

ON THE OPTIMAL PACKAGE FORMAT FOR ASSET SELLERS

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ABSTRACT

A seller who owns two common-value assets can choose to either sell them as a bundle or separately. In this paper, we present a theoretical model to select the optimal selling option when there is asymmetric information between the seller and the buyers. Our main finding is that separate selling makes the seller fall into a bilateral monopoly environment, in which the assets are sold through bargaining, while bundled selling leads to a competitive bidding environment. When the seller's bargaining ability is given, the difference between the two assets' values increases, so the seller's incentive to sell as a bundle decreases. On the other hand, given the values of both assets, when the seller's bargaining power increases, the incentive to sell as a bundle decreases.

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INTRODUCTION

Consider a large corporation with several divisions or branches being sold. The seller can either sell those branches as a bundle or take them apart and sell them separately. The seller may have more information about the assets being sold than potential buyers. Some of the potential buyers may have specialized knowledge of particular branches being sold, but may be unfamiliar with other branches. This situation is not only common in the sale of corporations but is frequently seen in the sale of financial assets. Of interest is to determine the optimal decision for the seller—to sell these assets as a bundle, or to sell them separately.

Earlier studies, such as Stigler (1968), Adams and Yellen (1976), and McAfee et al. (1989) among others, have already dealt with bundling problems. However, most of these studies focused on cases in which the value of goods varies with buyers' preferences or purposes. This is not consistent with the common-value characteristic of assets. To fill this gap, this paper examines the bundling problem in regard to the sale of assets.

We provide a model to consider the optimal package format for the seller of assets. The common-value characteristic of assets and asymmetric information between the seller and the buyers are considered in this model. The seller owns two different but symmetric assets that are for sale, and has complete information about these assets. On the other hand, each buyer has information regarding only one of the assets, and each buyer's information concerns a different asset. When the assets are sold separately, an individual buyer will not participate in the transaction for the object where he has inferior information. Thus, separate selling makes the seller fall into a bilateral monopoly environment, in which the object is sold through bargaining.

When the assets are sold in a bundle, where every buyer possesses some information about the object, and bidding in accordance with this information enables them to generate positive revenue, bundled selling leads to a competitive bidding environment. We find that given the seller's bargaining ability, the smaller that the difference between the two assets' values is, the stronger the tendency is for the seller to sell the assets as a bundle. On the other hand, given both assets' values, the seller's increasing bargaining power

weakens his/her incentive to sell the assets in a bundle. The rest of the paper is organized as follows. In section 2, we discuss the relevant literature. Section 3 provides the introduction to the model. Section 4 explores how the seller decides the optimal selling format. Section 5 is the conclusion.

RELATED LITERATURE

Some studies that focus on asset-backed securities provide various explanations for the reasons why people often adopt a bundling strategy for selling mortgages. Boot and Thakor (1993) concluded that bundling improves the precision of the information of informed traders, increases their return on information, and encourages higher informed demand. This makes the seller of a high-quality firm better off. Others, such as Riddiough (1997) and Glaeser and Kallal (1997), suggested that packaging mortgages is a strategy that can soften the problem of a lemon-related subordinated effect, and therefore increase liquidation proceeds. Since bundling effectively weakens the problem that results from information asymmetry, most of these studies regard bundling as a strategy that dominates unbundled sales. However, as we can observe, unbundled sales are not uncommon in practice. In our model, we explore an environment where both formats are possible. In this environment, the relative value of assets determines the seller's choice on package format.

Studies focusing on auctions of multiple products note that a bundling strategy should be adopted conditionally. Palfrey (1983) suggested that a bundling strategy is optimal when there are few bidders, and that a seller tends to prefer a separate auction when there are more bidders. Avery and Hendershott (2000) noted that the optimal auction format favors allocations that bundle the product to the bidders who are interested in both products, and the form of bundling can be different in an auction. Probabilistic bundling – where the bidder's chance of winning the product increases with her value of the other product – is usually adopted in an auction. Armstrong (2000) suggested that the optimal auction takes one of two formats: the objects are sold at independent auctions or with a degree of bundling, and are in a sense similar to probability bundling. The optimal format of an auction depends on the number of bidders, as reflected by Palfrey's results (1983). In our paper, we only consider a two assets case, so the problem of choosing package format is simpler compared to these papers. However, we emphasize the impact of information asymmetry between the seller and buyers. This situation is not considered by earlier authors, but is quite general in the case of financial or several real assets.

