AN EMPIRICAL STUDY OF VOLATILITY AND TRADING VOLUME DYNAMICS USING HIGH-FREQUENCY DATA

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ABSTRACT

This paper examines the dynamic relationship of volatility and trading volume using a bivariate vector autoregressive methodology. This study found bidirectional causal relations between trading volume and volatility, which is in accordance with sequential information arrival hypothesis that suggests lagged values of trading volume provide the predictability component of current volatility. Findings also reveal that trading volume shocks significantly contribute to the variability of volatility and then volatility shocks partly account for the variability of trading volume.

JEL: C01, G0, O16, O30

KEYWORDS: Trading volume; Volatility; Sequential information arrival hypothesis; Mixture of distribution hypothesis

INTRODUCTION

Four relevant information theories in the literature relate volume and volatility, namely, the mixture of distributions hypothesis (MDH), the sequential arrival of information hypothesis (SAIH), the dispersion of beliefs hypothesis, and the noise trader hypothesis. Information mainly determines the theories of volume and volatility. For example, according to the MDH, information dissemination is contemporaneous. Stock prices and trading volume change only when information arrives and evolve at a constant speed in event time. MDH suggests that daily price changes and trading volume are driven by the same underlying information flow. MDH implies only a contemporaneous relationship between volume and returns. The SAIH argues that each trader observes information sequentially. Hence, different types of traders will receive information sequentially. The econometric results show that past trading volume provides information on current volatility or absolute returns. Several studies find that a positive correlation exists between volume and volatility, including Lee and Rui (2002), Andersen (1996), Manganelli (2005), Xu et al. (2006) and Kim (2005). This investigation studies the dynamic relation between return volatility and trading volume on the Taiwan stock market.

This study differs, as follows, from other studies on the volume-volatility relationship. First, in this paper the measure of volatility is calculated by the sum of intraday 1-min returns. Minute-by-minute transaction data are used. The economic rationale is as follows: Martens (2002) shows the sum of squared intraday and intranight returns are better than using the daily return to measure stock market volatility. Andersen et al. (1999) and Martens (2001) show that intraday returns can improve not only measuring of volatility, but also the forecasting of volatility. The removal of microstructure bias makes the results in this paper more reliable. Second, in addition to using a vector-autoregressive (VAR) model to answer the question about the relationship of trading volume-volatility, The VAR model can consider the endogeneity of volume-volatility relations and capture the impact of volume (volatility) shock on volatility (volume) in the Taiwan stock market. The proposed model also provides the dynamic intraday-volume relation. Third, this paper reveals a volume-volatility relationship from the Taiwan stock market, whereas most empirical studies come from developed countries. Therefore, results from this study can complement and contrast with previous studies to assess whether the volatility-volume relationship is robust in different markets.

Empirical results show a significant relationship between the past trading volume and return volatility and current trading volume or volatility. The causality tests show a clear bidirectional relationship between trading volume and return volatility. Our results support the SAIH. The findings presented in this study demonstrate that the shock to trading volume has a significant effect on volatility. The contribution of trading volume shock to the variability of volatility accounts for 40%. Only about 8% of changes in volatility can be attributed to the shock in trading volume. The impulse response function shows that one standard deviation increase in trading volume is followed by an increase in volatility. As regards the effect on trading volume, there is a downward effect of a shock to volatility. Trading volume responds much more sluggishly.

The remainder of this paper is organized as follows. Section 2 briefly reviews prior literature. Section 3 discusses the empirical model and estimation methodology. Section 4 describes the data. Section 5 provides the main results including results from the analysis of regression parameters, the Granger causality test, variance decomposition (VDC) and the impulse response function (IRF). Section 6 concludes the study.

