THE USE OF TERM STRUCTURE INFORMATION IN THE HEDGING OF JAPANESE GOVERNMENT BONDS

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ABSTRACT

This paper employs the Kalman filter to explore the impact of term structure variables in the hedging of Japanese Government Bonds (JGBs) with treasury futures. The term structure factors (level parameter β_0, slope parameter β_1, and curvature parameter, β_2) are based on Nelson and Siegel (1987) model. The out-of-sample hedging performance is also provided by moving window technology. The empirical results show the existence of significant relationships among the term structure factors, the earlier hedge ratio, and the optimal hedge ratio. However, the time-varying hedge ratio (which includes the term structure variables from the information set) did not provide good out-of-sample hedging effectiveness. Nevertheless, the out-of-sample results did demonstrate that the performance of the time-varying hedge ratio with term structure variables is better than a hedge ratio with a naive hedge or OLS model in the 7–10-year Japanese Government Bond index.

JEL: G12, G15, G32

INTRODUCTION

The so-called 'term structure of interest rates' (TSIR), which is also known as the yield curve, shows the expected yield from zero-coupon government bonds under a given default risk. For many large financial institutions, information derived from the TSIR has played an important role in the valuation and hedging of interest-rate-dependent instruments. Moreover, the shape of the TSIR provides a good predictive indicator of future economic activity, with consequent implications for estimations of GDP and inflation rates (Estrella and Mishkin, 1998; Estrella and Hardouvelis, 1991) Investors can, therefore, make judgments on the future impact of financial and economic activities by observing changes in the TSIR, and can thus adjust their investment and hedging strategies.

Because the spot price of bonds is a function of interest rates and the movement of the yield curve is not parallel within different maturities, Litterman and Scheinkman (1991) decomposed the curve of the TSIR into three phases: level, slope, and curvature. Diebold and Li (2006) used β_0, β_1 and β_2 in the Nelson and Siegel (1987) model to designate these three phases, which they interpreted as the long-term, short-term, and middle-term phases in the interest rate. In addition, apart from these three phases, the factor τ from the Nelson and Siegel (1987) model governs the exponential decay rate. When the value of τ is small, slow decay is produced and there is a better fit with the yield curve at long maturities; conversely, when the value of τ is large, rapid decay is produced and there is a better fit with the yield curve at short maturities. Moreover, as Dolan (1999) and Diebold et al. (2006) have pointed out, the Nelson and Siegel (1987) model is well suited to describing the dynamic process of the TSIR and providing good forecasts.

To promote bond portfolio performance, bond investment managers obviously wish to manage interest rate risks efficiently, and they need to understand the dynamic process of the TSIR. In this regard, Markowitz (1991) decomposed portfolio risk into: (i) system risk; and (ii) non-system risk. According to this view, bond investment managers can eliminate non-system risk by diversification of investment;

however, the hedge function of futures contracts or other derivatives can diversify the system risk in the bond portfolio. Management of the interest risk with futures contracts has become an important issue in bond portfolio management.

The Tokyo Stock Exchange (TSE) has offered trading on 10-year Japanese Government Bond (JGB) Futures contracts since October 1985. The trading volume of these contracts has boomed, and the JGB Futures market is now regarded as one of the most active in the Asian financial system. These futures bonds are popular because they can be used to transfer risk and thus provide bond investors with a hedge against interest rate risks. In these circumstances, the determination of the optimal hedge ratio becomes a crucial issue in the hedge strategy of investors; however, their ability to do this effectively has been inhibited by the fact that most empirical studies of these issues have focused only on the question of stock portfolio hedge. The present paper addresses this gap in the literature by analysing the hedge function of interest rate futures in the JGB market.

Because the duration of a bond portfolio is fundamentally a function of interest rates, it is reasonable to assume that the returns of a bond portfolio will fluctuate with movements in the yield curve. In accordance with the approach adopted by Fink et al. (2005), the present study investigates the performance of an optimal hedge ratio using moving window technology based on the Kalman filter; however, the approach of Fink et al. (2005) extended here through a more accurate estimation of the yield curve factors. In this regard, Dolan (1999) pointed out that the parameters of the yield curve, estimated using the Nelson and Siegel (1987) model, could be predicted; indeed, Dolan (1999) presented forecasts of how the level, slope, and curve could have significant effects on bond portfolio performance. The main contribution of the present work is to combine the yield curve factors of the JGB, using the Nelson and Siegel (1987) model, with the Kalman filter to generate the optimal hedge ratio.

The remainder of this paper is organised as follows. In the next section, we discuss the relevant literature. The next section introduces the estimation of yield curve factors, and the calculation of the time-varying optimal hedge ratio, using the Kalman filter. The third section proceeds to an empirical analysis - describing: (i) the detailed data; (ii) the estimation of the model of yield curve and the Kalman filter; and (iii) the in-sample and out-of-sample performance of a number of hedges. The final section presents the conclusion.