THE MODEL

Consider a simple economy with one seller who issues two different assets, *asset 1* and *asset 2*, that will generate nonnegative random cash flows of \tilde{v}_1 and \tilde{v}_2 in the future, respectively. There are also two different kinds of buyers in the economy; named by *buyer 1* and *buyer 2*. We assume that \tilde{v}_1 and \tilde{v}_2 are identically and independently distributed between 0 and \bar{v} according to a density function $f(v)$, which is common knowledge to all agents in the economy. We further assume that the seller has complete information about the realized cash flows generated by the two assets, denoted by v_1 and v_2 , respectively. By contrast, the buyers have less information regarding the assets. Buyer 1 is informed about the realized value of asset 1 only, whereas buyer 2 is informed about the realized value of asset 2. The buyers have no information about the other asset except its density function $f(v)$. For all agents in our economy, the values of assets satisfy the additive condition, that is, the total value for an agent who owns both assets is the sum of the two asset values.

Market interest is normalized to be zero. Therefore, in the absence of market imperfections, the total reservation price of the two assets for the seller is $v_1 + v_2$. However, we assume that the market has some imperfections. For example, most banks and institutions in the financial service industry may face

credit constraints or minimum-capital constraints. As a result, the seller is willing to sell these assets at a discount in order to raise cash. In particular, we assume that the seller discounts future cash flows at a rate of $\delta \in [0,1]$. Consequently, if the seller retains the assets, they have a private value of $\delta(v_1 + v_2)$. In this model, assets are sold at an auction. The standard sealed-bid second-price auction is considered in this preliminary model.

Moreover, since the seller is more informed than the buyers, the latter may make inferences from the seller's selection of package format to get more specific information about the realized values of the two assets. Hence, the interaction and the information delivery between the seller and buyers should be considered when solving this model.

EQUILIBRIUM AND OPTIMAL SELLING FORMAT

In this section, we compare bundled selling with separate selling. We find that when the seller decides to sell the two assets separately, the buyer without information regarding the target will not participate in the transaction of the given asset, which will lead to a bilateral environment where the seller has to bargain with the informed buyer. On the other hand, if the seller sells both assets as a bundle, bidding competitively is profitable for the buyers. Finally, we will show the conditions that determine the seller's choice of package format. Since the seller is more informed about the assets than the two buyers, the choice of package formats can be regarded as a tactic to take advantage of the buyers' inferior information.

Selling the Assets Separately

In this subsection, we consider the situation in which the seller sells the assets separately. Because of the symmetry between the two assets, we start with an analysis of the sale of asset 1. When the second-price auction is held, it is easy to check that truth-bidding is a weak dominant strategy for buyer 1, which drives buyer 2, who has no information about asset 1, out of the auction. From the viewpoint specifically of buyer 2, he knows that buyer 1 will submit the true value v_1 as his bid. If he wins in the auction, buyer 2 will pay v_1 and collect zero profit. If he loses in the auction, the profit will also be zero. Since the best result for buyer 2 in the auction is zero, he has no incentive to put in his bid for asset 1. By the same token, buyer 1 does not participate in the auction in which asset 2 is on sale.

Having got the point when the seller decides to sell the assets separately, only the informed buyer would like to participate in the transaction. We know that the seller and the informed buyer are dwelling in a bilateral monopoly scenario. In this market, the asset is traded through bargaining between the informed buyer and the seller. Since the value of asset i for the seller and informed bidder i is δv_i and v_i , respectively, where $i = 1, 2$, the final deal price in the transaction will be somewhere between the two values, depending on both parties' bargaining power. When the seller's bargaining power is stronger, the price will be closer to v_i . In the opposite case, the price will be closer to δv_i . As a result, the final deal price can be the weighted average between δv_i and v_i , and we denote it as

$$p_i = \eta v_i + (1 - \eta) \delta v_i \tag{1}$$

where the weights η and $1 - \eta$ represent the bargaining power of the seller and the informed buyer, respectively. Note that $\eta \in [0,1]$, and that the closer η is near to 1, the stronger the seller's bargaining power will be.