LITERATURE REVIEW

Much literature exists on volatility-volume relationships in the stock market microstructure. Since the original work by Clark (1973), Epps and Epps (1976), and Harris (1986), a number of empirical papers have examined different aspects of the linkage between trading volume and volatility. Grammatikos and Saunders (1986), in early studies, found that price variability and trading volume are positively correlated in futures markets. More recently, Wang and Yau (2000) report evidence of a positive relation between trading volume and price volatility in futures markets. For a VAR Framwork, Garcia, Leuthold and Zapata (1986) document a lead-lag relationship between trading volume and volatility. Luu and Martens (2003) use US stock index futures market data and find a bi-directional causal relationship between volatility and trading volume. Xu et al. (2006) and Manganelli (2005) also find a strong contemporaneous and dynamic relationship between volume and volatility. However, some dissimilar results also appear in previous literature. For example, Pilar and Rafael (2002) argue that a decrease in volatility and increases trading volume. Watanabe (2001) suggests there is no relationship between price volatility and volume. The dynamic relationship between trading volume and volatility is unclear that depend on the market and time period we studied

Some studies consider the various types of trader volume and volatility. For example, Daigler and Wiley (1999) employ type of trader volume to study contemporaneous volume-volatility relationships. They primarily focus on the theory of sequence of information arrival and how different types of traders interpret and react to information. Chen and Daigler (2008) provide an integrated picture of the volume and volatility relationship by investigating the dynamic linear and nonlinear associations between volatility and the volume of informed and uninformed traders. The results of Chen and Daigler (2008) shows a one-way Granger causation from volatility to volume. Informed traders react less to lagged information than do uninformed traders for the sequential arrival of information framework, and public's trading volume creates excess volatility. Chen (2007) uses the data of four futures markets to investigate the effect of trader types on the intraday volatility-volume relationship. Chen's (2007) results from a VAR model show that the dynamic volatility-volume relationship depends on the trader types involved. The positive contemporaneous volatility-volume relationship is driven mainly by volume from trading between floor traders and customers. Alternatively, several studies focus on the effect of expected and unexpected volume shocks on volatility. Bessembinder and Seguin (1993) find that unexpected volume shocks have a larger effect on volatility in futures markets than expected volume. Daigler and Wiley (1999) find that the unexpected volume series is more important than the expected volume series in explaining volatility.

DATA

The data of the current empirical study consists of Taiwan stock exchange (TWSE) (http://www.twse.com.tw/ch/index.php) index transaction prices (represented by market index) and trading volume for the period 1st January 2005 to 31 December 2007. This study derives the daily trading volume from the TWSE database. There are 743 days (observations) in our sample. Andersen et al. (1999) and Martens (2001) show that intraday returns can improve not only the measuring of volatility, but also the forecasting of volatility. Therefore, our empirical analyses use intraday returns from each 1-min interval to measure returns and avoid market microstructure problems. There are 65310 intraday 1-min interval trading data in our sample. The continuously compounded returns of every minute are calculated as $r_{i,t} = 100 \times (\log(P_t) - \log(P_{t-1}))$, where $r_{i,t}$ and P_t are the return and market index at time t.

The daily returns are computed as $R_i = \sum_t r_{i,t}$.

Unfortunately, volatility is not directly observable. A popular approach to measure daily volatility is to use the daily squared return. Andersen and Bollerslev (1998) argue that in most financial applications, the asset price is assumed to follow a continuous time diffusion process, and the correct measure for daily volatility is

$$\sigma_t^2 = \int \sigma_{t+\tau}^2 d\tau \tag{1}$$

Andersen and Bollerslev (1998) show that the daily squared return is an unbiased estimator of true volatility. Martens (2002) also compares various measures and forecasts of volatility in daily volatility and find the best daily volatility measure is the sum of intraday squared returns. This implies that using the sum of squared intraday returns is better than using daily squared returns to measure stock market volatility. Hence, we use equation (1) to compute volatility. Table 1 provides basic statistics of volatility and trading volume.

Table 1: Basic Statistics of Sample

	Volatility(σ_t)	**Trading Volume(v_t)**
Mean	0.0004	15.104
Median	0.0007	15.086
Maximum	0.0512	16.157
Minimum	-0.0467	14.458
Standard deviation	0.0107	0.303
Skewness	-0.692	0.489
Kurtosis	4.0351	3.031

Note: The basic statistics of volatility (σ_t) and trading volume (v_t) are presented in this table. For the volatility, we analyzed with 1-minute intervals. Trading volumes, measured by nature logarithm, are from the TWSE database. The descriptive statistics have some clues for the behaviors of Taiwanese stock market.