LITERATURE REVIEW

Estimating Term Structure of Interest Rates

Many methods exist to estimate the yield curves. Generally speaking, there are two distinct approaches to estimate the term structure of interest rates: the equilibrium models and the empirical models**.** The equilibrium models are formalized by defining state variables characterizing the state of the economy (relevant to the determination of the term structure) which are driven by these random processes and are related in some way to the prices of bonds. It then uses no-arbitrage arguments to infer the dynamics of the term structure. Examples of this include Vasicek (1977), Dothan (1978), Brennan and Schwartz (1979), Cox Ingersoll and Ross (CIR, 1985) and Duffie and Kan (1996). Unfortunately, in terms of the expedient assumptions about the nature of the stochastic process driving interest rates, the term structure of interest rates derived by those models could only exist theoretically in an efficient market and do not conform well to the observed data on bond yields and prices.

In contrast to equilibrium models, the empirical models focusing on obtaining a continuing yield curve from cross-sectional coupon bond data based on curve fitting techniques are able to describe a richer variety of yield patterns in reality. The resulting term structure estimated from the statistical techniques can be directly put into interest rate models, such as the Ho and Lee (1986), the Heath et al. (1992) and Hull and White (1990) models, for pricing interest rate contingent claims. Since a coupon bond can be considered as a portfolio of discount bonds with maturities dates consistent with the coupon dates, the discount bond prices thus can be extracted from actual coupon bond prices by statistical techniques. Examples of this approach include McCulloch (1971, 1975), Schaefer (1981), Vasicek and Fong (1982) and Steely (1991). The major advantage of the empirical models is able to characterize a plenty variety of reasonable yield curve patterns which are consistent with real market yield curves. Furthermore, among the empirical models, the Nelson and Siegel (1987) model is most suited to the ultimate purpose of determining the optimal hedge ratio. There are three main superior features of the Nelson and Siegel (1987) model. First, it has only three major parameters, and which can be well used to explain the meaning of the yield curve shape in real market condition; Second, it has been proved to be good at fitting yield curves (Willner, 1996; Dolan, 1999; Diebold and Li, 2006; Diebold et al., 2006); Finally, each parameter derived from the Nelson and Siegel (1987) model can separately present the level, slope and curvature changes, which are three significant contributions of varied yield curve shape.

Revolution of Hedge Ratios Estimation

Traditionally, hedge ratios have been estimated by regression analysis. However, Myers (1991) has argued that this method of estimating hedge ratios encounters two problems: (I) the estimated value of hedge ratios using this methodology does not involve all relevant information; and (ii) the hedge ratio derived by this method is not time-dependent because the covariance matrix of spot and futures prices will not change with time. As a result, the assumption of a fixed covariance matrix could induce investors to take unacceptable risks with futures.

To resolve these difficulties in the traditional model, Engle (1982) provided the autoregressive conditional heteroscedasticity (ARCH) model, which predicts the conditional variance by taking a weighted average of past errors. In this model, recent information has more influence on the error term than information from the distant past. This ARCH model was extended in the Generalized ARCH model (known as the GARCH model), which was developed by Bollerslev (1986). The GARCH model assumes that variance is a weighted average between previous variance and error terms. An even more generalized model has been proposed by Engle et al. (1988), who took into consideration the feedback relationship between spots and futures. All of these ARCH-type models represent a significant advance on previously used methods because they all assume that hedge ratios are time-variant, and many studies have used them to estimate time-varying hedge ratios (Baillie and Myers, 1991; Kroner and Sultan, 1993; Koutmos, 2001; Rossi and Zucca, 2002). In order to avoid the difficulty of deciding the initial value of the GARCH model, Fink et al. (2005) utilised the Kalman filter to estimate a time-varying optimal hedge ratio.

As noted above, the traditional assumption that hedge ratios are time-invariant is not sustainable; rather, it is necessary to describe the dynamic relationship between yield-curve factors and hedge ratios. To incorporate information on the level, slope, and curvature of the yield curve into the estimation of a time-varying optimal hedge ratio, the present study utilises the Kalman filter, which avoids the difficulty of deciding the initial value of the GARCH model.

METHODOLOGY: CHOOSING A MODEL OF YIELD CURVE AND KALMAN FILTER

The parsimonious model of the yield curve used in this paper is that built by Nelson and Siegel (1987). Willner (1996) contended that this model is a useful method for approximating the sensitivity of a bond portfolio to yield-curve level, slope, and curvature. In a similar vein, Diebold and Li (2006) and Diebold et al. (2006) argued that the well-known Nelson-Siegel (1987) model is well suited to approximating

yield-curve dynamics and providing good predictions. The model is used in the present study to estimate the level, slope, and curvature of the yield curve with Japanese government coupon bonds.
The theoretical price of a coupon bond is equal to the sum of the present value of the future coupon and the principal payments according to the following relationship:

$$\hat{B}_i = \sum_{j=1}^{M_i} C(t_{i,j}) \exp\{-t_{i,j} R(t_{i,j})\}, \quad (1)$$

where:

$\hat{B}_i$ is the i_{th} theoretical price of coupon bond;

M_i is the maturity of the i_{th} bond;

$C(t_{i,j})$ is the cash flow of the i_{th} bond at time t_j; and

$R(t_{i,j})$ is the spot rate at time t_j in the i_{th} bond.