We are now in a position to calculate the total revenues that the seller will receive when he sells the assets separately. By imposing the additive assumption, the total revenues will be

$$R_s = p_1 + p_2 = \eta(v_1 + v_2) + (1 - \eta)\delta(v_1 + v_2) \quad (2)$$

As we can see in Equation (2), the total revenues collected from the sales of the assets increase with the seller's bargaining power. Equation (2) seems to imply that the issuer has the same bargaining power for each asset. Actually, it is allowed to let the issuer have different bargaining power when selling different assets to different buyers. We can regard η as the average bargaining power of the issuer for each asset. To be specific, suppose that η_1 and η_2 represent the bargaining power of the issuer for each asset, the total revenues from the sale of the two assets are $R_{ds} = \eta_1 v_1 + (1 - \eta_1)\delta v_1 + \eta_2 v_2 + (1 - \eta_2)\delta v_2$. If we replace η by $[\eta_1 v_1 + \eta_2 v_2] / [v_1 + v_2]$ in Equation (2), we get the same result as R_{ds} , where η is the average of η_1 and η_2 weighted by the ratio of each asset's value relative to the aggregate value of total assets.

Intuitively, it is unfavorable for the seller to sell the assets separately when his bargaining power is weak. At this time, bundled selling may be a wise decision for the seller to redeem himself from his weak bargaining ability because it stimulates buyers to compete. From now on, for the sake of simplicity, we focus on the case where the seller has zero bargaining ability, i.e. $\eta = 0$. The case where $\eta > 0$ will be discussed at the end of this paper.

Bundling the Assets

In this section we consider an alternative case where the seller bundles the assets and sells them as a composite by holding a second-price auction. When the assets are sold as a bundle, since each buyer has partial information about the true value of the whole bundle and offers his bid according to this information with profitable revenue, bundled selling makes both buyers bid competitively in the auction.

The bundling environment can be modeled as a affiliated common value auction, where the realized value of the object for each buyer is $v = v_1 + v_2$. To take the true value of the object as the average or summation of the signals of bidders is usually adopted in the common value auction literatures, such as Bikhchandani and Riley (1991), Albers and Harstad (1991), Krishna and Morgan (1997), Klemperer (1998), Bulow and Klemperer (1996), and Georee and Offerman (2003).

Since buyer 1 and buyer 2 are symmetric, we can just analyze the bidding behavior of bidder 1 and then extend the result to the behavior of bidder 2. In the affiliated model, $v(x, y) = E[v | v_1 = x, y_1 = y]$, which refers to the expected value function of the object conditional on bidder 1's own signal x and the highest signal of the remaining bidder's signal y , plays an important role in featuring the bidders' bidding strategies. In our model, the expected value function can be expressed as $v(x, y) = x + y$.

Following Milgrom and Weber (1982), the optimal bidding strategy for a bidder who receives the signal x is $b_s = v(x, x)$ in the second-price auction. This means that a bidder, say buyer 1, with signal x is asked to bid an amount such that if he were to just win the auction with that bid he would just break even. Thus, in our model a bidder with signal v_i submits a bid equal to

$$b_s(v_i) = 2v_i \quad (3)$$

However, since the seller is more informed, his decision for the package format of assets may reveal extra information to the buyers, and the revealed information may affect the bidders' bidding strategy. In Proposition 1 we have the result that bidders will still follow the bidding strategy as in Equation (3) even if the effect of the revealed information is considered. Furthermore, it is easy to check that the expected revenue for each buyer is positive (The proof in detail can be found in Milgrom and Weber (1982)).

Proposition 1: Each buyer's optimal bidding strategy will follow Equation (3), even if the information revealed by the seller's selling decision is considered.

Proof of Proposition 1: When bundling is held by the seller, buyer 1 infers that the revenue from auction $2v_2$ is at least as large as that from negotiation, δv , where $v = v_1 + v_2$. Thus, he knows that $2v_2 \geq \delta(v_1 + v_2)$ if he wins. The relationship between the two values can be further rearranged as $\frac{\delta}{2-\delta}v_1 \leq v_2$, meaning that bundling signals to him that the lower bound of bidder 2's signal is rising from zero to $\frac{\delta}{2-\delta}v_1$. Hence, buyer 1's revenue can be expressed as

$$R_1 = \int_L^{\hat{b}^{-1}(b_1)} (v_1 + v_2 - b(v_2))f(v_2)dv_2, \quad \text{where } L = \frac{\delta}{2-\delta}v_1.$$

To find the optimal bidding strategy, the first-order condition should be zero when $\hat{b}(v_1) = b_1$. After rearrangement, we have the following equation

$$(v_1 + \hat{b}^{-1}(\hat{b}(v_1)) - \hat{b}(\hat{b}^{-1}(\hat{b}(v_1))))f(\hat{b}^{-1}(\hat{b}(v_1))) = (2v_1 - \hat{b}(v_1))f(v_1) = 0.$$

It is easy to verify that the solution is $\hat{b}(v_1) = 2v_1$, which is the same as the strategy derived without considering the information revealed by the seller's selling decision.

