the other portfolio covariance terms are zero, given our assumptions of independence.

How a portfolio manager performs depends upon her ability to identify skilled managers. The smaller is her w (the smaller her proportion of unskilled managers), the higher is her α_p, the lower is her σ_p^2, and therefore the higher is her information ratio. Focusing our attention on w allows us to model the second level of superior ability that is necessary for positive active returns: the ability of the portfolio manager to identify fund managers who can consistently deliver alpha.

If a portfolio manager has no special skill at identifying these fund managers, then we would expect that w would equal $w_U/(w_U + w_S)$, which makes α_p zero. (Thus, as Waring and Siegel, 2003, note, the portfolio manager should stick to passive investments if she has no skill at picking fund managers.) Values of w higher than this will clearly make the situation even worse, and α_p will be negative. However, as w approaches zero, α_p approaches $\bar{\alpha}_S$, and more importantly, σ_p^2 approaches σ^2/n. This is the dominating case: a diversified portfolio of skillful managers (the case to which Waring and Siegel undoubtedly refer).

The key to the choice between our concentrated portfolio and a diversified portfolio therefore lies in a portfolio manager's expectation about her realization of w, our proxy for her ability to identify skillful fund managers. A consideration of the factors that make a portfolio manager inherently successful in this identification is beyond the scope of this paper. However, by evaluating our model's implications about w, we can consider the market factors that imply a payoff to this identification.

We will begin our characterization of w by finding its break-even value, the value of w at which we are indifferent between a diversified portfolio and a concentrated one. This is most easily accomplished by equating the squares of the two strategies' information ratios:

$$(\bar{\alpha}_S/\sigma)^2 = [w*(-w_S/w_U)*\bar{\alpha}_S + (1-w)*\bar{\alpha}_S]^2/[n*(1/n)^2*\sigma^2 + n*w*(n*w - 1)*(1/n)^2*\sigma^2*\rho_{UU}] \quad (10)$$

Rearranging (10) leads to the following quadratic equation:

$$w^2*[n*\rho - n*r^2] + w*[2*n*r - \rho] + (1-n) = 0, \quad (11)$$

where r equals $(w_U + w_S)/w_U$, or the inverse of the proportion of active managers who are unskilled. Solving for w, we find the following unwieldy expression for w_U^{max}, the break-even proportion:

$$w_U^{max} = \{\rho - 2*n*r + \{\rho^2 - 4*n*[\rho*(r + 1 - n) - r^2]\}^{1/2}\}/2*n*(\rho - r^2) \quad (12)$$

By evaluating the comparative statics of this expression, we can investigate the environmental factors that influence the relative payoff to a diversified portfolio of fund managers. If these factors result in a break-even w that is too low, the portfolio manager may conclude that her skill level is insufficient to expect success in creating a superior, diversified portfolio. Before examining these relationships, however, we will first consider a simple numerical example.

A Basic Example

The values we will use for this example are presented in Table 1.

Table 1: Numerical Example

Variable	Example Value
n	10
σ	0.04
ρ	0.2
w_S	0.1
w_U	0.3
$\bar{\alpha}_S$	0.02

The table presents the variable values that we will use to demonstrate the influences from equation (12).

We should motivate these choices briefly. First, we assume that the portfolio manager will use 10 fund managers. As a "satellite" strategy to a passive core, the active portfolio should not be so large as to be unmanageable. Ten active funds is within the usual range for small- to medium-sized pension funds, and it is the number of funds at which Nesbitt *et al.* (2003) show the strongest benefit to diversification. However, the n variable, more than being an influence on the outcome of equation (12), is primarily a reflection of the active opportunities available to the portfolio manager. (Given that it might then be a function of the active strategy employed, there may be a functional relationship between n and ρ; we will ignore these types of relationships.)

The correlation between unskilled active managers' returns, ρ_{UU}, is positive. Similar trading strategies or holdings can induce correlations among fund returns, which will reduce the value of portfolio-level diversification. For the universe of thousands of hedge funds, Kosowski *et al.* (2007) assert that most of their return distributions are likely to be "sparsely overlapping" (page 230); similarly, Ross (2003) notes that, from 1993 to 2002, hedge fund portfolio strategies had risk that was "markedly lower than that for any individual strategy, confirming that the returns on the strategies are relatively uncorrelated" (page 7). However, this evidence does not preclude significant overlap among subsets of these returns. To quantify this relationship, then, we turn to Waring and Siegel (2003), who note that historical alphas have correlations falling usually between -0.2 and +0.2 (page 14; see also Ross, 2003, Table 3). This informs our choice of 0.2 for ρ_{UU}. These authors also note that "pure" alphas are often assumed to be independent; this is what we have done for all correlations involving skilled managers.

