EVALUATING ALTERNATIVE WEIGHTING SCHEMES FOR STOCKS IN A "BEST IDEAS" PORTFOLIO

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ABSTRACT

As institutional investors have become more aggressive in deploying their capital, fund managers have become more creative with their product offerings. In this paper, we consider a new institutional fund of mutual funds, a portfolio that combines the "best-idea" stocks from two underlying primary funds. The sponsor of this portfolio has chosen to weight all of the stocks equally, even those chosen by both of the underlying funds' managers. However, stocks chosen by both managers may be more likely to outperform. We propose an alternative weighting scheme, where these "confirmed" stocks are weighted more heavily. We show that this overweighting strategy leads to a higher expected portfolio return than does the equally weighted scheme used by the sponsor.

JEL: G11, G23

KEYWORDS: Funds-of-funds, Mutual Funds

INTRODUCTION

As the interest rate environment has become more challenging and investment products have become more sophisticated, institutional portfolio managers have become increasingly aggressive in their search for superior returns. Many have turned to funds of funds (FOFs), since these embody the conventional institutional wisdom that active risk should be diversified to improve portfolio performance.

In this paper, we consider a new fund-of-funds product that combines stocks from two underlying mutual funds: an active fundamental fund (itself composed of several large-cap portfolios) and a quantitative fund (a large-cap enhanced index fund, again composed of several underlying portfolios). The new product is a "best ideas" portfolio, created using the twenty-five most overweighted and widely held stocks from each of the component funds. The idea is to skim the cream from each fund, combining the results to diversify active risk. The resulting portfolio is expected to generate a superior risk-return profile.

In this paper, we evaluate the proposed weighting scheme for this new fund-of-funds. As proposed, the institutional investor is free to choose the relative weighting for each of the underlying funds (e.g., 50%/50%; 60%/40%). However, the stocks in the funds will not be double-counted: any stock occurring in both the fundamental fund and the quantitative fund will be counted only once. Thus, if one stock is held in common, there will be only 49 different stocks, each of which (in the 50%/50% case, on which we will focus, and which is highlighted by the sponsor) will be weighted at $1/49^{th}$ of the portfolio.

Such a weighting scheme discounts the extra vote of confidence that a shared stock receives. If our underlying funds' managers are truly skilled, is it not more likely that a stock is "good" if they both choose it? Would we not be better of by weighting such "confirmed" stocks more heavily than those that received only one vote?

In this paper, we consider such an alternative "confirmation" weighting system, and compare it to the proposed equally weighted scheme. We find that, given simple assumptions, the proposed portfolio is likely to underperform our alternative. Stocks chosen by both managers are more likely to perform well, and should be overweighted in the overall portfolio.

The paper proceeds as follows. We first review the literature that bears on the weighting schemes for funds-of-funds. We then develop the model, first describing the weighting schemes for the proposed equally weighted portfolio and our alternative, then considering the relative probabilities that a stock is good, given that it is chosen by one or both managers. Using this framework, we then determine the returns we expect from the two weighting schemes. Finally, before concluding, we relax some of our basic assumptions, considering different combinations of the underlying funds and different relationships between the managers' choices.

RELEVANT LITERATURE

We are concerned with the proposed weighting scheme for a fund of actively managed mutual funds. Our work is therefore informed by literature on the value of active management and on the efficacy of creating funds of funds. More directly related, of course, would be studies evaluating the relative weightings of funds within larger portfolios. Given the dearth of research on this point, however, we will consider studies of index weights used for benchmarking hedge funds. (Many of the studies we discuss involve hedge funds, which are relevant here because the active—and expensive—strategy we are considering is designed for the same institutional investors who employ hedge funds.) In this section, then, we briefly consider three strands of research: the potential for excess returns from active management; the potential improvement of those excess returns from the creation of portfolios of actively managed funds; and the prevalence of various weighting schemes for measuring those excess returns using hedge fund benchmarks.

A lot of fees ride on the belief that active management can deliver superior performance, and, as a result, the industry literature is generally supportive of the idea. Waring and Siegel (2003) provide a good summary of the rationale for active management, explaining how, "[u]nder a couple of fairly easily satisfied conditions, you *can* beat the market": "As long as the market is not completely efficient (and we believe that none are) and as long as there are native differences in human intelligence and skill levels (of course there are), *some managers will outperform through real skill, not just by virtue of random variation*" (emphasis original). The sponsor of the fund-of-funds that we are considering similarly "fundamentally believe[s] in the value of active management," so that, "over time, our funds will deliver performance benefits" (Wainscott, 2005). Anson (2003) is also enthusiastic, noting that hedge fund indexes (which he describes as indexes of "almost pure bets on the skill of a specific manager") exhibit higher Sharpe ratios than do traditional asset classes. While—given the skewed nature of hedge fund returns—Sharpe ratios may not be the best performance assessment metrics for them (see Malkiel and Saha, 2005), Anson's work nonetheless demonstrates the profound institutional enthusiasm for active management.

The academic literature is less convinced of the efficacy of active management. For example, Malkiel and Saha (2005) adjust hedge fund index returns for various biases (such as backfill and survivor bias), and find that "after correcting for these biases, hedge fund returns appear to be lower than the returns from popular equity indexes and look very similar to mutual fund returns." On the other hand, Kosowski, Naik, and Teo (2007) use a bootstrap method to rank hedge funds by their (Bayesian posterior) alpha t-statistics, and find persistent, superior returns. There is, therefore, some academic support for the benefits of active management.

However, when considering the potential of our sponsor's strategy in particular, we must first clarify what, specifically, is "active" about its management. The sponsor of this FOF is selling its ability to choose successful underlying fund managers. When we develop our model, we will assume that it has been successful at this task.

It is important to recognize that funds of active funds involve two levels of active management: security selection at the underlying fund level, and fund selection at the portfolio level. In this paper, we are concerned only with the second of these—fund selection. Gehin and Vaissie (2004) emphasize that the value-added from funds-of-funds requires that portfolio managers exhibit "excellent *fund picking* ability" (emphasis added), and that these managers concentrate their efforts on this task. The sponsor of our fund concurs about its mission, defining its job as "find[ing] skilled money managers with great stock selection skills" (Frank Russell, 2005).

FOF managers—like our sponsor—advertise the active processes that are supposed to generate this success. For example, Waring and Siegel (2003) describe their "manager structure optimization" process, in which they advise portfolio managers to treat each of their underlying fund managers "like a stock": they should describe the expected excess return and active risk of each fund manager, characterize the correlations among the managers' performance, then utilize a manager "efficient frontier." Similarly, consultant Wilshire Associates has described its general "manager due diligence" as a five-step evaluation that draws on both external and proprietary sources to evaluate fund managers along 41 "key criteria" (Napoli, 2004; see also Foresti, 2005). When discussing their evaluation of fund-of-fund managers in particular, Wilshire emphasizes those managers' skills at picking and monitoring underlying fund managers. We stress this point because, if our sponsor is successful at its job, then it has chosen skillful managers. We should be able to take this skill as given when evaluating the weighting scheme the sponsor has chosen for the FOF.

While our analysis does take this skill as given, there are caveats. In order for our portfolio manager to be able to choose successful fund managers, there first must *be* successful fund managers, and then our sponsor must be able to identify them. Thus, both levels of active management must be successful. However, the literature suggests that, for at least three reasons, this success is neither guaranteed nor sufficient for a profitable investment.

First, we assume that our underlying fund managers are good stock pickers. While the professional literature touts active management, its great expectations to not extend to all asset classes. Our funds' stocks are chosen from the large-cap domestic equity universe, which is acknowledged to be a difficult space for active managers. Most of the professional literature advises that skilled managers are best deployed into markets generally viewed as less efficient. Indeed, Bonafede, Foresti, and Toth (2004) say that, in general, "[a]n experienced fund-of-funds manager adds value by identifying and gaining access to the top managers in an industry that is quite inefficient, while avoiding costly blowups." (For a similar industry argument, see Anson, 2003; Khandani and Lo, 2007, provide an academic argument.) However, the large-cap domestic equity space is "acknowledged as the most efficient asset class and is the most challenging for active managers" (Wainscott, 2005).

Second, we assume that our manager can identify these successful stock pickers. However, as Gehin and Vaissie (2004) note, "an overwhelming proportion of [fund of hedge fund] managers do not have any fund picking ability." Similarly, Malkiel and Saha (2005) find that, while "fund of funds managers will often claim that the manager can select the best hedge funds for inclusion in the portfolio," their results show that FOFs perform much worse than the average fund. "Clearly, the typical Fund of Funds is not able to select the best performing individual Hedge Funds."