RESEARCH METHODOLOGY

The VAR approach provides a framework and has been used widely in the literature for the issue in our research (e.g. Luu and Martens (2003), and Fujihara and Mougoue (1997)). VAR modeling requires that all times series be stationary. As a first step, trading volume and volatility and their first differences were tested for stationarity using Augmented Dickey-Fuller tests. If the calculated ADF statistic is less than its critical value, then the variable is said to be stationary or integrated to the order zero. If they are non-stationary, then the issue is to what degree they are integrated. In practice, a number of econometric packages can perform this test, which gives the critical value of the ADF statistic. Computations were performed using Eviews 6.0 and the number of lags or augmentation in ADF regressions were selected by Akaike Information Criterion. Table 2 lists the conclusion.

As a result, the following VAR(k) model is estimated, in which the Akaike Information Criterion (AIC) is used to determine the optimal lag length (k). The VAR model used in this study is shown in equation (2) and (3) below.

$$\sigma_t = c_0 + \alpha_1\sigma_{t-1} + \alpha_2\sigma_{t-2} + \beta_1 v_{t-1} + \beta_2 v_{t-2} + \varepsilon_{1t} \qquad (2)$$

$$v_t = a_0 + a_1 v_{t-1} + a_2 v_{t-2} + b_1\sigma_{t-1} + b_2\sigma_{t-2} + \varepsilon_{2t} \qquad (3)$$

Where σ_t is the vector that represents the volatility and v_t is the vector that represents the trading volume. The optimal lag length (k) in the VAR model is selected by the Akaike Information Criterion (AIC) (i.e., $k = 2$).

The next step is to determine the direction of Granger causality. Under the assumption of stationarity of variables and the null hypothesis of no Granger causality, the standard F-test is used to examine Granger-causality between variables in the VAR system. If the F-test rejects the null hypothesis that the lag coefficients of variable $v_t\,(\sigma_t)$ are jointly zero when variable $\sigma_t\,(v_t)$ is the dependent variable in the VAR system, then variable $v_t\,(\sigma_t)$ Granger-causes variable $\sigma_t\,(v_t)$.

Once the VAR system was estimated, this study employed two short-run dynamic analyses: variance decomposition and impulse response functions. Forecast error variance decomposition separates the variation in an endogenous variable into the component shocks to the VAR system. The variance decomposition is an estimate of the proportion of the movement of the n-step-ahead forecast error variance of a variable in the VAR system that is attributable to its own shock and that of another variable in the system. However, the recursive ordering of the variables in the VAR system for this study follows this order. Volatility is first and trading volume is ordered next to volatility. The ordering reflects previous studies such as Chen and Daigler (2008). Forecast error variance decomposition can characterize the dynamic behavior of a VAR system. In addition, we derive impulse response functions, which show the dynamic effects on volatility (trading volume) of innovations to the trading volume (volatility). We estimate the VAR model and orthogonalize these shocks by resorting to a Choleski decomposition of the estimated variance-covariance matrix of the VAR residuals to generate impulse response functions. Figures 1 and 2 list the results.

ESTIMATION RESULTS

As the first step, all the two variables were tested for stationarity using Augmented Dickey-Fuller tests. Table 2 gives the results. It can be seen that for all of the level variable less than critical value at 95% level of confidence. An examination of test results shows that all the time series employed in this research are stationary at level. The null hypothesis of the unit root is rejected for all variables at the 5% significance level.

Table 2: ADF Tests for Unit Roots

Variable	Without trend		With trend	
	Test statistic	Critical value	Test statistic	Critical value
Volatility(σ_t)	-3.649	-2.865	-3.999	-3.416
Trading volume(v_t)	-3.613	-2.865	-4.028	-3.416

Note: σ_t and v_t represents volatility and trading volume, respectively. Computations were performed by using Eviews 6.0 and the number of lags or augmentation in ADF regressions are selected by Akaike Information Criterion. The ADF test rejects the null of a unit root for both series in this table.

Table 3 shows the VAR estimation results. Results indicate that the past trading volume and volatility significantly affect the current volatility or trading volume. This conclusion is very important as it gives useful information about trading volume and forecasts of returns and volatility. Table 4 presents causality test results obtained by VAR estimation using equations (1) and (2). The results indicate the trading volume of the Taiwan stock index significantly Granger-causes volatility. Volatility also strongly Granger-causes the trading volume of the Taiwan stock index. Furthermore, the Granger-causality between two variables is in both directions. The results also show the past market information about volatility and trading volume has an ability to predict the volatility and trading volume in the future in Taiwan. According to some theoretical papers, both the MDH and the sequential arrival of information hypothesis support a positive and contemporaneous relationship between trading volume and absolute returns. Our results supports the mixture of distributions hypothesis (MDH). Furthermore, a bi-directional causality test was found between volatility and trading volume, which is consistent with the findings of Luu and Martens (2003) and Chen (2007).