The standard deviation of 4% comes from a broad characterization used by pension consultants: 1% to 3% active risk implies a strategy like enhanced indexation; 4% and up means a truly active strategy (see, for example, Foresti *et al*., 2006, page 17). This is comparable both to Waring and Siegel's (2003) assertion that active risk usually falls in the range from 4% to 25%, averaging 5-6%, and to Ross's (2003) evidence that hedge fund risks tend to fall between 3% to 10.5% (see his Table 2). Similarly, the 2% $\bar{\alpha}_S$ was suggested by the premium above index that is a common benchmark for active managers. Together, these choices of $\bar{\alpha}_S$ and σ imply an information ratio of 0.5, the value Waring and Siegel (2003) say is the lower threshold for top-quartile active managers.

Finally, the .1 and .3 values for w_S and w_U, respectively, imply that the market in our example has 60% passive managers and 40% active. In the active space, there are three times as many unskilled managers as skilled, so that a random choice of active manager has a 75% chance of drawing someone unskilled. This is the ratio that the portfolio manager has to beat: she must choose her fund managers so that her realized proportion of unskilled active managers, her w, is lower than 75%.

How much lower? Given the values assumed in Table 1, equation (12) gives a break-even value for w of 47%. In a market where unskilled managers outweigh skilled by 3-to-1, our portfolio manager must be able to create her active satellite with at least 53% skilled managers. If she cannot, she is better off with a concentrated portfolio with one skilled manager. (Of course, this requires that she be able to identify at least *one* skilled manager. Given the distribution of fund managers' skills, even a random grab gives her a 25% chance of choosing one skilled manager. However, she would have only a 7.8% chance of choosing *five* or more of them.)

Having considered this basic example, we now consider the general implications of the model.

Comparative Statics and Implications

In this section we will review the effects on w_U^{max} of three variables: r, n, and ρ. To facilitate the discussion, we will simplify equation (12) by making the following substitution:

$$z \equiv \{\rho^2 - 4*n*[\rho*(r + 1 - n) - r^2]\} \quad (13)$$

This allows us to express w_U^{max} as:

$$w_U^{max} = \{\rho - 2*n*r + z^{1/2}\}/2*n*(\rho - r^2) \quad (14)$$

Expressing z as $\rho^2 + 4*n*\rho*(n-1) + 4*n*r*(r-\rho)$, and noting that $\rho \leq 1$ while $r \geq 1$, we can see that z is positive. In fact, $z > 1$, as is its square root. These relationships will help us sign the partial derivatives of w_U^{max}.

We begin by considering the effect of r on w_U^{max}. Remember that r is defined as $(w_S + w_U)/w_U$, so that higher values of r imply that a larger relative proportion of active fund managers is skilled. With higher values of r, it is easier to choose a skilled fund manager by chance, making it more likely that a portfolio manager would prefer a diversified portfolio to a concentrated one.

But what of the effect of r on w_U^{max}? Using (14), we see that:

$$\delta w_U^{max}/\delta r = [.5*z^{1/2}*(\delta z/\delta r) - 2*n]/(2*n*\rho - 2*n*r^2) + (\rho - 2*n*r + z^{1/2})*4*(n*r)/(2*n*\rho - 2*n*r^2)^2 \quad (15)$$

The denominator of the first term is negative. The first term will therefore be positive if $.5*z^{1/2}*(\delta z/\delta r)$ is greater than 2*n. (Note that $\delta z/\delta r$, which equals $[8*n*r - 4*n*\rho]$, is positive.) Simplifying, this requires that $(2*r - \rho)*z^{1/2} > 1$. We can verify easily that this inequality holds by substituting 1 as the minimum for r and the maximum for ρ. Thus, the first term in equation (15) is (+/-), or negative.