Finally, even if our sponsor can identify superior stock pickers, that does not necessarily imply that the FOF will outperform. For example, Kosowski, Naik, and Teo (2007) do not even include FOFs in their performance tests, since "it is well known that Funds of Funds have lower average returns than individual hedge funds." This may have nothing to do with the efficacy of active management; instead, these authors suggest that the poor performance could be caused simply by the extra layer of fees charged by the FOF managers. Bonafede, Foresti, and Toth (2004) present a similar analysis.

While we acknowledge these caveats about the general strategy of the proposed FOF, we begin our analysis at a much later stage. Granting our sponsor the benefit of the doubt about its skill, its fund managers' skill, and its fee structure, we ask: Even if a FOF manager can choose the best-performing underlying funds, how should he weight those funds? Our sponsor has already selected two underlying funds, from which it will identify the most overweighted stocks. In the new portfolio, each of these chosen stocks will be equally weighted, including stocks chosen by both underlying managers. Our purpose is to consider whether a heavier weighting for these "confirmed" stocks could improve the FOF's performance.

There is some hedge fund literature that might shed light on this question. It is only obliquely helpful, however: we would prefer literature specific to mutual funds, but, in general, FOFs are not as common among mutual funds as they are among hedge funds (Fung and Hsieh, 2000). We would also prefer considerations of weighting within FOFs; however, the literature concentrates on weighting schemes used to create hedge fund benchmarks, not specific funds. Even these benchmark studies are difficult to apply to our situation, since hedge fund reporting is subject to numerous biases that do not apply to mutual funds (such as self-selection bias, backfill bias, and end-of-life reporting bias). However, we can use the hedge fund benchmark literature to demonstrate how common equal weighting is in the active, institutional fund world, rendering our sponsor's choice of this weighting strategy unsurprising (if uninspired).

Equal weighting is essentially the default position for hedge fund FOF indexes. For example, Anson (2003) notes that equally weighted hedge fund indexes are not overly sensitive to large funds or "flavor-of-the-year" funds. Thus, he notes that "[m]ost hedge fund index providers argue that a hedge fund index should be equally weighted to reflect fully all strategies." Of the ten benchmarks he studies (and the eleven studied by Gehin and Vaissie, 2004), seven are equally weighted (and one more is calculated using both equal and asset weights). Similarly, all of the averages of hedge fund performance studied by Malkiel and Saha (2005) are equally weighted.

When considering hedge fund benchmarks, however, there is a tension between describing current funds' performance and modeling the *investable* universe. As Fung and Hsieh (2000) note, the "observable" proxy for the hedge-fund market portfolio is equally weighted, since a market value-weighted index would require a complete record of hedge fund performance and asset data, which does not exist ("…assets under management are frequently incomplete or simply not available in hedge fund databases, so the equally weighted construct is the only proxy that can be calculated from individual hedge funds"). However, Gehin and Vaissie (2004) point out that matching an equally weighted index would require an unlikely contrarian strategy (selling winners to buy losers), and almost certainly cannot describe the true performance of an industry in which 75% of assets are concentrated in 25% of the funds. Nonetheless, Amo, Harasty, and Hillion (2007) use equal weighting when analyzing the terminal wealth generated from randomly selected hedge funds (and find that their nonparametric approach suggests that single hedge funds are much riskier than usually supposed).

The lesson from this benchmark literature is that equal weighting is a common institutional construct. Combined with conventional wisdom such as Waring and Siegel's (2003)—who note that traditional managers often hold "more or less equal weighted" portfolios—the weighting scheme chosen by our FOF sponsor is unsurprising. But does it offer the optimal combination of the stocks of the underlying funds? We begin to explore this question in the next section.

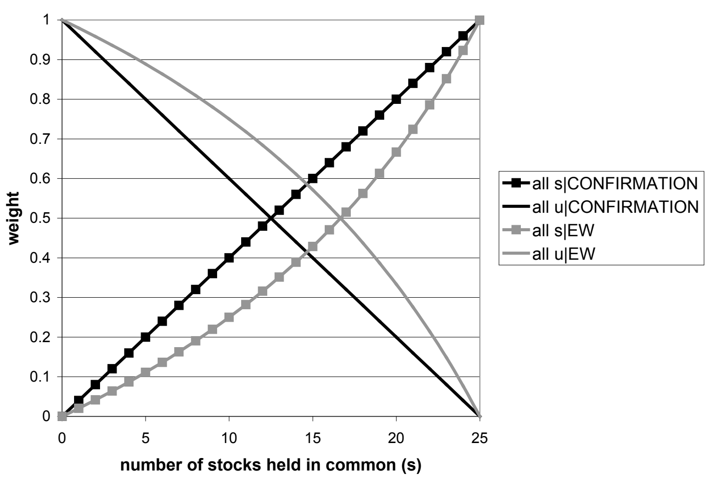
THE MODEL

Weighting Schemes

In this section, we clarify the distinctions between the new portfolio's proposed weighting scheme (the "equally weighted" portfolio) and our alternative (the "confirmation" portfolio). We will create portfolios of two underlying funds, fund 1 and fund 2. Assume that there are n_1 and n_2 stocks chosen from each fund, respectively, so that the maximum number of stocks held in each of our portfolios is $(n_1 + n_2) \equiv$ N. However, of these N stocks, s are chosen from both funds. (We will call these the "shared" stocks.) Thus, there are (N - s) different stocks chosen from both funds, but only (N – 2*s) $\equiv u$ "unique" stocks chosen from only one fund. For example, if $n_1 = n_2 = 25$, and one stock is chosen from both funds, then N = 50, s = 1, and u = 48. Twenty-four unique stocks are chosen from each underlying fund. The total number of different stocks held in our portfolio, (N - s), is 49.

We will create two portfolios of these funds, using two different weighting schemes. For the "confirmation" portfolio, we will weight each stock in each fund by [1/N]. This means that the s stocks held in both funds will be treated as separate assets, and therefore will be weighted twice, giving each a total portfolio weight of [2/N]. In the "equally weighted" portfolio, we will count each stock only once, even if it is held in both funds. We will therefore have (N-s) different stocks, each weighted by 1/(N-s). For $0 < s < N$, the equally weighted portfolio will overweight the unique stocks and underweight the shared stocks, relative to the confirmation portfolio. This is illustrated below in Figure 1.

Figure 1: Relative Weights of Unique and Shared Stocks in the Two Portfolios



In the figure, the smooth curves show the weights for the portfolios' unique stocks, while the boxed curves show the weights for the shared stocks. The equally weighted portfolio underweights the shared stocks and overweights the unique stocks, relative to the "confirmation" portfolio.

We want to determine the conditions under which the equally weighted portfolio will outperform the confirmation portfolio. To do this, we must first characterize the returns of the portfolios. For the confirmation portfolio, return can be written as:

$$R_{CONF} = \frac{1}{N}\left[\sum_{i=1}^{u} R_i + 2 * \sum_{k=1}^{s} R_k\right], \qquad (1)$$

where the R_i are the returns for the (N-2*s) unique stocks, and the R_k are the returns for the s shared stocks. For the equally weighted portfolio, return is:

$$R_{EW} \quad = \quad \frac{1}{(N-s)}\ [\sum_{i=1}^{u} R_i + \sum_{k=1}^{s} R_k\ ]. \tag{2}$$

We can use these characterizations to describe the situations in which the equally weighted portfolio will outperform the confirmation portfolio. This will happen when R_{EW} is greater than R_{CONF}, so that:

$$(R_{EW} - R_{CONF}) \quad > \quad 0, \tag{3}$$

which implies:

$$[\sum_{i=1}^{u} R_i\ ]*[\frac{1}{(N-s)} - \frac{1}{N}] + [\sum_{k=1}^{s} R_k\ ]*[\frac{1}{(N-s)} - \frac{2}{N}] \quad > \quad 0, \tag{4}$$

or:

$$[\sum_{i=1}^{u} R_i\ ]*[\frac{s}{(N-s)*N}] + [\sum_{k=1}^{s} R_k\ ]*[\frac{2*s-N}{(N-s)*N}] \quad > \quad 0. \tag{5}$$

Rearranging (5), we find that this simplifies to the straightforward requirement that

$$s*[\sum_{i=1}^{u} R_i\ ] \quad > \quad (\mathrm{N} - 2*s)*[\sum_{k=1}^{s} R_k\ ]. \tag{6}$$

Using our example values for s and N, this implies that the equally weighted portfolio will outperform the confirmation portfolio if:

(sum of returns on 48 unique stocks) > 48*(return on stock held by both funds)

Thus, if the average return for the 48 unique stocks is greater than the return for the one shared stock, we are better off with the equally weighted portfolio. But is it likely that the stock chosen by both fund managers is no better than the ones that were chosen only once? We consider this question in the next section.