Table 3: VAR Estimation Results

Dependent variable	σ_t	v_t
Constant	-6.82E-06	303898.7
	(-3.5497)***	(4.1735)***
σ_{t-1}	0.9029	-7.52E+09
	(24.5974)***	(-5.4091)***
σ_{t-2}	0.0675	7.14E+09
	(1.8581)*	(5.1885) ***
v_{t-1}	5.80E-12	0.6166
	(6.2697)***	(17.6085)***
v_{t-2}	-3.08E-12	0.3154
	(-3.2616)***	(8.8062)***

*Note: 1. σ_t and v_t represents volatility and trading volume, respectively. 2. t statistics are indicated in the parentheses. 3. "***", "**" and "*" indicate significance at the 1, 5 and 10 percent levels, respectively.*

Table 4: Granger Causality Tests for Volatility and Trading Volume

Causality relation	Statistics	P-value
$\sigma_t \rightarrow v_t$	29.393	0.000(<5%)***
$v_t \rightarrow \sigma_t$	59.371	0.000(<5%)***

*Note: 1. σ_t and v_t represents volatility and trading volume, respectively. 2. $\sigma_t \rightarrow v_t$ means the volatility Granger-causes volume. $v_t \rightarrow \sigma_t$ denotes the volume Granger-causes volatility. 3. "***" represents the causal relationship being significant at 1% level.*

Table 5 illustrates the estimation results of variance decomposition to examine dynamic relationships in volatility and trading volume further. In Table 5, the stock volatility variance decomposition analysis reveals that the largest share of shock to volatility, apart from its own shock, trading volume accounted for about 40% during the 24-day period (about one month), while trading volume accounted for 21% of the shock during the 12-day period (about two weeks). The shock to trading volume has a significant effect on volatility. In addition, the movement in trading volume is explained by its own shocks rather than by the shocks to volatility. Clearly, volatility does not explain a large part of the variance decomposition of trading volume. The variance of volatility accounts for approximately 6% during the 4-day period and 8% in the 24-day period. This shows the small proportion of volatility shocks on the variability of trading volume.

Table 5: Estimates of Variance Decomposition

Lags (n)	Percentage of the movement in the σ_t explained by shocks to :		Percentage of the movement in the v_t explained by shocks to :	
	σ_t	v_t	σ_t	v_t
1	100	0	0.703	99.297
4	94.681	5.319	5.705	94.295
8	87.050	12.950	6.658	93.342
12	78.820	21.180	7.185	92.815
16	71.269	28.731	7.559	92.440
20	64.867	35.133	7.842	92.158
24	59.654	40.346	8.059	91.941

Note: σ_t and v_t stand for the volatility of Taiwan market index and trading volume, respectively. To further examine dynamic relationships in σ_t and v_t, this table provides the percentage of the movement in the σ_t explained by shocks to v_t and the percentage of movement in the v_t explained by shocks to σ_t .

The second use to which we put the VAR model was the derivation of impulse response functions, which show the dynamic effects between volatility and trading volume. Figure 1 and 2 depict the estimated impulse response functions. The time horizon extends to 30 days, over which the dynamic adjustment paths of volatility are plotted following the innovations to each of the trading volumes. One standard deviation increase in the trading volume is followed by an increase in the volatility. The effects on volatility peak after 17 days. As regards the effect on trading volume, there is a downward effect of a shock to volatility. Trading volume responds much more sluggishly. One standard deviation increase in volatility is followed by a decrease in the trading volume. The effect on trading volume peaks after 3 days. The results in Figures 1 and 2 show that past information about trading volume has an ability to predict volatility.