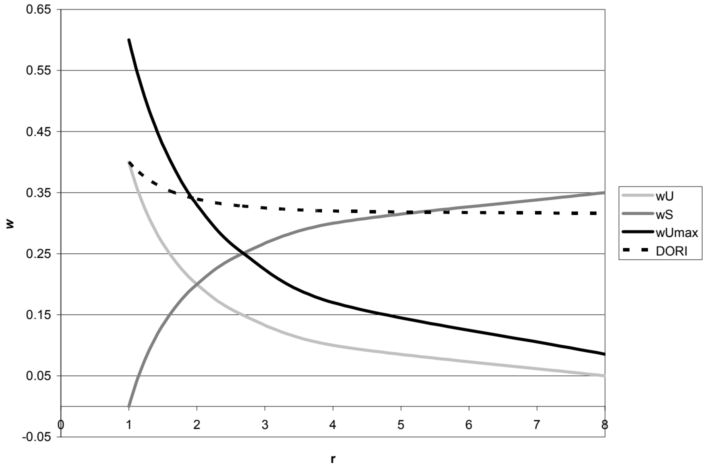
In the second term, the denominator is positive. Therefore, this term will be negative if $(2*n*r - \rho) > z^{1/2}$. Squaring both sides and simplifying, this leads to the requirement that $n*(r^2 - \rho) - n^2*(r^2 - \rho) < 0$. As this is obviously true, the second term in equation (15) is negative. Thus, $\delta w_U^{max}/\delta r$ is negative. This relationship between r and w_U^{max} is illustrated in Figure 1, using the values from Table 1.

Intuition suggests that, as the proportion of skillful managers in the active pool increases, a fund manager can more confidently diversify. On the other hand, we have just shown that as r rises, a portfolio manager has a *lower maximum* proportion for the number of unskilled managers that she can choose for a successful diversified portfolio. However, this negative relationship between r and w_U^{max} does not mean that she is less likely to diversify when r is high. Instead, it reflects the fact that the proportion of unskilled managers in the active pool is lower when r is higher. We can see this by looking at the "degree of required improvement" (DORI) which is also plotted in Figure 1. The degree of required improvement is the (absolute value of) the percentage difference between w_U^{max}, the maximum possible proportion of unskilled managers in a successful diversified portfolio, and the actual proportion of unskilled managers in the active pool. Thus,

$$\text{DORI} \equiv |[w_U^{max} - (1/r)]/(1/r)| \quad (16)$$

The lower is the DORI, the less skill a portfolio manager needs in order to benefit from diversification. If a manager's expected deliverable degree of improvement is less than DORI, she should not attempt to diversify. DORI falls as r increases. This means that having more skilled managers in the active pool puts less pressure on a portfolio manager to choose well—her job is easier, requiring less skill. It is, as we would expect, easier to create a successful diversified portfolio when the average skill level of active fund managers rises.

Figure 1: Effect of r on w_U^{max}



A higher value of r implies a larger proportion of skilled managers in the active pool (w_U falls and w_S rises). The maximum proportion of unskilled managers in a successful diversified portfolio (w_U^{max}) falls, but only because the total proportion of unskilled managers falls. Successful diversification is actually easier, as illustrated by the falling level of the degree of required improvement (DORI).

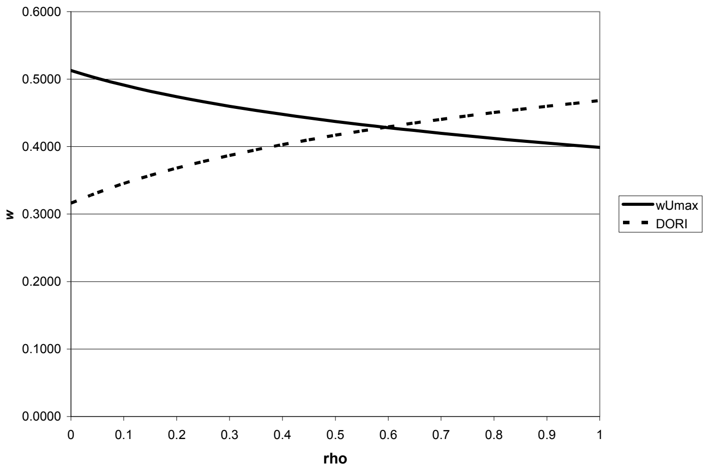
We now consider the effects of ρ and n on w_U^{max}. Both $\delta w_U^{max}/\delta\rho$ and $\delta w_U^{max}/\delta n$ are quite convoluted, so signing them directly is difficult. Fortunately for us, the intuition in these cases is straightforward: it is easier to diversify successfully when n is higher and ρ is lower. We will consider this intuition more fully after a brief discussion of the mechanics.