Probabilities

We now consider a possible "confirmation effect" from having both managers choose the same stock. Let us assume that there are n_T different stocks in the relevant universe, from which both managers will pick. These stocks, S_i, are either "good" (G) or "bad" (B): $S_i \in (G, B)$; $i=(1\ldots n_T)$. (We will define "good" and "bad" more carefully in the next section.) There are n_G "good" stocks and $(n_T - n_G) \equiv n_B$ "bad" ones, so that the relative proportions of good and bad stocks are $(n_G/n_T) \equiv p_G$ and $(1- p_G) \equiv p_B$, respectively. The unconditional probability that a stock chosen at random will be good is therefore p_G, and the probability that a *specific* good stock, say stock Q, is chosen, is $prob(S_i{=}Q) = prob(S_i{=}Q \cap S_i{\in}G) = prob(S_i{=}Q|S_i{\in}G)*prob(S_i{\in}G) = (1/n_G)*p_G = 1/n_T$. This is the same as the unconditional probability that a specific bad stock will be chosen.

However, let us also assume that our managers are better stock pickers than average. The probability that one of them will choose a good stock is greater than p_G, say $(p_G + \varepsilon)$ (where $[1\text{-}p_G] > \varepsilon > 0$). Now, the probability that one of our managers will choose a specific good stock is $(p_G + \varepsilon)*(1/n_G)$, which is greater than the probability that a given bad stock is chosen, $(1\text{-}\ p_G - \varepsilon)*(1/n_B)$.

We will define $prob[ch(x, S_i)]$ as the probability that x of our managers (where $x \in [0, 1, 2]$) have chosen a specific stock, S_i. (For notational simplicity, we will abbreviate this to simply $ch(x)$.) Thus, $prob(ch(1)|S_i \in G) \approx 2/n_G$ (since a given stock could be chosen by either of our two managers), and $prob(S_i \in G|ch(1)) = (p_G+\varepsilon)$. Using this notation, we can determine the probability that a stock is good, given that it is chosen by both managers, as:

$$prob(S_i \in G|ch(2)) = prob[S_i \in G \cap ch(2)]/prob(ch(2))$$

$$= [prob(ch(2)|S_i \in G)*(p_G+\varepsilon)]/prob(ch(2)) \quad (7)$$

If we assume that the choices by the two managers are independent, then the probability that a given stock is chosen by both managers is simply $(1/n_G)^2$. Thus, we have:

$$prob(S_i \in G|ch(2)) = [(1/n_G)^2*(p_G+\varepsilon)]/prob(ch(2)) \quad (8)$$

Expanding the denominator, this becomes:

$$prob(S_i \in G|ch(2)) =$$

$$[(1/n_G)^2*(p_G+\varepsilon)]/[prob(ch(2)|S_i \in G)*(p_G+\varepsilon) + prob(ch(2)|S_i \in B)*(1-p_G-\varepsilon)]$$

$$= [(1/n_G)^2*(p_G+\varepsilon)]/[(1/n_G)^2*(p_G+\varepsilon) + (1/n_B)^2*(1-p_G-\varepsilon)] \quad (9)$$

Simplifying, we find:

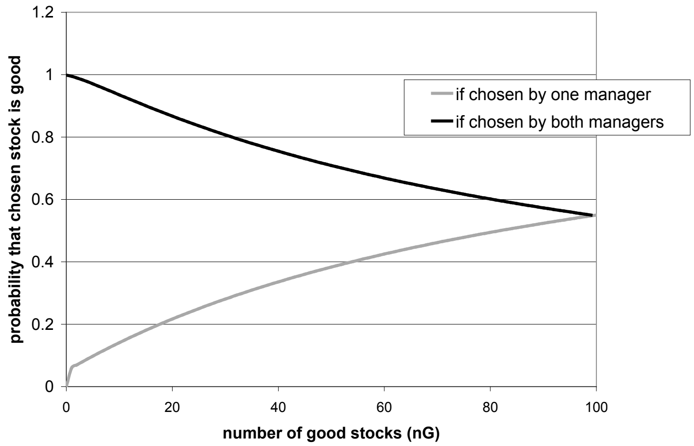
$$prob(S_i \in G|ch(2)) = \frac{(p_G+\varepsilon)*n_B^{\,2}}{(p_G+\varepsilon)*n_B^{\,2}+(1-p_G-\varepsilon)*n_G^{\,2}} \equiv p_{G2}. \quad (10)$$

To evaluate the relative value of the two portfolio weighting schemes, we need to determine whether this probability is greater than the probability that a stock is good, given that it is only chosen by one manager. That is, does having both managers choose a specific stock provide some additional confirmation that the stock is good?

Comparing p_{G2} to $(p_G+\varepsilon)$, we find that confirmation is indeed valuable: as long as $n_B > n_G$, $prob(S_i \in G|ch(2)) > prob(S_i \in G|ch(1))$. We can see this is Figure 2. In this figure, we have assumed that the number of bad stocks stays constant at 100, and have plotted the relative probabilities that a stock is good—given that it is chosen by one or both managers—against a changing number of good stocks. (Note that this implies that p_G rises as we move to the right across the graph, so that the unconditional probability that a stock is good is increasing—this is what is driving the bottom curve.) As long as $n_G < n_B$, the confirmation effect holds. This effect is particularly pronounced when the difference in the numbers of the two types of stocks is great (as we might expect it to be in the real world). If we assume instead that n_T is constant, so that n_B decreases as n_G increases, the shape of the relationship is different, but the effect is the same: confirmation has value as long as $n_G < n_B$.

Since the confirmation portfolio weighting scheme incorporates this additional vote of confidence by weighting shared stocks more heavily, we might expect it to perform better than the equally weighted portfolio. We will explore this prediction in the next section.

Figure 2: Relative Probabilities That a Chosen Stock is Good, Given That n_B is Fixed



In this figure, we assume that ε is .05 and that the number of bad stocks is fixed at 100. As the number of good stocks added to the universe increases, the probability that a chosen stock is good changes, rising if the stock is chosen by one manager, and falling if chosen by both.

Putting It All Together: Expected Excess Returns

Now that we have characterized both the two weighting schemes and the probabilities that a chosen stock is good, we can compare the expected returns from the two portfolio types. To simplify the discussion, we will evaluate expected *excess* returns to the portfolios, α. This is consistent with the stated goal of the strategy, which is "designed for alpha generation in domestic equity portfolios" (Junkin, 2007a).

α is a standard metric in portfolio analysis. Its original incarnation was Jensen's, who defined α as a portfolio's return above that predicted by the CAPM. The concept has been elaborated on and expanded since (for example, it has been expressed in continuous time, linked to a K-factor model, and recast in a Bayesian posterior form; see Bodie, Kane, and Marcus, 2008; Nielsen and Vassalou, 2004; Lo, 2007; and Kosowski, Naik, and Teo, 2007). For our purposes, we can abstract from the specific form of the underlying index model; however, our focus on α does require an assumption. Since we are considering adding a "satellite" to a diversified core portfolio, theory and practice assert that we should be comparing portfolios based on their information ratios, the ratios of alpha—active return—to active risk (see Bodie *et al.,* 1993; Kosowski *et al.,* 2007; and Bonafede *et al.,* 2004). We will therefore assume that the active risks in the confirmation and equally weighted portfolios are comparable, allowing us to concentrate only on return. (The actual risks of the two underlying funds is comparable: each has "12-month excess rolling risk" of approximately 2.8%; Junkin, 2007b. Of course, this does not imply that the two fund-of-fund portfolios we are considering will have the same risk. However, for example, the higher is the correlation between the two funds, the more likely is this result to hold.)