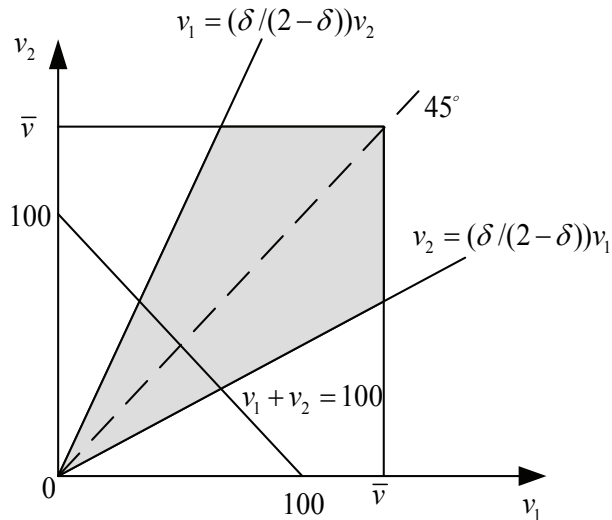
The buyers' bidding strategy is common knowledge and the seller has precise information about the buyers' signals, so that the revenue generated from the sale of the bundled assets is a certainty for the seller, which is represented as

$$R_b = \min\{2v_1, 2v_2\} \tag{4}$$

The Package Decision: Bundling vs. Separating

In what follows, we compare the revenue from bundling with that from separating, and try to find out how the seller picks the optimal selling option. Proposition 2 states the criteria that help the seller make the decision.

Figure 1: Relation between the Values of Assets and the Package Formats



This figure relates the optimal selling format to the values of the assets. The seller selects bundling strategy when both assets' values fall into the shaded area. This implies that the seller prefers bundling when the difference of assets' values is small.

Proposition 2: Let $v_m = \min\{v_1, v_2\}$ and $v_M = \max\{v_1, v_2\}$, where v_1 and v_2 are realized values of \tilde{v}_1 and \tilde{v}_2 . For all $0 < \delta < 1$, when $v_m > \delta v_M / (2 - \delta)$, the seller can be better off by selling the assets in a bundle. By contrast, when $v_m < \delta v_M / (2 - \delta)$, the seller will prefer to sell the assets separately. In the case with $v_m = \delta v_M / (2 - \delta)$, the seller will be indifferent between choosing the two selling formats.

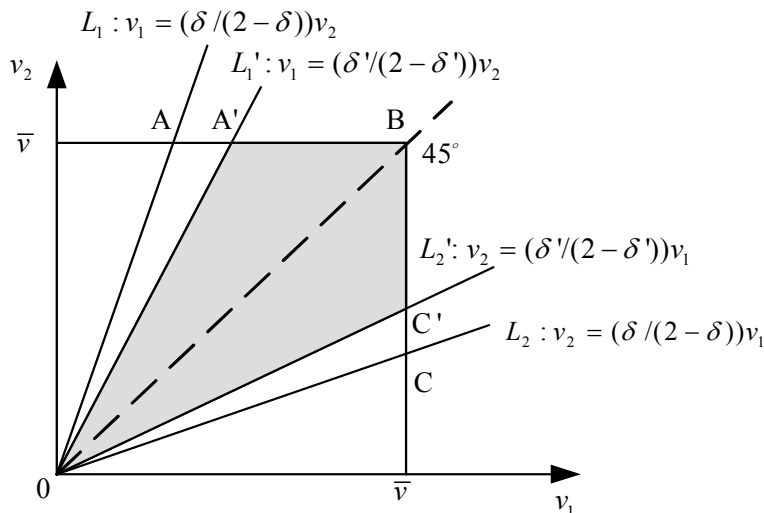
Proof of Proposition 2: Although bundling reveals information to the bidders, the bidding strategy of the bidder does not change, as shown by the results in Proposition 1. Thus, the revenue from selling assets in a bundle is $2v_m$, and that from selling assets separately is $\delta v_M + \delta v_m$. Since the seller sells assets in a bundle when bundling generates higher revenue than separating, we have the relationship denoted by $\delta v_M + \delta v_m < 2v_m$. Thus, the seller would rather sell the assets in a bundle when $v_m > \delta v_M / (2 - \delta)$. By the same token, separating is held by the seller while $v_m < \delta v_M / (2 - \delta)$.