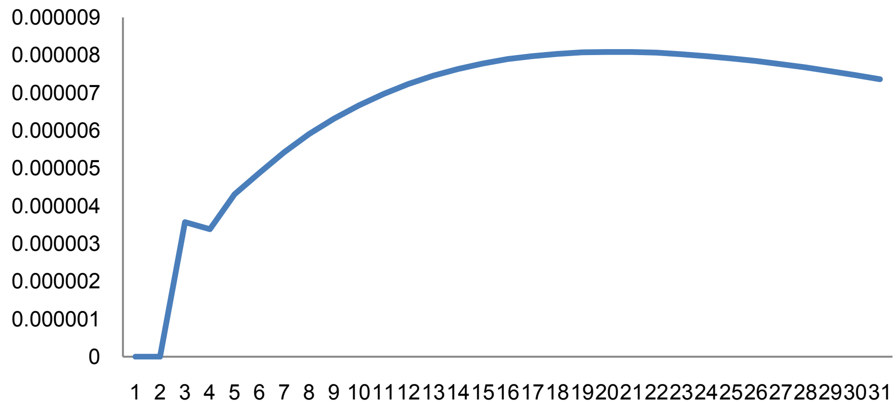
CONCLUSION

This paper aimed to investigate the dynamic relations between return volatility and trading volume on the Taiwan stock market. The use of the VAR model allowed us to trace the predictability of volatility and trading volume, and to account for the endogeneity between volatility and trading volume. The VAR model also enabled us to capture the economic interactions between those variables. We used intraday returns to measure volatility and avoid microstructure bias. This paper sheds further light on the dynamics between volatility and trading volume. First, we found a general bi-directional causal relationship. Because past market information about volatility and trading volume has an ability to predict volatility and trading volume in the future, our results supports both the mixture of distributions and the sequential arrival of information hypotheses.

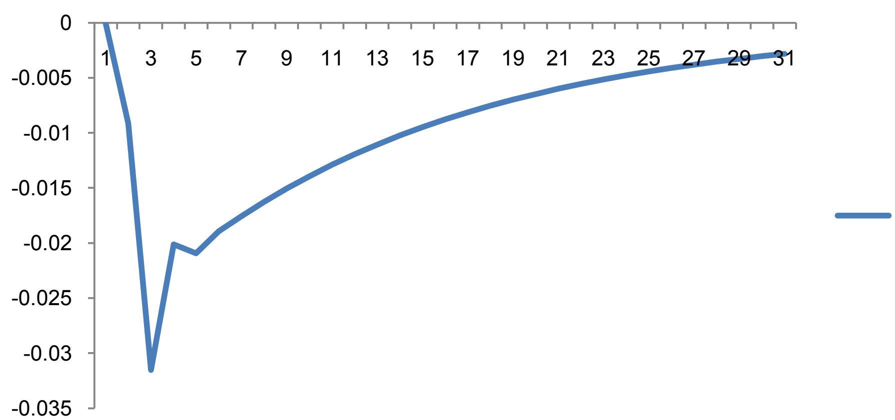
The forecast error variance decomposition was obtained with the aim of assessing how much such shocks contribute to the variability of the variables in the system. The result shows the trading volume shocks significantly contribute to the variability of volatility by accounting for about 40% of the shock during the 24-day period. However, the contribution of volatility shocks to the variability of trading volume only accounts for 8% of the shock during the 24-day period. This finding confirms that the variability in stock volatility is substantially explained by trading volume.

Figure 1:Estimation of Response Function

Response of σ_t to v_t



Response of v_t to σ_t



Note: σ_t and v_t represents volatility and trading volume, respectively. The impulse response function show responses of each variable in the VAR system to a one standard deviation shock to itself and to the other series. In this figure, the dynamic interrelation of σ_t and v_t can be shown.

The findings from the impulse response function show that one standard deviation increase in the trading volume is followed by an increase in the volatility. The effect on volatility peaks after 17 days. As regards the effect on trading volume, there is a downward effect of a shock to volatility. Trading volume responds much more sluggishly. One standard deviation increase in volatility is followed by an increase in trading volume. The effect on trading volume peaks after 3 days. These findings are helpful to financial managers dealing with the stock index or its derivations. The limitations to our model is sample size, additional research needs to collect different types of traders' data. The different types of traders may have distinct information. Recently, many studies begin to investigate SAIH to focus on the effect of different types of trader. Therefore, further results should need samples that are more detailed and many kinds of trader judgments.

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