Since the average excess return in the market is zero, the unconditional expected excess return for a stock, $E(\alpha_i)$, in equilibrium, is zero. However, we can now define a "good" stock as one with a positive α_i for a given period, and a good fund manager—like ours—as someone who has above-average skill at identifying such stocks. Defining the average excess return for good stocks as $\overline{\alpha_G}$ and the average excess return for bad stocks as $\overline{\alpha_B}$, it must be that

$$0 = p_G * \overline{\alpha_G} + (1-p_G) * \overline{\alpha_B}, \quad (11)$$

so that:

$$\overline{\alpha_B} \quad = \quad -p_G * \overline{\alpha_G} / (1-p_G) \quad < \quad 0. \tag{12}$$

Now, we can define the expected return on a stock, given that it is chosen by one of our fund managers, as $(p_G+\varepsilon)*\overline{\alpha_G} + (1-p_G-\varepsilon)*\overline{\alpha_B}$; this will be the expected return for the u unique stocks. If the stock is chosen by both managers, its expected return is $p_{G2}*\overline{\alpha_G} + (1-p_{G2})*\overline{\alpha_B}$; this is the expected return for the s stocks held in common. Substituting these expected stock returns into equations (1) and (2), we find the following expressions for expected portfolio returns for the confirmation and equally weighted strategies:

$E(R_{CONF}) \quad =$

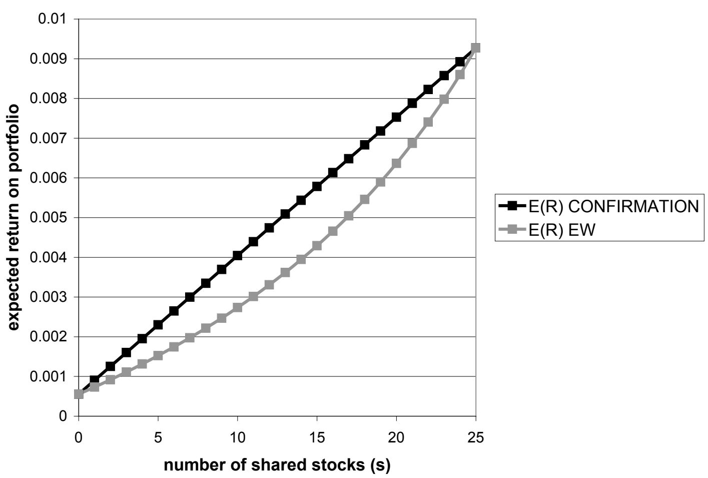
$$\frac{1}{N}\ \{\sum\nolimits_{i=1}^{u}[\overline{\alpha}_G*(p_G+\varepsilon)+\overline{\alpha}_B*(1-p_G-\varepsilon)]+2*\sum\nolimits_{k=1}^{s}[\overline{\alpha}_G*(p_{G2})+\overline{\alpha}_B*(1-p_{G2})]\}. \tag{13}$$

$E(R_{EW}) =$

$$\frac{1}{(N-s)}\ [\sum\nolimits_{i=1}^{u}[\overline{\alpha}_G*(p_G+\varepsilon)+\overline{\alpha}_B*(1-p_G-\varepsilon)]+\sum\nolimits_{k=1}^{s}[\overline{\alpha}_G*(p_{G2})+\overline{\alpha}_B*(1-p_{G2})]]. \tag{14}$$

In Figure 3 below, we graph these expected returns assuming $n_1 = n_2 = 25$, $p_G = .1$, $n_T = 100$, $\varepsilon = .05$, and $\alpha_G = .01$.

Figure 3: Expected Returns for the Two Portfolio Weighting Schemes



The figure shows that the expected returns for the confirmation portfolio exceed those for the equally weighted portfolio, unless the portfolios are either identical or completely distinct.

The expected return for the confirmation portfolio is higher unless the two portfolios are either identical or completely distinct. As long as there is only partial overlap, taking advantage of the confirmation effect—weighting stocks that get two votes more heavily, since they are more likely to be good—results in higher expected excess returns. The confirmation portfolio is preferable to the equally weighted one.

How sensitive is this result to the assumptions we have made? In the next section, we consider the effects on the portfolios' relative performance from several changes. First, we evaluate the macro weighting scheme of the strategy—the proportion of the allocation made to the underlying active fundamental and enhanced index funds. We have assumed a 50%/50% split, but the sponsor allows different allocations. We will determine whether different macro weights will change the portfolios' relative performance. Second, we briefly consider a hybrid approach, in which the portfolio manager is able to choose to weight more heavily only a subset of the shared stocks. Giving the portfolio manager the ability to select how many and which of the shared stocks to overweight adds another level of active management to the portfolio, which may result in superior performance. Third, we remove our assumption that fund managers' choices are independent. In fact, their strategies are likely to be correlated, changing our conclusion about p_{G2}. Fourth, we briefly consider the additional information that we may get when one of two managers does not choose a stock: $prob(S_i \in G|ch(1))$ really means that S_i was chosen by one manager, *but not by* the other. If the second manager's avoidance of the stock gives us material information, we should incorporate it into our expectation about the stock's excess return. Finally, we reevaluate our probabilities by explicitly recognizing that managers have multiple chances to match, since each chooses 25 times. We expect that managers who choose once are more likely to match on good stocks; is the same true when managers choose 25 times?

DISCUSSION AND ADDITIONAL CONSIDERATIONS

The Macro Weighting Scheme

We first consider the macro weighting scheme, the proportions that the portfolio manager allocates to the fundamental and quantitative funds, since the product allows participants to choose their relative exposures. So far, we have assumed a 50%/50% split, which is the mix highlighted by the sponsor, but we should determine whether our results would change under other weighting schemes. To begin, we will rewrite the return on the confirmation portfolio as:

$$R_{CONF} = \frac{w_1}{(n_1)}\left[\sum_{i=1}^{u1} R_i + \sum_{k=1}^{s} R_k\right] + \frac{w_2}{(n_2)}\left[\sum_{i=1}^{u2} R_i + \sum_{k=1}^{s} R_k\right], \tag{15}$$

where w_j and u_j are the weights in and the numbers of unique stocks held in each fund, j ($j \in (1,2)$). Since $n_1 = n_2$, we can rewrite this as:

$$R_{CONF} = \frac{1}{(n_1)}\left[w_1 * \sum_{i=1}^{u1} R_i + w_2 * \sum_{i=1}^{u2} R_i + \sum_{k=1}^{s} R_k\right]. \tag{16}$$

Similarly, we can write the return for the equally weighted portfolio as:

$$R_{EW} = w_1 * \left[\sum_{i=1}^{u1} \frac{R_i}{(N-s)}\right] + w_2 * \left[\sum_{i=1}^{u2} \frac{R_i}{(N-s)}\right] + (w_1 + w_2) * \left[\sum_{k=1}^{s} \frac{R_k}{(N-s)}\right] \tag{17}$$

In both cases, the contributions of shared stocks are not affected by different weighting schemes. Thus, although the returns of the portfolios will certainly be affected as we alter the mix of the component funds (as the unique stocks in the two funds perform differently), there will be no change attributable to the shared stocks. Since we are concerned only with the shared stocks' effect on the relative performance of the confirmation and equally weighted portfolios, we can ignore the weighting schemes assigned to the underlying funds.

Portfolio Manager's Selection of Subset of Shared Stocks to Overweight

The institutional portfolio we are considering uses equal weighting. We suggest that the "confirmation" weighting offers higher expected returns. For completeness, we now consider whether some mix of the two approaches might do even better: what if we allow the portfolio manager—the sponsor, who created this "best ideas" portfolio and chose the two underlying fund managers—to choose which of the shared stocks to overweight?

Looking at Figure 3, it seems implausible, all else equal, for such a strategy to dominate the confirmation portfolio, since we would essentially be averaging that portfolio's returns with the lower returns from the equally weighted portfolio. However, the question becomes more interesting if we allow the portfolio manager to have some skill at choosing from among the shared stocks. For example, let us assume that if the portfolio manager chooses to overweight one shared stock, the probability that the stock is good is (p_{G2} + δ)—higher than the unconditional probability for the shared stocks by the factor δ. However, also assume that the portfolio manager's stock-picking ability falls as she chooses to overweight more of the shared stocks, so that there is no additional confirmation if she chooses them all (since that would just imply that she had chosen the confirmation portfolio's weighting scheme). Let m denote the number of stocks the portfolio manager chooses to overweight, where $1 \le m \le s$. Letting the probability that her chosen stocks are good fall linearly (from [p_{G2} + δ] when m=1 to p_{G2} when m=s) implies that this probability (which we will call $p_{G\text{-}PM}$), is:

$$p_{G\text{-}PM} = p_{G2} + \left(\frac{\delta}{1-s}\right)*(m\text{-}s). \tag{18}$$

Now, we can consider the expected return for such a mixed portfolio. There will be three terms in this equation: one for the unique stocks (u), one for the overweighted shared stocks (m), and one for the rest of the shared stocks (s-m). For each of these three terms, we must consider both the appropriate weighting scheme and the expected return for a representative stock.