Nelson and Siegel (1987) chose a function for the forward rate curve that can be transferred by integrating process to spot rate curve as follows:

$$R(t_{i,j}) = \beta_0 + \beta_1\left(\frac{\tau}{t_{i,j}}\right)\left[1-\exp\left(\frac{-t_{i,j}}{\tau}\right)\right] + \beta_2\left(\frac{\tau}{t_{i,j}}\right)\left[1-\exp\left(\frac{-t_{i,j}}{\tau}\right)\left(\frac{t_{i,j}}{\tau}+1\right)\right] \quad (2)$$

Where

β_0, β_1, β_2 and τ are the parameters for a maturity of t years.

The Nelson and Siegel (1987) model implies an intuitive explanation of the parameters: (i) the value of β_0, which is regarded as a long-term interest rate, is represented by the level of the curve; (ii) the value of β_1, which is regarded as a short-term interest rate, is represented by the slope of the curve; (iii) the value of β_2, which is regarded as a medium-term interest rate, is represented by the curvature of the curve; and (iv) the parameter τ, which governs the exponential decay rate at which the short-term and medium-term factors decay to zero.

To generate these parameters of the yield curve, we added the function of spot rate into the theoretical price of the coupon bond function (1) as follows:

$$\begin{aligned}\hat{B}_i &= \sum_{j=1}^{z_i} C(t_{i,j})\exp((-t)(R(t))) \\ &= \sum_{j=1}^{z_i} C(t_{i.j})\exp\left((-t_{i,j})\times\left(\beta_0+\beta_1(\frac{\tau}{t_{i,j}})\left[1-\exp(\frac{-t_{i,j}}{\tau})\right]+\beta_2(\frac{\tau}{t_{i,j}})\left[1-\exp(\frac{-t_{i,j}}{\tau})(\frac{t_{i,j}}{\tau}+1)\right]\right)\right)\end{aligned} \quad (3)$$

The parameters can then be estimated by minimising the difference between the actual and theoretical bond price; that is:

$$Q = \frac{1}{n}\sum_{i=1}^{n}\left[(B_i - \hat{B}_i)\right]^2 \quad (4)$$

where n is the number of bonds.

Because the objective function is nonlinear, the Newton method can be used to approximate the parameters of the Nelson and Siegel (1987) model. One advantage of this method is that τ cannot assume to be a constant; rather, it varies with other parameters. In this regard it should be noted that Diebold and Li (2006) estimated the Nelson and Siegel (1987) model with a constant value of the τ, but

Hurn et al. (2005) argued that the curve from the Nelson and Siegel (1987) model is sensitive to the scale parameter τ, which cannot be fixed. Applying the Newton method to the Nelson and Siegel (1987) model for each day generates a time series of estimates of parameters, which can then be placed as yield-curve factors in the Kalman filter model to estimate the optimal hedge ratio.

The GARCH-based estimation method for time-varying hedge ratios requires the imposition of inequality restrictions on model parameters and the use of a wide range of starting values (Fackler and McNew, 1994; Harris and Shen, 2003). To overcome these negative features of the GARCH method, the present study utilised a Kalman filter to construct a state space specification to estimate the optimal hedge ratio. A state space representation of the relationship between spot and futures return is given by the following system of equations:

$$
\begin{aligned}
\Delta S_t &= \Delta f_t v_t + \mu_t \\
v_t &= \alpha + \lambda v_{t-1} + \gamma_1 \beta_0 + \gamma_2 \beta_1 + \gamma_3 \beta_2 + \zeta_t \\
\mu_t &\sim N\left(0, \sigma_\mu^2\right) \\
\zeta_t &\sim N\left(0, \sigma_\zeta^2\right)
\end{aligned}
\tag{5}
$$

where

ΔS_t is the log return of the bond index at time t;

Δf_t is the log return of the 10-year JGB futures contracts employed in the hedge portfolio at time t; and

v_t, the optimal hedge ratio at time t, determines the value of futures contracts purchased or sold to the underlying security.

In the state equation, λ measures the persistence of the optimal hedge ratio. Other coefficients that interpret the effect of the yield-curve shape are:

γ_1, which represents the level effect of the yield curve;

γ_2, which represents the slope effect of the yield curve;

γ_3, which represents the curvature effect of the yield curve.

The both error terms μ_t and ζ_t are assumed to follow a normal distribution and are independent of each other.