Looking at equation (14), we can see how ρ affects w_U^{max}. In the ratio for w_U^{max}, both the numerator and the denominator are negative. The derivative of the numerator with respect to ρ is $[1 + ½*z^{-1/2}*(\delta z/\delta\rho)]$. Given that n > (r+1), δz/δρ is positive. Thus, the numerator of w_U^{max} rises with ρ. The derivative of the denominator with respect to ρ is simply 2*n, obviously also positive. However, it is not immediately clear is how the ratio *w* behaves. We know that the denominator is larger in absolute value and that it increases linearly with ρ. However, the second derivative of the numerator with respect to ρ, $[4*z - (\delta z/\delta p)^2]/(4*z^{3/2})$, is negative. Over the relevant range, this causes the difference (relative to the denominator) between the two components to rise. The ratio w_U^{max} therefore falls.

We can observe this behavior using the numbers given above in Table 1. Setting all inputs to their Table 1 values except for ρ, we find that when ρ is zero, w_U^{max} is 0.5128. In this case, z is 71.11, the numerator of w_U^{max} is -18.23, and the denominator is -35.56. Raising ρ to 1, w_U^{max} falls to 0.3989. z is now 378.78, the numerator is -6.2, and the denominator is -15.56. The numerator rose by 66%, while the denominator only rose by 56%. This decreased the absolute difference between numerator and denominator, but increased the *relative* difference. The ratio, w_U^{max}, therefore fell. This behavior is clear from the graph in Figure 2.

The intuition informing the relationship between ρ and *w* is much more straightforward than the previous discussion would suggest. Increasing ρ means that diversification is more difficult, since unskilled active managers' returns are more closely related. A fund manager must have more skill in choosing active managers in this case—her realized w_U^{max} must be smaller. Thus, the degree of required improvement increases with ρ, as $\delta w_U^{max}/\delta\rho$ decreases.

Figure 2: Effect of ρ on w_U^{max}



If unskilled managers' returns are more closely correlated, it is harder to create a successful diversified portfolio. As ρ rises, the portfolio manager must choose a lower proportion of unskilled managers (lower w_U^{max})—delivering a higher degree of relative improvement—to succeed at diversification

Now let us consider the effect of n on w_U^{max}. The partial derivative of w_U^{max} with respect to n is:

$$\delta w_U^{max}/\delta n = \{[-2*r + .5*z^{-1/2}*(\delta z/\delta n)]*(2*n*\rho - 2*n*r^2) - (\rho - 2*n*\rho + z^{1/2})*(2*\rho - 2*r^2)\}/(2*n*\rho - 2*n*r^2)^2 \quad (17)$$

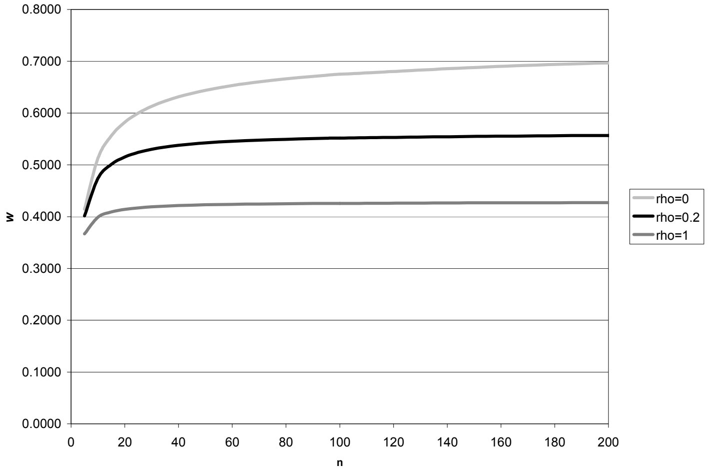
The partial of z with respect to n, δz/δn, equals $4*\{r^2 + \rho*[2*n - (r+1)]\}$. Since n > (r+1), δz/δn is positive, as are the z terms. However, every other bracketed term in the numerator of $\delta w/\delta n$ is negative, making this derivative hard to sign. Using numerical methods, we can determine that $\delta w_U^{max}/\delta n > 0$, while $\delta(w_U^{max})^2/\delta n^2 < 0$: w_U^{max} rises with n, but at a decreasing rate. The vast majority of n's effect on w_U^{max} comes at fairly low levels of n. In fact, once n exceeds about ten, there is very little marginal effect of increasing n on w_U^{max}, and w_U^{max} rapidly approaches its limit of:

$$\lim_{n \to \infty} w_U^{max} = (\rho^{1/2} - r)/(\rho - r^2) \quad (18)$$

However, as figure 3.A. shows, the value of having more managers depends upon the correlation among their returns. As we would expect, the lower the correlation, the greater the potential benefits of diversification, and the greater the effect of n on w_U^{max}. Considering the degree of required improvement from increasing n confirms these effects, as shown in Figure 3.B. The DORI falls with n, meaning that portfolio managers can more easily diversify successfully. However, this opportunity dissipates quickly as ρ increases.

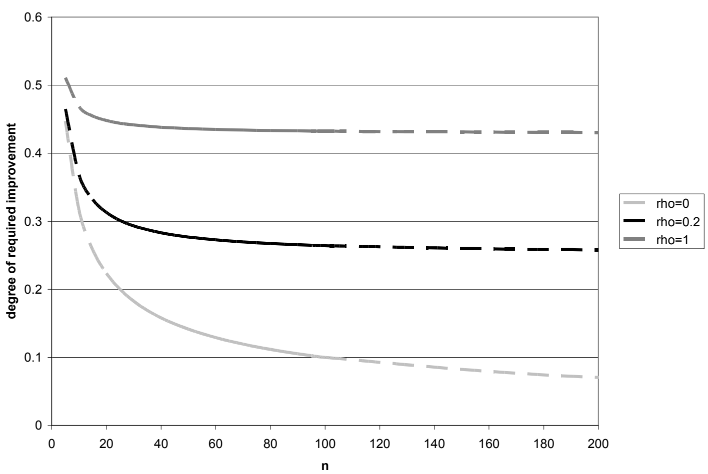
Figure 4 summarizes the major points of this section by plotting w_U^{max} against both ρ and r. The portfolio manager should diversify if she can keep her realized *w*, the proportion of unskilled managers she chooses, below the "ceiling" defined by the figure in the graph. If her realized *w* lies above this barrier, she would have been better off with a concentrated portfolio. Diversification is potentially most fruitful when the proportion of skillful managers in the active pool is relatively large, and when the returns of the unskilled managers are correlated less closely. Note that the correlation effect is particularly pronounced when r is low—when there are relatively more unskilled managers.

Figure 3.A : Effect of n on w_U^{max}



Successful diversification is easier (w_U^{max} is higher) with more funds, but this effect dissipates very quickly once n>10. Again we see that ρ is the crucial variable: when unskilled managers' returns are too closely correlated, increasing n does little to allow successful diversification.

Figure 3.B: Degree of Required Improvement with n

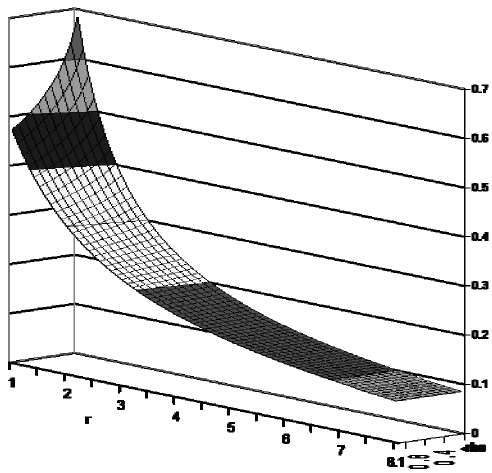


Successful diversification is easier (DORI is lower) as n rises, but higher correlations among unskilled managers' returns severely weaken this effect.

DISCUSSION

In a paper directly related to our model, Nesbitt *et al.* (2003) attempt to empirically determine the optimal number of hedge funds for a diversified portfolio. Using historical returns for 612 hedge funds, they form 1,000 equally weighted portfolios at each size *n*, where n ranges from 1 to 50 funds. They then compare the risk and return quartiles across sizes. Beginning with a one-fund standard deviation of 11.7% (the median annualized standard deviation for their sample), they show that portfolio risk falls to 6.5% when *n*=10. Further expanding the portfolios to 20 funds lowers risk marginally more, to 5.9%. Looking again at Figure 3.A., we see that, in our example, the beneficial effect of increasing the number of managers in a portfolio rapidly decreases above *n*=20, the same result that Nesbitt *et al*. find empirically. As noted earlier, however, their results lead them to the broad conclusion that it takes 20 to 30 individual funds to diversify optimally a hedge fund portfolio.