The portfolio weights will be determined by considering how many parts we will create by having the unique stocks and the (s-m) shared stocks weighted equally, while giving the m shared stocks double weight. Such a scheme divides the portfolio into (N – s +m) parts. Thus, the weight for the unique stocks and for the (s-m) shared stocks will be $\left(\frac{1}{N-s+m}\right)$; the weight for the overweighted shared stocks will be twice that. (For example, if N = 50, s = 5, and m = 2, so that the portfolio manager chooses to overweight two of the five shared stocks, the portfolio will be broken into $\left(\frac{1}{50-5+2}\right)$, or 47 parts. The [50 – 2*5] = 40 unique stocks will each receive a weight of (1/47), as will (5-2) = 3 of the shared stocks, while the 2 shared stocks chosen by the portfolio manager will each be weighted at (2/47).)

We now consider the expected return for a representative stock in each of the three groups. The expected return for the unique stocks will be the same as it was for the confirmation and equally weighted portfolios above, as shown in equations (13) and (14). For the shared stocks, however, we must use new probabilities that each stock is good or bad, since we now have more information: the portfolio manager's evaluation of each stock as either good (so that it is included among the m overweighted stocks) or bad (so that it is not). We have defined the probability that a shared stock is good, given that it is chosen by the portfolio manager, as $p_{G\text{-}PM}$; thus, the probability that a stock in this chosen group is bad is (1-$p_{G\text{-}PM}$). To determine the relative probabilities for the rest of the shared stocks that the portfolio manager did not choose, we consider the number of good stocks that we expect to have among this group. In the full set of 50 choices, we expect a total of p_{G2}*s good stocks. For a given number of m shared stocks chosen by the portfolio manager, we expect $p_{G\text{-}PM}$*m good stocks. The difference between these two expected values is the number of good stocks we expect to find in the shared/not-chosen group. Expressing this number as a proportion of the (s-m) stocks in this group gives us the probability that a shared stock is good, given that it is not chosen. (For example, when s=5, m=4, δ=.03, and p_{G2} = 0.9342, we have that the probability that a stock is good, given that the portfolio manager chose it, is $p_{G\text{-}PM}$ = 0.9421; the probability that a stock is bad, given that the portfolio manager chose it, is (1-$p_{G\text{-}PM}$) = .0579; the probability that a stock is good, given that it was not chosen by the portfolio manager, is 0.9046; and the probability that a stock is bad, given that it is not chosen by the portfolio manager, is 0.0658.) Calling this good/not-chosen probability p_{G*}, we have the following expression for the expected return of the mixed portfolio:

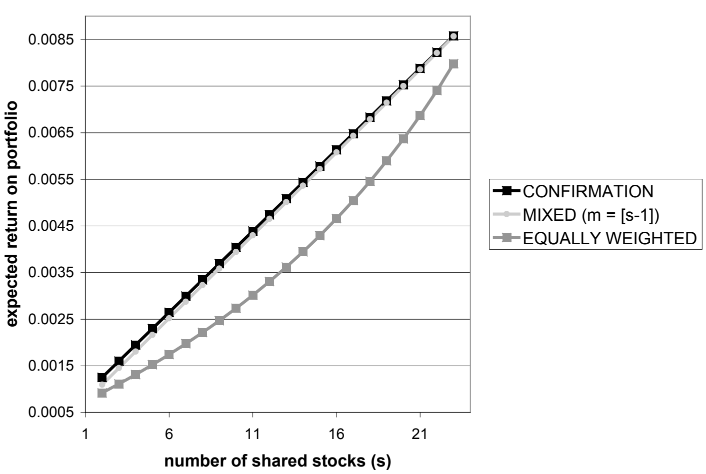
$$E(R_{MIXED}) =$$

$$\frac{1}{(N-s+m)}\{\sum_{i=1}^{u}[\overline{\alpha}_G*(p_G+\varepsilon)+\overline{\alpha}_B*(1-p_G-\varepsilon)]+2*\sum_{k=1}^{m}[\overline{\alpha}_G*(p_{G-PM})+\overline{\alpha}_B*(1-p_{G-PM})]$$

$$+\sum_{l=1}^{(s-m)}[\overline{\alpha}_G*(p_{G*})+\overline{\alpha}_B*(1-p_{G*})]])\}. \tag{19}$$

This expected return is increasing in δ, the measure of the portfolio manager's skill, and in m, the number of overweighted stocks. However, even at a maximum m of (s-1) and the highest possible δ (that at which $p_{G\text{-}PM} \approx 1$, which, for our example, is approximately 0.06), the mixed portfolio is outperformed by the confirmation portfolio. Figure 4 illustrates this relationship. As the number of shared stocks (and, in this case, the number of overweighted stocks) increases, the expected return of the mixed portfolio rises, approaching, but not exceeding, the expected return for the confirmation portfolio. Thus, allowing the portfolio manager to mix the two basic strategies does not result in better performance for the investors, even when the portfolio manager has some skill at choosing good stocks.

Figure 4: Comparison of the Expected Returns for the "Mixed" Weighting Scheme to the Two Schemes



Assuming a δ value of 0.03, the figure shows that the expected returns for the mixed portfolio—which allows the portfolio manager to choose a subset of the shared stocks to overweight—does not improve the overall expected return. Relative to the confirmation portfolio, the mixed portfolio puts more emphasis on the unique stocks and less on the shared stocks not chosen for overweighting. However, the overweighting that the mixed portfolio does use allows it to perform better than the equally weighted portfolio.

The expected return differences among the three portfolios are driven by both the weighting schemes and the expected returns of the shared stocks (whereas the earlier differences between the confirmation portfolio and the equally weighted were driven solely by the weighting schemes). Relative to both alternatives, the mixed portfolio puts less weight on the (*s*-*m*) shared stocks not chosen for overweighting, and more weight on the *m* chosen stocks. It also weights the unique stocks slightly more heavily than does the confirmation portfolio. These differences increase when *m* is smaller (since, as *m* approaches s, the mixed portfolio approaches the confirmation portfolio). The expected return for each unique stock is the same for all three portfolios. However, the expected return for each of the *m* chosen stocks in the mixed portfolio is higher than that used for the shared stocks in the other portfolios, and the expected return for the (*s*-*m*) unchosen shared stocks is lower. The net effect is that, relative to the confirmation portfolio (which dominates the equally weighted over the relevant range, and therefore provides the real comparison for the mixed strategy), the mixed portfolio overemphasizes the unique stocks and underemphasizes the shared stocks. The mixed strategy fails to take complete advantage of the confirmation effect—an effect that provides a much stronger signal of a stock's quality than does the marginal contribution of the portfolio manager's own skill.

We should emphasize here that even the benefit the mixed strategy appears to offer is, in all likelihood, illusory. That is because it is probably unrealistic to extrapolate the portfolio manager's skill at indentifying good managers to include the ability to pick specific stocks. We would not expect a portfolio manager to assert that he has skill at both levels, nor do we generally observe portfolio managers attempting to choose stocks. On the contrary, Fung and Hsieh's (2000) comment that "portfolio managers generally do not directly engage in trading" and Friedberg and Neill (2003) observation that '[o]nly a few fund-of-funds managers make direct investments" suggest a concentration of portfolio manger effort at the fund level. Thus, given that the portfolio manager is unlikely to be able to add value when she has skill, and given that–in the real world—we have no reason to suspect that this fund-of-funds portfolio manager even *has* any stock-picking skill, we expect that the mixed strategy is dominated by the confirmation approach.

Correlation of Managers' Choices

In the initial analysis, we assumed that the fund managers' choices were independent, so that $prob[ch(2)|S_i \in G) = prob(ch(1)|\ S_i \in G)^2$. However, what if their choices were correlated? Would that change our conclusion that stocks chosen by both managers are more likely to be good than stocks only chosen by one manager?