The Kalman filter procedure takes into account the serially correlated and heteroscedastic disturbance in the relationship between changes in the spot return and changes in the futures return. In addition, the Kalman filter is a recursive algorithm for sequentially updating the time-varying hedge ratios (given new information during the time series). For instance, consider a dataset that includes T observations with the former state vector $\lambda_{1|0}$ defined as the optimal hedge ratio at time one (which is estimated at time zero). In these circumstances, the later state variable $P_{1|0}$ represents the covariance matrix of the conditional distribution of the state vector $\lambda_{1|0}$ (given information available at time zero). Given that the information parameters λ 、 γ_1 、 γ_2 、 γ_3 、 σ_μ^2, and σ_ζ^2 are assumed to be known, the one-step ahead predictor of state terms $\lambda_{1|0}$ and $P_{1|0}$ can be expressed as:

$$\hat{v}_{t|t} = \hat{v}_{t|t-1} + \left(P_{t|t-1}\right)^2 \left(\Delta f_t\right)^2 F_t^{-1} \left(\Delta S_t - \lambda \hat{v}_{t|t-1}\right) \tag{6}$$

$$P_{t|t} = P_{t|t-1} - \left(P_{t|t-1}\right)^2 \left(\Delta f_t\right)^2 F_t^{-1} \tag{7}$$

$$F_t = (\Delta f_t)^2 P_{t|t-1} + \sigma_\mu^2 \tag{8}$$

Therefore, the optimal hedge ratio λ is predicted one step ahead in the following way:

$$\hat{v}_{t+1|t} = (\lambda - K_t \Delta f_t)\hat{v}_{t|t-1} + K_t \Delta S_t + (\alpha + \gamma_1 \beta_0 + \gamma_2 \beta_1 + \gamma_3 \beta_2) \tag{9}$$

$$K_t = \lambda P_{t|t-1} \Delta f_t F_t^{-1} \tag{10}$$

$$P_{t+1} = \lambda^2 \left\{ P_{t|t-1} - (P_{t|t-1})^2 \lambda^2 F_t^{-1} \right\} + \sigma_\zeta^2 \tag{11}$$

To complete the Kalman filter, the unknown elements of the system matrices must be replaced by their estimates. Given the assumption of the normality of μ_t and ζ_t, the parameters of the system equations can be estimated by formulating the log likelihood function as follows:

$$\log L = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log|F_t| - \frac{1}{2}\sum_{t=1}^{T}\frac{\theta_t^2}{F_t} \quad ,\theta_t = \Delta S_t - \Delta f_t \hat{v}_{t|t-1} \tag{12}$$

Data

The data used in the empirical study referred to daily 10-year JGB nearby Futures contracts traded on the Tokyo Security Exchange (TSE). The 10-year JGB nearby Futures contracts settlement prices were obtained from Datastream. The daily JGB price index was calculated by JP Morgan and collected from Datastream. The data transferred to the daily log return covered the period from 30 May 2002 to 18 April 2007. The total number of time-series observations in the data set was 1275. For estimating the JGB yield curve, the daily JGB price was plotted. This consisted of 179 observations (on average) per day from 30 May 2002 to 18 April 2007. A Newton method was used to extract these yield-curve factors embedded in the Nelson and Siegel (1987) model.

EMPIRICAL RESULTS

From these data, it was possible to derive the parameters and variables as described in the previous section. Table 1 provides some descriptive statistics of these time-series parameters and variables. The left column of the table shows the means, mediums, maximums, minimums, and standard deviations. Among the yield-curve factors:

(1) the mean of daily β_0 was 0.0327, which shows that the long-term interest rate level tended to 3.27%;
(2) the mean of daily β_1 was -0.0336, which represents the positive slope of the yield curve on average;
(3) the mean of daily β_2 was 0.0036, which shows that the slope of the yield curve was not only positive, but also had a hump in the JGB market.

In addition, the maximum β_2 and the minimum β_2 were not all larger than zero, which shows that the shapes of the yield curves in the JGB market involved different patterns. Therefore, the risk of yield-curve changes should be taken into account in interest risk management.

Table 1 also shows the statistics of the JGB spot and futures log return. The average log return of the JGB spot (–0.0017%) was less than that of the JGB futures (–0.0032%). Similarly, the standard deviation of spot log return (0.1453%) was also lower than that of the futures log return(0.2414%), which implies that the volatility of the futures market was greater than that of the spot market.

Table 1: Descriptive Statistics for Yield Curve Factors, Spot and Futures Returns

	β_0	β_1	β_2	JGB_Spot	JGB_Futures
Mean	0.0327	(0.0336)	0.0036	-0.0017%	-0.0032%
Median	0.0296	(0.0320)	0.0038	0.0000%	0.0000%
Maximum	0.0671	(0.0141)	2.1192	0.7086%	1.0525%
Minimum	0.0034	(0.0526)	(0.1376)	-0.8620%	-1.7891%
Std. Dev.	0.0123	0.0079	0.0866	0.1453%	0.2414%

The parameters β_0, β_1 and β_2 are the yield curve factors embedded in the Nelson and Siegel model. The daily returns of JGB price index and futures settlement price are transferred to log returns.