The optimal selling format involves a tradeoff between a bargaining cost and a bidding cost for the seller. When selling separately, the seller suffers a bargaining cost, which results from the fact that the seller has to sell the assets at a discounted price of δv . The cost decreases when δ increases. On the other hand, when selling as a bundle, the seller faces a bidding cost, which results from the fact that the high-valued asset is sold at a price equal to the low-valued asset. This cost can be estimated by the relative value of both assets. The bigger the difference between the values of both assets, the higher the bidding cost will be. Because the optimal format is the one that costs the seller less, the seller's incentive to use bundled (separate) selling increases when δ decreases (increases) or the difference between the values of both assets decreases (increases).

Figure 1 relates the optimal selling format to the values of the assets. In this figure, the seller selects bundling when both assets' values fall into the shaded area. It is verified that the decision for the selling format depends on the relative value of both assets. Moreover, we find that the total value of assets does not affect the seller's decision. To be clearer, we provide a numerical example for illustration. Given

that $\delta = 0.5$, we compare the following two cases. We let $v_1 = 90$ and $v_2 = 10$ in case (1), while $v_1 = 50$ and $v_2 = 50$ in case (2). In case (1), the revenue is 50 from the separate selling, while it is only 20 from the bundled selling. Thus, separate selling is better in case (1). In case (2), the revenue is also 50 from separate selling. However, the seller receives 100 from the bundled selling. Thus, in case (2) the seller prefers to sell the assets as a bundle. Therefore, the seller may choose different selling formats even when the aggregate value of the assets is the same. The result can be shown in Figure 1, and it is obvious that the line $v_1 + v_2 = 100$ pass through both areas.

Figure 2: The Relation between the Seller’s Bargaining Ability and the Package Formats



This figure relates the optimal selling format to the seller’s bargaining ability. The increase in the seller’s bargaining ability reduces the bundling area from OABC to OA’BC’. This implies that the increase in the seller’s bargaining power weakens his tendency to adopt a bundling format.

Now, we go back to the situation where the seller’s bargaining ability η is not zero. After rearranging Equation (1) and replacing $\eta + (1 - \eta)\delta$ with δ' , we have

$$R_s = p_1 + p_2 = \eta(v_1 + v_2) + (1 - \eta)\delta(v_1 + v_2) = (\eta + (1 - \eta)\delta)(v_1 + v_2) = \delta'(v_1 + v_2) \tag{5}$$

which implies that the increase in the seller’s bargaining ability can be regarded as the increase in the discounted factor value from δ to δ' . In Figure 2, we can see that the increase in the seller’s bargaining ability reduces the bundling area from OABC to OA’BC’. Consequently, we can conclude that the increase in the seller’s bargaining power weakens his tendency to adopt a bundling format.

CONCLUSION

In this paper, we consider how a seller, who owns multiple assets, increases his advantage through packaging strategies. We set up an environment where the seller has two assets and wants to sell them because of some financial constraints. The seller is precisely informed all of the values of his assets while each buyer has only partial information concerning the assets. In this environment, selling separately drives out the buyer who has no information regarding the item being sold, which leads to a bilateral environment where the seller has to bargain with the informed buyer. If the seller’s bargaining ability is weak, separate selling may not be a wise decision for him. On the other hand, if the seller sells both assets as a bundle, since each buyer possesses some information regarding the object, and bidding according to this information enables the buyers to generate positive revenue, bundled selling leads to a

competitive bidding environment.

We establish a theoretical model in this study and show that package formats selected by the seller can be regarded as tactics for him to take his information advantage when a monopolistic seller is better informed than the buyers. Our study provides two primary results. First, by giving the seller's bargaining ability, we show that the relative value of assets determines the selling format. In specific, the smaller the difference between the two assets' values, the greater the tendency will be for the seller to sell the assets as a bundle. Second, given the assets' values, the increase in the seller's bargaining power weakens the seller's incentive to use bundled selling.

We consider only the sealed-bid second-price auction to obtain the preliminary results. To arrive at a clearer picture in the scenario we set, it is necessary to discuss the effects of introducing other auction formats, e.g., the first-price auction and English auction. This will be left to our future research.

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