If the managers' choices were positively correlated, the probability that a given stock was chosen by both managers would be greater than $prob(ch(1)|\ S_i \in G)^2$. To make our analysis simple, let us assume that the new probability is higher than the old via some positive function of a variable η (for example, we could simply say that the new probability equals the old probability plus η). Interpreting equation (7) using this new assumption, we can take its derivative with respect to η and find that:

$$\delta prob(S_i \in G|ch(2))/\delta\eta =$$

$$[\delta prob(ch(2)|S_i \in G)/prob(ch(2)|S_i \in G)] - [\delta prob(ch(2)|S_i \in B)/prob(ch(2)|S_i \in B)] \quad (20)$$

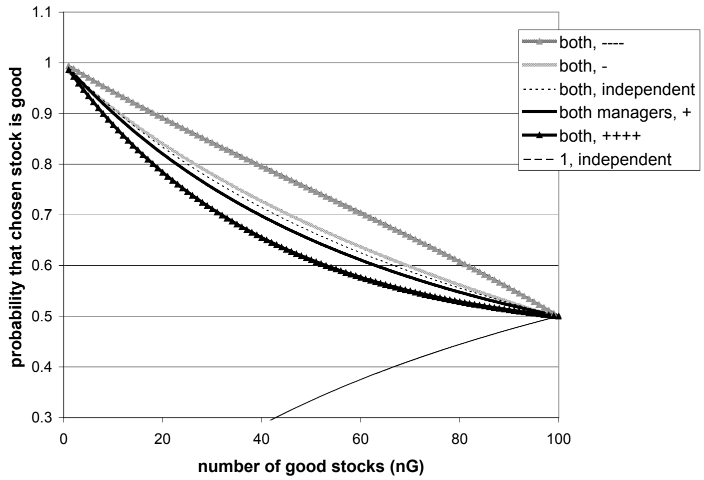
Using the simple form for the change in probability—adding η—the numerators in these terms are just 1, and the sign of this derivative depends only on the relative sizes of $prob(ch(2)|S_i \in G)$ and $prob(ch(2)|S_i \in B)$. Thus, since $prob(ch(2)|S_i \in G) > prob(ch(2)|S_i \in B)$, $\delta prob(S_i \in G|ch(2))/\delta\eta$ will be negative: higher correlation between managers' choices means we can derive less comfort from their common selections. Each choice by one manager gives us less independent information than before. (For example, imagine the case when the two managers always chose the same stocks. The probability that a given stock was good would then be simply (p_G+ε).) (We could assume that correlation causes one manager to be more likely to duplicate the other's bad choices rather than his good ones; this could reverse the sign of the inequality. However, such an assumption does not seem justified, especially given that unconditional probabilities favor duplication among good stocks. It is more likely that the same tendencies would lead to the same result when there are fewer choices—from the smaller pool of good stocks.) On the other hand, if we assume that the managers' choices are negatively correlated, we would have the opposite result: a duplication would make it more likely that the chosen stock was good.

Negative correlation between the funds' *returns* would be ideal from the portfolio manager's perspective on diversification grounds alone. However, the fund returns are vastly more likely to be positively correlated, as are the managers' stock *choices*, which are our concern. Our funds employ nominally distinct strategies. However, Khandani and Lo (2007) note that if funds use techniques based on common historical data, they will make similar bets, whether they are quantitative or active fundamental: "the same historical data… will point to the same empirical anomalies to be exploited… [M]any of these empirical regularities [used by quantitative funds] have been incorporated into non-quantitative equity investment processes, including fundamental 'bottom-up' valuation approaches." In our particular case, our two funds both choose stocks from the same domestic, large-cap universe. Both underlying funds are themselves composed from holdings of multiple managers, and are constructed by selecting the most widely held and overweighted stocks from those managers, relative to their benchmarks. By construction, then, the stocks used are shared at a sublevel. Looking at the actual stock selections, we can see that sector allocations of the two funds show similar concentrations (Junkin, 2007a). Finally, we have an estimate of the actual excess return correlation of our funds: according to the proposal, this correlation is between .2 and .3 (including back-tested data for the enhanced index fund, which is about 9 months younger than the fundamental fund; Junkin, 2007a). Although these returns could be correlated even if the stock selections are not, this evidence nonetheless suggests that a positive relationship between stock choices is more likely than a negative one. (If this were not the case, why would the sponsor explicitly account for duplications in the most basic description of the strategy?) Thus, we expect that the probability that a stock is good, given that is chosen by both managers, is not as high as we found when we assumed that their choices were independent.

We can see these effects below in Figure 5. In this figure, the curves labeled "independent" duplicate the values from Figure 2. To this baseline, we have added adjusted values, assuming both large and small, positive and negative values of η. The large, negative η (labeled "both, ----"), as expected, gives us the most favorable confirmation effect; the small negative η also improves over the independent value, although not nearly as significantly. On the other hand, positive correlation makes confirmation less valuable, and this effect is magnified for the larger correlation ("both, ++++"). However, the important point is this: even if managers' choices are positively correlated, as ours probably are, it is still more likely that a given stock is good if both managers choose it. The confirmation effect, while muted with positive correlation, does not disappear.

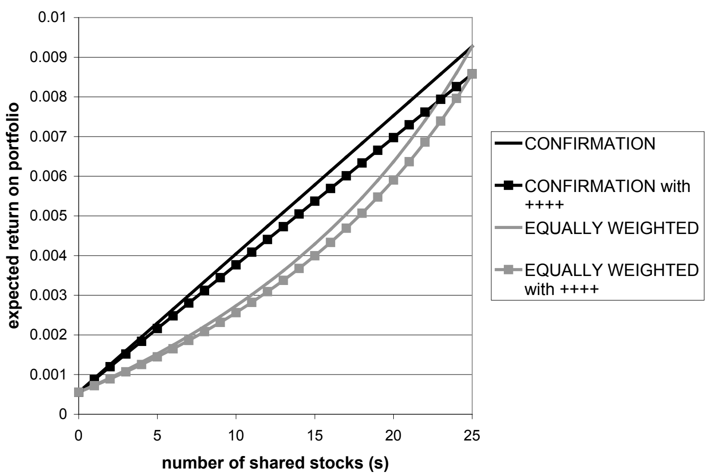
We can see this by looking at what is really important—expected returns. To our earlier portfolio comparison (from Figure 3), Figure 6 adds the expected confirmation and equally weighted portfolio returns adjusted for the higher, positive η (the worst-case scenario we have used). Expected returns for both portfolios fall, as we would expect. But the relative story has not changed: the confirmation portfolio still outperforms the equally weighted. There is still information to be gained from confirmation.

Figure 5: How Probabilities Vary with Correlations between Managers' Choices



This figure adds four curves to those from Figure 2, illustrating the effects of correlation between the managers' choices. When the managers' choices are negatively correlated, the confirmation effect is magnified, as shown in the two uppermost grey curves ("both, ----" and "both, -"). When their choices are positively correlated, the confirmation effect is muted: the two "+" curves lie below the initial "both, independent" curve from Figure 2. However, even when choices are correlated, a stock is more likely to be good if chosen by both managers. (The figure assumes η values were $\pm.00001$ and $\pm.00004$. These were chosen to be comparable to, but smaller in magnitude than, $1/(n_B)^2$, which was .0001.)

Figure 6: Effect of Positive Correlation on Expected Returns



When managers' choices are positively correlated, expected returns for both portfolio weighting schemes fall. However, the full portfolio still improves over the adjusted portfolio.

Unique Stocks Are Not Chosen by the Second Manager

Figure 6 shows that, given our assumptions, we would be better off with the confirmation portfolio even when managers' choices are positively correlated. Stocks chosen by both managers have received two "good" signals, and are therefore more likely to be good. However, we have not considered fully the information that we receive when a stock is chosen by only one of our two managers. Given that we have two possible signals, receiving only one "good" signal out of two is not the same as having a single manager identify a stock as good. That is, when we have two signals, but only one is good, we actually have the event (chosen$\cap$not chosen): the second manager has *not* chosen the stock. Does this give us useful additional information?

Returning to our basic assumption of independence, we have that *prob*(S_i chosen by #1$\cap S_i$ not chosen by #2) = *prob*(S_i chosen by #1)**prob*(S_i not chosen by #2). We have already characterized the first of these probabilities. The second is $[(n_G - 1)/n_G]*(p_G+\varepsilon) + [(n_B - 1)/n_B]*(1-p_G-\varepsilon)$. Using the same numbers as in Figure 3, we have an example of the magnitude of this probability: (9/10)*(.15) + (89/90)*(.85) = .98. Thus, incorporating the second manager's avoidance of the stock should not make a meaningful difference in our qualitative results.