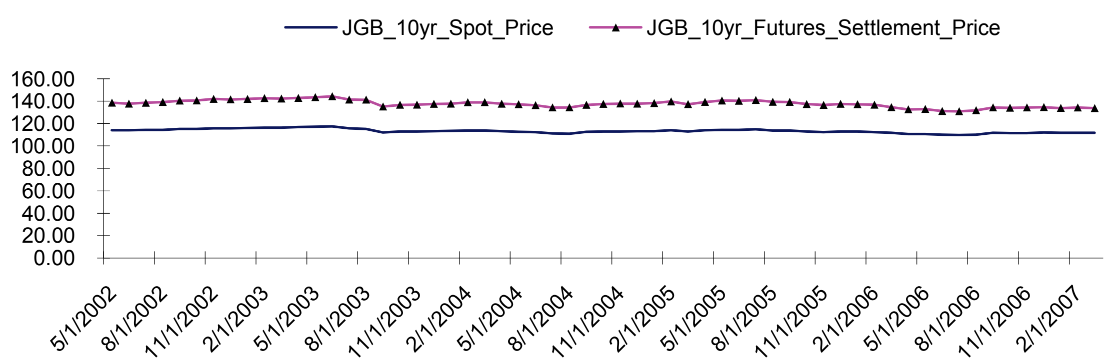
Pearson correlation analysis was employed to investigate the yield-fitting ability of the Nelson and Siegel (1987) model and the degree of relationship between the JGB spot and futures returns (as shown in Table 2). The correlation coefficient between the 10-year JGB price index and 10-year JGB futures settlement price was quite high (98.189%), which implies that the 10-year JGB price index return was more strongly correlated to the 10-year JGB futures return. This relationship is also clear from Figure 1. The strong correlation between these factors indicates that the JGB 10-year futures could provide a good hedge function for the JGB 10-year price index.

Table 2: Correlation of JGB Spot and Futures Price with JGB Yield and NS Yield

	10yr JGB_Spot Index Price	10yr JGB_Futures Settlement Price	10yr JGB_Yield	10yr JGB_NS_Yield
10yr JGB_Spot Index price	1	0.98189	-0.92197	-0.90868
10yr JGB_Futures Settlement Price	0.98189	1	-0.89346	-0.89210
10yr JGB_Yield	-0.92197	-0.89346	1	0.95757
10yr JGB_NS_Yield	-0.90868	-0.89210	0.95757	1

10-year JGB futures settlement price was quite high (98.189%), which implies that the 10-year JGB price index return was more strongly correlated to the 10-year JGB futures return.

Figure 1: The Price of 10 Year JGB versus the Settlement Price of 10 Year JGB Futures

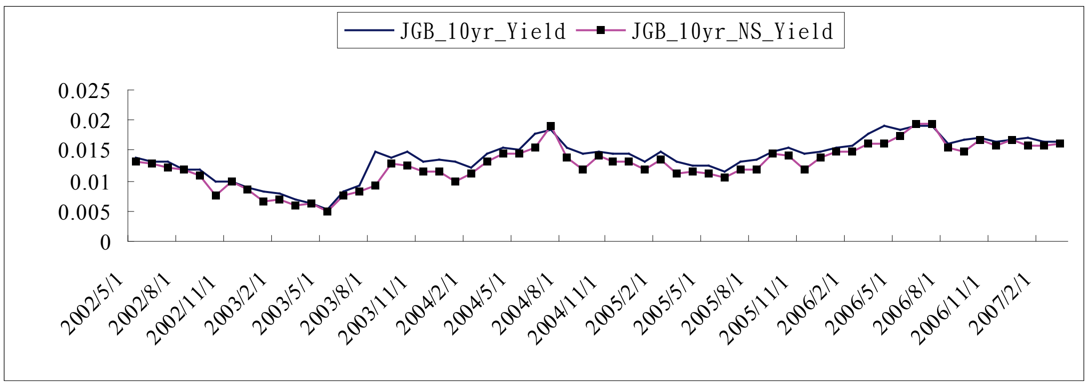


The strong correlation between these factors indicates that the JGB 10-year futures could provide a good hedge function for the JGB 10-year price index.

As shown in Table 2, the correlation coefficient between the 10-year JGB yield and the 10-year JGB price index was negative (–92.197%). Table 2 also presents the correlation between the 10-year JGB yield and the 10-year JGB futures settlement price (–89.346%). The negative relationship between the JGB yield and price is in accordance with the intuitive perception of bond pricing.

Finally, both Table 2 and Figure 2 show that the 10-year JGB yield estimated by Nelson and Siegel (1987) had a correlation of 95.757% with the actual 10-year JGB yield, which suggests that the Nelson and Siegel (1987) model could fit the 10-year JGB yield well. For this reason, the parameters of the model should involve some information to explain the variety of the 10-year JGB yield.