Figure 4: Effects of Both ρ and r on w_U^{max}



Successful diversification requires that the portfolio manager's proportion of unskilled managers falls below the illustrated "ceiling" (w_U^{max}). This will be easier when ρ is low and when there are more skillful managers in the active pool.

Here, then, is an institutional argument in favor of diversification. How do these empirical results inform our model, especially at the lower levels of *n*, where our conclusions may vary? Let us try to reconcile their results with ours.

Nesbitt *et al*. do not distinguish between skilled and unskilled managers. Assume, then, that the proportion of unskilled managers in the active pool is .75, while the skilled proportion is .25 (this will give us the same r value as we used earlier). Assume also that σ = .117. A portfolio manager randomly selecting ten funds from this universe would achieve a median portfolio standard deviation of 6.5% if ρ_{UU} = .5 (using equation (9), and assuming a binomial distribution from which fund managers are drawn). We will therefore take this ρ value as the model input implied by the authors' results.

Nesbitt *et al*. suggest that a skillful portfolio manager should be able to deliver top-quartile returns; in this case, such a result implies that σ = 5.85%. Using the binomial distribution, we see that a manager achieving this σ could choose at most six unskilled managers (a w_U^{max} of .6), meaning that she would need a degree of improvement of only .2. On the contrary, our model implies that this manager should concentrate: the implied w_U^{max} is only .44, with a DORI of .42. Given the high implied correlation among unskilled fund managers' returns, the model predicts little improvement from diversification.

The different conclusions of our model and Nesbitt *et al.*'s empirical results therefore hinge on the assumed correlations among active managers' returns. They acknowledge the importance of this input when they repeat their analysis on subsamples. Nesbitt *et al.*'s primary sample included all types of hedge funds, maximizing the potential for diversification (so that the actual ρ was almost certainly lower than our implied .5). However, when they separated funds by type, this potential fell. Macro manager benefited the most from diversification, "probably due to the non-correlated positions within their individual portfolios" (page 3). These authors summarize their findings by saying that "a good deal of diversification comes from mixing hedge fund styles, more so than from simply adding managers from within any one hedge fund style" (page 3). In the subsamples, then, their results are broadly consistent with ours—diversification is less beneficial when correlations are high.

Khandani and Lo (2007) echo this emphasis on correlations, and warn that increasingly correlated returns may decrease the potential for diversification across hedge funds in the future. Their study examines the

August, 2007 behavior of statistical arbitrage, quantitative equity market neutral, long/short equity, and 130/30 ("active extension") hedge-fund strategies. They show that the returns of these long/short strategy funds over this volatile period suggest that these types of funds are becoming much more highly correlated. For example, they note that "the necessarily quantitative nature of 130/30 strategies creates an unavoidable commonality between them and quantitative equity market-neutral strategies" (page 23). They note that if funds…

> …use similar quantitative portfolio construction techniques, then more often than not, they will make the same kind of bets because these techniques are based on the same historical data, which will point to the same empirical anomalies to be exploited… Moreover, the widespread use of standardized factor risk models… by many quantitative managers will almost certainly create common exposures among those managers to the risk factors contained in such platforms…. But even more significant is the fact that many of these empirical regularities have been incorporated into non-quantitative equity investment processes, including fundamental "bottom-up" valuation approaches like value/growth characteristics, earnings quality, and financial ratio analysis. (page 27)

Similarly, Lhabitant and Learned (2004), in explaining the significant increase in kurtosis and decrease in skewness from diversifying within fixed-income arbitrage and event-driven strategies, note that, "[i]t is our assumption that many of these managers have heavily invested in the same underlying assets, and are, therefore exposed to the same underlying risks…By diversifying among them, we are, in a sense, sure to capture these risks" (page 8). Correlation again is the enemy of successful fund-of-fund diversification.

Even the popular press acknowledges this increasing correlation. Describing the same period as Khandani and Lo, the *Wall Street Journal* noted that, "The reliance on models can be especially problematic because many quant hedge funds have very similar models. That means they are often doing the same trades and buying the same shares" (8/9/07). Again, the next day, the paper reported that, "Since market-neutral funds often are guided by similar computer models and share similar holdings, the actions magnified moves in asset prices. The last week has been the worst on record for many large hedge funds focusing on this strategy, worrying traders across Wall Street, many of whom look to these firms for signs of stability in difficult markets" (8/10/07; see also Cox *et al*., 9/4/07).