Managers Make Multiple Choices

Having considered possible interactions among managers' choices, we now turn to their *number* of choices. The derivations and figures discussed so far assume that the managers choose a stock once. This assumption simplifies calculation and results in expressions that are easily interpretable. However, each manager actually makes 25 choices, so that there are multiple opportunities for the two managers to choose the same stocks. Does having multiple opportunities to "match" change our conclusions, making the adjusted portfolio preferable?

Thoroughly characterizing the probabilities involved here is complex, but we can easily explore this question with some simplifying assumptions. Let us describe the number of matches as a binomial variable, where a match is a success. (Note that, in reality, the stocks are chosen without replacement, so that the probability of a match varies as choices are made. However, since there are fewer good stocks than bad, properly accounting for this fact would simply strengthen our conclusion.) Given that managers #1 and #2 have chosen a certain number of good stocks, (say n_{1G} and n_{2G}, respectively), the probability that manager #2 will choose one of the same good stocks as #1 is $\left(\frac{n_{2G}!}{(n_{2G}-1)!}\right)*\left(\frac{n_{1G}}{n_G}\right)*\left(\frac{n_G - n_{1G}}{n_G}\right)^{24}$.

The probability that they will match on a bad stock in similarly defined. We can use these probabilities to examine whether matches are more likely to be made on good stocks (so that we would prefer to double-count matches, as does the confirmation portfolio) or on bad stocks (so that we would prefer the equal weighting scheme).

As an example, assume that there are 1000 bad stocks (n_B) and 100 good stocks (n_G). The unconditional probability that someone would choose a good stock is therefore approximately 9%. Each of our managers chooses 25 stocks ($n_1 = n_2 = 25$). Table 1 shows the relative probabilities of a single match on a good or bad stock, given varying numbers of good stocks chosen by the two managers. That is, for each combination of n_{1G} and n_{2G}, the values in the table show the probability of making a match on either a good or bad stock. So, for example, assume that manager #1 chooses three good stocks and manager #2 chooses two. If manager #2 is choosing a bad stock, he has a (22/1000) chance of matching one of manager #1's, and a (978/1000) of not matching. The comparable probabilities for good stocks are (3/100) and (97/100). Given these probabilities, and the fact that manager #2 will choose two good stocks and 23 bad ones, Table 1 shows that the probability of a match on a bad stock is .310, while the probability of a match on a good stock is only .058. Thus, if (n_{1G}, n_{2G}) = (3, 2) is a likely outcome, we would prefer the equally weighted weighting scheme.

The (n_{1G}, n_{2G}) combinations for which a good match is more likely are boldfaced and highlighted in Table 1. In this area—more than half of the table, stretching up and left from the lower right-hand corner—a match is more likely to be good than bad. Thus, if we expect that our managers' choices will put us in this area, we would be better off with the confirmation-portfolio weighting scheme. We can assess the likelihood that we will be in this area by using the numbers above the main body of the table. These give us the probability that a manager will choose the specified number of good stocks, using a range of values for their skill, ($p_G+\varepsilon$).

Table 1: Relative Probabilities That a Match is a Good Stock or a Bad Stock

p_G+e =																		
0.1	0.1994	0.2659	0.2265	0.1384	0.0646	0.0239	0.0072	0.0018	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
cumulative	0.2712	**0.5371**	0.7636	0.9020	0.9666	0.9905	0.9977	0.9995	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.3	0.0014	0.0074	0.0243	0.0572	0.1030	0.1472	0.1712	0.1651	0.1336	0.0916	0.0536	0.0268	0.0115	0.0042	0.0013	0.0004	0.0001	0.0000
cumulative	0.0016	0.0090	0.0332	0.0905	0.1935	0.3407	**0.5118**	0.6769	0.8106	0.9022	0.9558	0.9825	0.9940	0.9982	0.9995	0.9999	1.0000	1.0000
0.5	0.0000	0.0000	0.0001	0.0004	0.0016	0.0053	0.0143	0.0322	0.0609	0.0974	0.1328	0.1550	0.1550	0.1328	0.0974	0.0609	0.0322	0.0143
cumulative	0.0000	0.0000	0.0001	0.0005	0.0020	0.0073	0.0216	0.0539	0.1148	0.2122	0.3450	**0.5000**	0.6550	0.7878	0.8852	0.9461	0.9784	0.9927
0.7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0004	0.0013	0.0042	0.0115	0.0268	0.0536	0.0916	0.1336	0.1651	0.1712
cumulative	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0005	0.0018	0.0060	0.0175	0.0442	0.0978	0.1894	0.3231	**0.4882**	0.6593

# good chosen by #1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
# good chosen by #2																		
1																		
bad	0.329	0.323	0.317	0.309	0.302	0.293	0.284	0.275	0.265	0.254	0.243	0.231	0.218	0.205	0.190	0.175	0.160	0.143
good	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080	0.090	0.100	0.110	0.120	0.130	0.140	0.150	0.160	**0.170**	**0.180**
2																		
bad	0.323	0.317	0.310	0.303	0.295	0.287	0.278	0.268	0.258	0.247	0.236	0.224	0.212	0.198	0.184	0.170	0.154	0.138
good	0.020	0.039	0.058	0.077	0.095	0.113	0.130	0.147	0.164	0.180	0.196	0.211	**0.226**	**0.241**	**0.255**	**0.269**	**0.282**	**0.295**
3																		
bad	0.317	0.310	0.303	0.296	0.288	0.279	0.270	0.261	0.251	0.240	0.229	0.217	0.205	0.192	0.178	0.164	0.149	0.133
good	0.029	0.058	0.085	0.111	0.135	0.159	0.182	0.203	0.224	**0.243**	**0.261**	**0.279**	**0.295**	**0.311**	**0.325**	**0.339**	**0.351**	**0.363**
4																		
bad	0.309	0.303	0.296	0.288	0.280	0.272	0.263	0.253	0.243	0.233	0.222	0.210	0.198	0.185	0.172	0.158	0.143	0.128
good	0.040	0.075	0.110	0.142	0.171	0.199	0.225	0.249	**0.271**	**0.292**	**0.310**	**0.327**	**0.342**	**0.356**	**0.368**	**0.379**	**0.389**	**0.397**
5																		
bad	0.303	0.296	0.288	0.281	0.272	0.264	0.255	0.245	0.236	0.225	0.214	0.203	0.191	0.178	0.165	0.152	0.125	0.116
good	0.048	0.092	0.133	0.170	0.204	0.234	**0.262**	**0.287**	**0.309**	**0.328**	**0.345**	**0.360**	**0.372**	**0.383**	**0.392**	**0.398**	**0.319**	**0.301**
6																		
bad	0.294	0.287	0.280	0.272	0.264	0.256	0.247	0.237	0.227	0.217	0.206	0.195	0.183	0.171	0.159	0.145	0.132	0.117
good	0.057	0.108	0.155	0.196	0.232	**0.264**	**0.292**	**0.316**	**0.337**	**0.354**	**0.369**	**0.380**	**0.389**	**0.395**	**0.399**	**0.401**	**0.402**	**0.400**
7																		
bad	0.286	0.279	0.271	0.264	0.255	0.247	0.238	0.229	0.219	0.209	0.198	0.187	0.176	0.164	0.152	0.139	0.126	0.112
good	0.066	0.124	0.175	0.219	**0.257**	**0.290**	**0.317**	**0.340**	**0.358**	**0.372**	**0.383**	**0.390**	**0.395**	**0.396**	**0.396**	**0.393**	**0.389**	**0.383**
8																		
bad	0.277	0.269	0.262	0.254	0.246	0.238	0.229	0.220	0.210	0.200	0.190	0.179	0.168	0.157	0.145	0.132	0.120	0.106
good	0.075	0.139	0.194	0.240	**0.279**	**0.311**	**0.337**	**0.357**	**0.372**	**0.383**	**0.389**	**0.392**	**0.392**	**0.390**	**0.385**	**0.378**	**0.369**	**0.359**
9																		
bad	0.267	0.260	0.252	0.244	0.236	0.228	0.219	0.210	0.201	0.191	0.181	0.171	0.160	0.149	0.138	0.126	0.113	0.101
good	0.083	0.153	0.212	**0.260**	**0.299**	**0.329**	**0.353**	**0.372**	**0.381**	**0.387**	**0.390**	**0.388**	**0.384**	**0.377**	**0.368**	**0.357**	**0.345**	**0.331**
10																		
bad	0.256	0.249	0.242	0.234	0.226	0.218	0.209	0.201	0.191	0.182	0.172	0.162	0.152	0.141	0.130	0.119	0.107	0.095
good	0.091	0.167	0.228	**0.277**	**0.315**	**0.344**	**0.364**	**0.378**	**0.385**	**0.387**	**0.385**	**0.380**	**0.371**	**0.360**	**0.347**	**0.333**	**0.318**	**0.302**
11																		
bad	0.245	0.238	0.231	0.223	0.215	0.207	0.199	0.190	0.182	0.173	0.163	0.154	0.144	0.133	0.123	0.112	0.101	0.089
good	0.099	0.180	**0.243**	**0.293**	**0.329**	**0.355**	**0.373**	**0.382**	**0.386**	**0.384**	**0.377**	**0.368**	**0.355**	**0.341**	**0.325**	**0.308**	**0.290**	**0.272**
12																		
bad	0.233	0.226	0.219	0.212	0.204	0.196	0.188	0.180	0.171	0.163	0.154	0.144	0.135	0.125	0.115	0.105	0.094	0.084
good	0.107	0.192	**0.258**	**0.306**	**0.341**	**0.365**	**0.378**	**0.384**	**0.383**	**0.377**	**0.366**	**0.353**	**0.337**	**0.320**	**0.301**	**0.282**	**0.263**	**0.243**
13																		
bad	0.220	0.214	0.207	0.200	0.192	0.185	0.177	0.169	0.161	0.152	0.144	0.135	0.126	0.117	0.107	0.098	0.088	0.078
good		0.204	**0.271**	**0.319**	**0.351**	**0.371**	**0.381**	**0.382**	**0.377**	**0.367**	**0.353**	**0.336**	**0.318**	**0.298**	**0.277**	**0.257**	**0.236**	**0.216**
14																		
bad	0.207	0.200	0.194	0.187	0.180	0.173	0.165	0.158	0.150	0.142	0.134	0.125	0.117	0.108	0.099	0.090	0.081	0.072
good	0.123	**0.215**	**0.283**	**0.329**	**0.359**	**0.376**	**0.382**	**0.379**	**0.370**	**0.356**	**0.339**	**0.319**	**0.298**	**0.276**	**0.254**	**0.232**	**0.211**	**0.191**
15																		
bad	0.193	0.187	0.180	0.173	0.167	0.160	0.153	0.146	0.138	0.131	0.123	0.116	0.108	0.100	0.091	0.083	0.074	0.066
good	0.130	**0.226**	**0.294**	**0.339**	**0.366**	**0.378**	**0.380**	**0.373**	**0.361**	**0.343**	**0.323**	**0.301**	**0.278**	**0.254**	**0.231**	**0.209**	**0.188**	**0.168**
16																		
bad	0.178	0.172	0.166	0.159	0.153	0.147	0.140	0.133	0.127	0.120	0.113	0.105	0.098	0.091	0.083	0.075	0.068	0.060
good	0.138	**0.236**	**0.304**	**0.347**	**0.371**	**0.379**	**0.377**	**0.366**	**0.350**	**0.329**	**0.306**	**0.282**	**0.258**	**0.233**	**0.210**	**0.187**	**0.166**	**0.147**
17																		
bad	0.162	0.156	0.151	0.145	0.139	0.133	0.127	0.121	0.114	0.108	0.101	0.095	0.088	0.081	0.075	0.068	0.061	0.053
good	0.145	**0.246**	**0.313**	**0.354**	**0.374**	**0.379**	**0.373**	**0.358**	**0.338**	**0.315**	**0.290**	**0.264**	**0.238**	**0.213**	**0.189**	**0.167**	**0.147**	**0.128**
18																		
bad	0.145	0.140	0.135	0.129	0.124	0.119	0.113	0.107	0.102	0.096	0.090	0.084	0.078	0.072	0.066	0.060	0.053	0.047
good	**0.152**	**0.255**	**0.322**	**0.360**	**0.376**	**0.377**	**0.367**	**0.349**	**0.326**	**0.300**	**0.273**	**0.246**	**0.219**	**0.194**	**0.170**	**0.149**	**0.129**	**0.111**