Figure 2: The Yield of 10 Year JGB versus the NS Yield of 10 Year JGB



Nelson and Siegel (1987) model could fit the 10-year JGB yield. Thus, the parameters of the model should involve some information to explain the variety of the 10-year JGB yield.

Effect of TSIR Factors on the Hedge Ratio

To compare the influence of different parameters on the determination of hedge ratio, a number of constrained alternatives are specified. Table 3 shows results of a Kalman filter for unrestricted and restricted models. The following observations can be made.

First, it is apparent that the persistence parameter λ was significantly positive in all models, which implies that the movement of the hedge ratio displayed persistency. Secondly, the level coefficient γ_1 (2.1324) in model 1 was significantly positive with respect to the hedge ratio. This phenomenon might be due to investors increasing their hedge position as the level of interest rate increases. Thirdly, the slope coefficient γ_2 (2.3873) in model 1 was significantly positive with respect to the hedge ratio, which implies that the difference between the short-term and long-term interest rates pushed investors to increase their hedge position. Finally, the curvature coefficient γ_3 (0.0499) was not significant. These findings demonstrate that the early hedge ratio and the level and slope of the yield curve affect the next optimal hedge ratio.

To gain a better understanding of the distribution of each parameter from the empirical model, different constraints of the parameters are shown as models 2 to 8 in Table 3. The following observations can be made.
Model 2 was estimated with a constraint of γ_3=0. The persistent coefficient λ (0.1316) of model 2 remained significantly positive. The yield-curve coefficients γ_1 (2.0956) and γ_2 (2.3896) also had a

significant influence on the optimal hedge ratio. Model 3 shows an alternative restriction, in which γ_2=0. The coefficients λ and γ_1 were significantly positive. Model 4 imposed a constraint of γ_1=0. The coefficient λ remained significantly positive. Two constraints were imposed to compare the contribution of each parameter in models 5 to 7. The coefficient λ was still significantly positive in these three models, but only γ_1 was significantly positive in model 5. These models returned similar results to those of previous models. As shown in models 1 to 7, it is apparent that the coefficients λ and γ_1 were important parameters for explaining the hedge ratio.

Finally, model 8 had a constraint of λ=0 (which enabled consideration of the effect of the yield-curve factor without the coefficient λ). The coefficients γ_1 and γ_2 were still significantly positive with respect to the hedge ratio, which implies that the level and slope factors provide additional information of importance in explaining the determination of the optimal hedge ratio.

Table 3: Results of Kalman Filter for Unrestricted and Restricted Model

	Model 1 (not restricted)	**Model 2 (γ_3=0)**	**Model 3 (γ_2=0)**	**Model 4 (γ_1=0)**	**Model 5 (γ_3=0) (γ_2=0)**	**Model 6 (γ_3=0) (γ_1=0)**	**Model 7 (γ_2=0) (γ_1=0)**	**Model 8 (λ=0)**
λ	0.1312*** (0.0006)	0.1316*** (0.0006)	0.1383*** (0.0002)	0.1380*** (0.0002)	0.1340*** (0.0002)	0.1394*** (0.0002)	0.1398*** (0.0002)	
γ_1	2.1324** (0.0110)	2.0956** (0.0126)	1.1054* (0.0572)		1.0506*** (0.0701)			2.4291*** (0.0065)
γ_2	2.3873** (0.0384)	2.3896** (0.0379)		0.3173 (0.6920)		0.3422 (0.6672)		2.8098** (0.0215)
γ_3	0.0499 (0.4338)		0.0407 (0.5709)	0.0233 (0.7745)			0.0245 (0.7624)	0.0624 (0.3153)
σ_μ	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
σ_ζ	0.1622	0.1624	0.1634	0.1645	0.1633	0.1644	0.1643	0.1670
Log likelihood	8286.8470	8286.5490	8284.3550	8282.4010	8284.1580	8282.3360	8282.3160	8284.7400

This table provides parameter estimates in the following model:

$$\Delta S_t = \Delta f_t \nu_t + \mu_t$$
$$\nu_t = \alpha + \lambda \nu_{t-1} + \gamma_1 \beta_0 + \gamma_2 \beta_1 + \gamma_3 \beta_2 + \zeta_t$$
$$\mu_t \sim N(0, \sigma_\mu^2)$$
$$\zeta_t \sim N(0, \sigma_\zeta^2)$$

The ΔS_t is log returns of the JGB price index at time t. The variable Δf_t is the log returns of the JGB futures settlement price, that it can structure the hedge portfolio. The term structure factors β_0, β_1 and β_2, form Nelson and Siegel model, show level movement, slope change, and curvature shift separately. The ν_t presents the appropriate hedge ratio at time t, and the coefficient λ is parameter of persistence to determine the appropriate hedge ratio. Finally, the coefficients γ_1, γ_2, and γ_3 demonstrate the effect of term structure factor on hedge ratio. The sample period is daily between May 30, 2002 and April 18, 2007 with 1275 empirical data. Model 1 through model 8 exhibit various restrictions of parameters.