Khandani and Lo (2007) help quantify this correlation. In a summary comparison of the correlations among 13 hedge-fund indexes, 49 of these authors' coefficients were above .25, while 23 were above .50. Mean and median 36-month rolling-window correlations were between .50 and .60. (See their Figures 7 and 8.) Perhaps, then, our model's implied ρ of .5 was not so far off; perhaps diversification is not as efficacious at the fund level as it is at the security level.

But what of return? Nesbitt *et al.* suggest that, while expected return declines with n, it does so more slowly than risk. Diversification therefore benefits the portfolio manager.

However, their results track return and risk separately. Combining the median return measure and the median risk measure does not necessarily give the median information ratio. If we assume that a skilled manager delivers an average alpha of 2% and an unskilled manager delivers 0%, while continuing other assumptions above, we find that IR declines significantly as n increases: at the 25^{th} percentile, IR is approximately .14, down from .54 when n=1 (however, the probability of achieving the IR of .54 is 0.0000009537). Considering both risk and return, diversification can impose a significant cost on a naïve portfolio manager.

In a consistent finding, Kosowski *et al*. (2007) report that the best and worst funds in their hedge fund sample are Directional Trader funds, which make aggressive bets on the market's direction. Either the big bets pay off, or they do not. On the other hand, the overall worst type of investment strategy is the fund-of-funds: even their best performances are less statistically significant than those of other strategies. In fact, these authors assert that "it is well known that Fund of Funds have lower average returns than individual hedge funds"—and not just because of the added layer of fees—a conventional wisdom that their empirical work verifies (page 250; see also Bondafede *et al*., 2004). Concentration may be better than diversification.

Nesbitt *et al*. admit this possibility when they discuss "slippage," which they define as the additional decrease in expected return that comes from combining managers in the face of decreasing portfolio manager skill. They explain that slippage could occur for many reasons: "...One is that it becomes much more difficult to identify good managers… Another could be the cost associated with researching and monitoring more managers. Fund sponsors' experiences with long-only managers suggest that slippage is more likely to appear as the number of managers increases" (page 5). Similarly, Foresti and Rush (2007) note that a manager's success at choosing active weights should fall as the number of these bets rises (page 12). These acknowledgments support the basic premise of our model: sometimes diversification attempts can do more harm than good.

CONCLUSION

Alpha is an enticing concept. Portfolio managers want to believe that they can add "juice" to their portfolios by adding active managers, just as active managers want to believe they can deliver it. However, the average excess return in the market must be zero. If there are fund managers who can add juice, there must also be those who drain it.

The job of the portfolio manager, should she chose to leave behind the relative safety of indexing, is to identify those fund managers who can deliver alpha because they are skillful, while avoiding those who may simply get lucky. Only the skillful fund managers will be worth what they cost. If a portfolio manager cannot distinguish luck from skill, she should stick with a passive portfolio.

However, even if she does believe that she can identify skillful fund managers, she may not be able to diversify profitably. Waring and Siegel (2003) prescribe that active fund managers should "[b]ias toward diversified portfolios, away from concentration," and that active portfolio managers should do the same: portfolio managers should "build efficient, or optimized, portfolios of managers just as they should of asset classes." However, these authors also assume that manager skill levels are "generally equal" (pages 19 and 20). What if they aren't?

According to Bonafede *et al.* (2004), the "Fundamental Law of Active Management" states that the information ratio is "approximately equal to skill times the square root of breadth." That is, current institutional practice suggests that we should be able to add breadth through diversification. However, if by doing so we sacrifice skill for size, we lose: skill, not size, is the input that enters the performance expression at full strength.

In this paper, we present a model of the choice between diversification and concentration. Three variables influence this choice: the number of funds used, the relative proportion of skilled managers in the active pool, and the degree of correlation in unskilled managers' returns. Of the three, correlation is the most potent. As fund managers' returns become more highly correlated—and recent evidence suggests this is happening—portfolio managers will need to exhibit more skill in selecting funds. If they cannot reliably separate the good from the lucky, portfolio managers may learn to prefer concentration to "deworsification."

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