The values in the body of the table show the probabilities of making a match on a good stock (lower half of cell) or on a bad stock (upper half of cell) for various combinations of good choices by managers 1 and 2. Cells with shading identify combinations of numbers of good choices by our managers that make the confirmation portfolio weighting scheme preferable to the equally weighted scheme.

They also show the cumulative probabilities, highlighting the middle of the distribution. For example, if we expect that our managers are no better than average, we can assume that (p_G+ε) = .1. In this case, the most likely number of good stocks chosen is 2, and 95% of the time, (n_{1G}, n_{2G}) will fall in the "bad" northwest corner of the table. Matches are more likely to be on bad stocks than on good, and we would be better off with the equally weighted portfolio.

But why pursue active management at all if we assume our managers have no skill? Instead, let us consider what happens if our managers choose good stocks half the time. In this case, the expected number of good stocks for each manager is 12, and we are almost certain to fall in the "good" part of the table. If our managers are truly skilled, we expect them to choose good stocks with high probability, so that any matches are more likely to be on good stocks than on bad. Thus, if we trust our managers, we would prefer to double-count their confirmed choices in our portfolio: we would prefer the confirmation weighting scheme.

CONCLUSIONS

Successful active management is hard, especially in the large-cap domestic equity space. For example, in a Wilshire Associates study of the relative performance of the S&P500 against active large-cap core managers, the authors find that most active managers underperformed the index during the entire second half of the 1990s (Foresti and Toth, 2006). However, in a low interest rate environment, the search for yield leads many pension fund managers toward active strategies and their promise of alpha. As demand increases, fund managers design new products in response. In this paper, we consider one of these new products: an institutional fund of mutual funds combining the most heavily weighted stocks from an active fundamental fund and from an enhanced index fund. After the sponsor identifies each of his two fund managers' 25 "best ideas" stocks, he will put them together in an equally weighted portfolio. Any stocks held in both underlying funds will be counted only once. Our goal was to assess the proposed equally weighted scheme against an alternative "confirmation" weighting scheme, in which stocks among the best ideas of both managers are more heavily weighted in the fund of funds.

The fund of funds we are considering involves two levels of active management: the stock-picking ability of the two fund managers, and the manager-picking skill of the portfolio manager. We enter the process at the end. The fund managers have chosen their stocks, and our portfolio manager has identified them, from the universe of managers working in the relevant parts of the large-cap space, as the two he believes are most highly skilled. If he is right—if he really has the skill to choose managers, as he asserts he does with his very job description—why treat stocks with two votes of confidence like all of the others? Two votes may make it more likely that a stock is good. If so, the portfolio's expected return will be higher if this stock is more heavily weighted.

Our model assumes that the skill of the fund managers is real. This implies a confirmation effect for stocks chosen by both, so that our proposed weighting scheme performs better than the sponsor's equally weighted scheme. This superior performance is robust to changes in many of our assumptions. For example, the result holds regardless of the relative macro weights assigned to the underlying funds. If the fund manager's choices are negatively correlated—which is unlikely—the confirmation effect is strengthened; on the other hand, when the correlation is positive, the effect is mitigated, but not eliminated. Similarly, when we account for the fact that one manager did *not* choose stocks unique to the other manager, or for managers' multiple opportunities to choose the same stocks, the conclusion remains: we can increase the portfolio's expected return by overweighting confirmed stocks.

Moreover, allowing the portfolio manager to add a third level of active management, by choosing which of the shared stocks to overweight, does not add value to the process. Even when the portfolio manager is himself a skilled stock picker (which we would not expect, since these managers are hired to choose managers, not stocks), the gain in expected return from his additional level of confirmation is outweighed by the loss in expected return from the lower weighting for the shared stocks he does not choose. Again, the confirmation weighting scheme is the dominant performer.

The critical assumption underlying this result is that the underlying fund managers are skilled stock pickers. This is the assertion that is made by the portfolio manager, too, as he pitches his new fund-of-funds product to institutional buyers. In future research, it would be interesting to attempt to quantify the size and persistence of ε, our measure of the fund managers' stock-picking skill. If ε is "too small," then we would not expect our proposed weighting scheme to add enough value to justify any extra implementation costs. Of course, if ε is too small, the whole rationale for the fund of funds falls apart in any case.

Thus, unless we expect our managers to have no skill in choosing stocks—in which case, why would we pay the higher fees for them?—we should capitalize on the stronger signal we get when both identify the same stock as desirable. Portfolio returns should be higher when we recognize that two heads really are better than one.

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