The Wald test was used to examine the null hypothesis implied in Table 3. The results are shown in Table 4. Model 2 demonstrated an insignificant coefficient (γ_3), as expected. The findings for models 3, 4, and 8 could not lead to a rejection of their null hypotheses, which indicates that, the coefficients, γ_1, γ_2 and λ, all involve rich information in deciding the next hedge ratio. In contrast, the results from models 5, 6, and 7 provide grounds for rejecting their null hypotheses. Model 8 also rejects the null hypothesis, which supports the contention that coefficient λ has a significant relationship with the hedge ratio.

Table 4: WALD Test

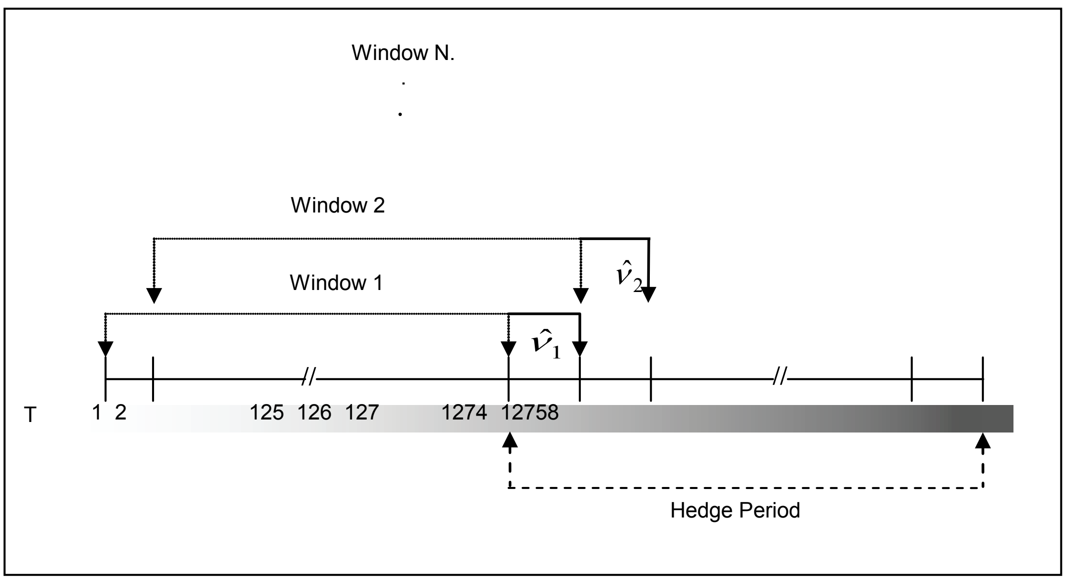
	Model 2 (γ_3=0)	Model 3 (γ_2=0)	Model 4 (γ_1=0)	Model 5 (γ_3=0) (γ_2=0)	Model 6 (γ_3=0) (γ_1=0)	Model 7 (γ_2=0) (γ_1=0)	Model 8 (λ=0)
Chi-square	0.6101 (0.4348)	4.3229** (0.0376)	6.4667** (0.0110)	4.7847* (0.0914)	7.0372** (0.0296)	6.5890** (0.0371)	11.8578*** (0.0006)

The equation in parenthesis exhibits hypothesis.
**** indicates significant at the 1%. ** indicates significant at the 5%. * indicates significant at the 10%.*

Results of Out-of–Sample Performance

To examine the performance of one-step-ahead hedge portfolios, the optimal hedge ratios (based on models 1 to 8) were estimated, beginning from the first period, proceeding to 125 days, and concluding with the end period day (as shown in Figure 3).

Figure 3: Out of Sample Hedge for Moving Window Estimation



The hedge portfolio consisted of a long position on the JGB 10-year price index and a short position on the JGB 10-year futures, which multiplied by the one-step-ahead optimal hedge ratio ($\nu_{t+1|t}$). Table 5 shows the standard deviation of returns of the simulated hedge portfolios when various restrictions were imposed.

A comparison of the various models shows that model 9 (OLS method) had a greater standard deviation (0.0084) than the other models. This result demonstrates that the yield-curve factors and persistence factor can provide additional explanatory power to decrease the standard deviation of the hedge portfolio.

Table 5: Out of Sample Result for Standard Deviation of Hedge Position

	Model 1 (no restricted)	Model 2 (γ_3=0)	Model 3 (γ_2=0)	Model 4 (γ_1=0)	Model 5 (γ_3=0) (γ_2=0)	Model 6 (γ_3=0) (γ_1=0)	Model 7 (γ_2=0) (γ_1=0)	Model 8 (λ=0)	Model 9 OLS
Standard Deviation of Hedge Position	0.0050	0.0051	0.0054	0.0054	0.0056	0.0058	0.0053	0.0052	0.0084

The estimation, used in model 1 through model 9 within 125 daily data, is set to forecast the hedge ratio on next day, so we can construct a hedge portfolio. With this procedure of estimation, we can get 1149 hedge portfolio returns and standard deviation between the 126th day and 1275th day.

Table 6 presents a methodology suggested by Ederington (1979) for testing hedge effectiveness, as follows:

$$h_e = 1 - \frac{\sigma^2_{hedge}}{\sigma^2_{unhedge}} \qquad (13)$$

where:

σ^2_{hedge} is the variance of the hedged portfolio; and

$\sigma^2_{unhedge}$ is the variance of the unhedged portfolio.

As the value of h_e trends to higher levels, the hedged portfolio becomes more effective.

As shown in Table 6, it is apparent that all models had impressive out-out-sample hedge effectiveness (up to 90%). This result demonstrates that the JGB futures market could provide an excellent environment for hedge investments. This is likely to be due to the high dependence between the spot and futures prices and the high liquidity of market trading.

Table 6: Out of Sample Result for Hedge Efficiency

	Restriction	Hedge Efficiency
Model 1	No restricted	97.1874%
Model 2	(γ_3=0)	97.0002%
Model 3	(γ_2=0)	96.6053%
Model 4	(γ_1=0)	96.6546%
Model 5	(γ_3=0) (γ_2=0)	96.3564%
Model 6	(γ_3=0) (γ_1=0)	96.0964%
Model 7	(γ_2=0) (γ_1=0)	96.7635%
Model 8	(λ=0)	96.8751%
Model 9	OLS	91.8141%
Model 10	v_t=1	95.8989%

The estimation, used in model 1 through model 9 within 125 daily data, is set to forecast the hedge ratio on next day, so we can construct a hedge portfolio. With this procedure of estimation, we can get 1149 hedge portfolio returns and standard deviation to calculate the hedge efficiency between the 126th day and 1275th day. The model 10 has taken the estimation with Perfect Hedge Method, or called Naive Hedge, which means the hedger can buy and sell the same amount of futures contract in contrast to his holding spot position.

It should be noted that the models that involved the yield-curve factors produced greater hedge effectiveness. This was especially apparent in models 1 and 2, which exhibited the greatest hedge effectiveness. This result shows that the models that include information about the level and slope factors provide significantly greater benefits. In addition, our empirical results are quite different with Fink et al. (2005), which indicate that both the level of interest rates and the slope of the yield curve are unimportant variables in determining the empirically optimal hedge ratio between mortgage-backed securities and Treasury futures contracts.

To summarise the results of out-of-sample testing, it has been demonstrated that the yield-curve information (such as level and slope factors) can improve the determination of the optimal hedge ratio and thus improve the effectiveness of the hedge. The persistence factor, λ, is also an important factor in determining the optimal hedge ratio.

CONCLUSION

The present study has incorporated factors from Nelson and Siegel (1987) with a Kalman filter approach to investigate hedge effectiveness between Japanese Government Bond (JGB) spot and futures. The study has demonstrated statistically significant effects from the persistent, level, and slope factors from an in-sample test. An analysis of out-of-sample predicted performance has demonstrated that the use of yield-curve information (such as persistence, level and slope factors) in determining the optimal hedge ratio can improve the effectiveness of the hedge. The findings also contribute to the literature by revealing that the term structure information need to be accounted for directly in the hedging of the government bonds with interest rates futures contracts.

Fink, et al. (2006) find that both the level of interest rates and the slope of the yield curve are unimportant variables in determining the empirically optimal hedge ratio between MBS and Treasury futures contract. On the contrary, this article concludes the yield-curve information, intuition suggests them to be relevant determinants, should play a significant role in the determination of the time-varying hedge ratio between Treasury bonds and Treasury futures. Since the chief source of basis risk comes from the prepayment of mortgages underlying the MBS, the basis risk in Treasury futures and its underlying asset is much lower than that of Treasury futures and MBS. Thus, it seems reasonable that our empirical findings could be generalised to other government bond markets. Furthermore, this paper compares the hedging effectiveness in hedging 10-year bonds with a Kalman filter approach. Also, our JGB futures contracts are based on Japanese government bonds with a term to maturity of 10 years. Thus, the improvement in hedge effectiveness based on yield-curve information will be limited when we choose the 7-year or the 5-year cash bonds as hedging objects.

More research should be done to assess the bond features: Is the type of issuer important when comparing the hedging effectiveness with a Kalman filter approach? What is the difference between the developed and developing bond market? How should the liquidity risk affect the hedging effectiveness？ How should the optimal hedge ratio be measured. Answers to these questions will contribute the literature in helping the computation of a more reliable hedge ratio between Treasury bonds and Treasury futures.

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