THE LIQUIDITY EFFECT IN OPTION PRICING: AN EMPIRICAL ANALYSIS

Shih-Ping Feng, National Taiwan University

ABSTRACT

This paper empirically examines whether asset's liquidity can help resolve the known strike-price biases of the Black-Scholes model for different liquidity measures based on trading volume, bid-ask spread and the Amihud's ILLIQ. Our results indicate that, when the underlying asset or its derivative exhibit lower liquidity, the degree of curvature of the strike-price biases will tend to increase, regardless of the liquidity measures used. Furthermore, inspection of R^2 *reveals that the stock's liquidity has an excellent ability in explaining the strike-price biases compared with the option's liquidity in terms of the liquidity measures based on trading volume and the Amihud's ILLIQ.*

JEL: G10; G12; G13

KEYWORDS: Option Pricing; Liquidity; Stock's Liquidity; Option's Liquidity; Strike-Price Biases

INTRODUCTION

The purpose of this paper is to empirically examine whether the underlying asset's liquidity and its derivative's liquidity have potential to help resolve the strike-price biases associated with the Black-Scholes model (Black and Scholes (1973)) as depicted in Figure 1. While most attempts to explain these pricing errors focus on relaxing the Black-Scholes assumption of constant volatility (Heston, 1993 and Heston and Nandi, 2000), later examinations of stochastic volatility indicate that they cannot fully explain the pattern of the pricing errors and conclude that there is a need for a new explanation for this apparent pricing bias (Bakshi *et al*., 1997 and Eraker, 2004).

Liquidity, a new perspective on asset pricing, captures our attention on its potential to resolve the known strike-price biases. Cetin *et al*., (2006) studied the pricing of option in which the stock is not perfectly liquid and concluded that liquidity cost comprises a significant component of an option's price. Liu and Yong (2005) reported that the imperfect stock' liquidity would affect the replication of an option. These results motivated this research. Along the idea of Pena *et al*. (1999), who indicated that explain directly the determinants of the volatility smile is necessary to capture the important reasons behind the apparent failure of Black-Scholes model, we direct our analysis to further investigate the role of the stock's liquidity and its derivative's liquidity in explaining the strike-price biases.

While recent studies point out the choice of liquidity measures may have a significant effect on research outcome (Aitken and Comerton-Forde, 2003), that provides us a strong motivation to employ several different dimensions of liquidity measures to examine whether the liquidity can provide a new explanation for this apparent pricing bias of the Black-Scholes model. Furthermore, this enables us to test whether the effect of liquidity is robust enough for different liquidity measures. This is surely the first contribution of this paper over existing theories.

The second contribution of this paper is that we consider two types of liquidity risk, the underlying asset's liquidity and its derivative's liquidity, into our analysis. It is quite important to notice that these two types of liquidity risk would affect the option price in different way (Liu and Yong, 2005 and Brenner *et al*., 2001). A possible concern is whether these two types of liquidity risk need to be considered into option pricing model simultaneously. However, the existing empirical literature mostly deals with the effect of the stock's liquidity on the option pricing (Cetin *et al*., 2006 and Liu and Yong, 2005). In contrast, such research on the option's liquidity effect is still lacking. This paper fills in this gap. In addition, we provide evidence to show that the option's liquidity is attributed partially to the illiquidity present in its underlying asset.

Figure 1: Absolute Pricing Errors across Strike Prices For General Electric (GE)

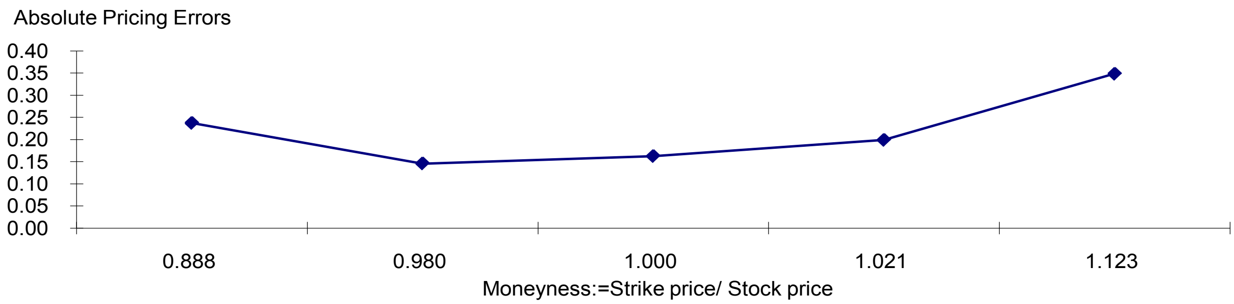

Figure 1 presents the relationship between the absolute pricing errors and the degree of moneyness for GE from 1996/1/1 through 2007/12/31. We employ five intervals for the degree of moneyness. The absolute pricing error is defined as the median of the absolute difference between the market price and the model price for each of the option price data in each moneyness classification.

Empirically, we employ the liquidity measures proposed by Cao and Wei (2010) to investigate the relationship between the degree of curvature in the absolute pricing errors and the asset's liquidity during the period from January 1996 to December 2007. In general, our results support the view that when the underlying asset or its derivative exhibit lower liquidity, the degree of curvature of the strike-price biases will tend to increase, regardless of the liquidity measures used. This means that the underlying asset's liquidity and its derivative's liquidity are both the informative determinants of the pattern of the strike-price biases. On the other hand, inspection of the R^2 reveals that the stock's liquidity has an excellent ability in explaining the strike-price biases compared with the option's liquidity in terms of the trading volume, and the Amihud's ILLIQ, especially after the sample period April 2001. Furthermore, our results indicate that despite the fact that the stochastic volatility can nicely improve the pricing performance of the Black-Scholes model, it is necessary to take both the option's liquidity and the stock's liquidity into account for further development of a more general pricing model. All empirical results regarding the liquidity effect on the curvature of the strike-price biases is the third contribution of this paper.

The rest of the article proceeds as follows. Section II introduces the literature review concerning liquidity proxies. Section III shows the liquidity measures used in this paper. Sections IV describes the data and methodology in the empirical study. Sections V and VI present the empirical results and conclusion.

LITERATURE REVIEW CONCERNING LIQUIDITY PROXIES

There are many liquidity proxies in empirical study. These measures may be divided into two broad categories: trade-base measures and order-base measures. The former is related to trading volume, dollar trading volume and the turnover ratio, while the latter refers to bid-ask spreads. For example, Dater, Naik and Radcliffe (1998) proposed a share turnover ratio as a liquidity proxy. Amihud and Mendelson(1986) used bid-ask spread as liquidity proxy to study the effect of the bid-ask spread on asset pricing. In addition to these two main categories of liquidity measures, Amihud's AILLIQ (2002) which is defined as the ratio of the absolute stock return on volume is also commonly used as liquidity proxy. Acharya and Pedersen (2005) adopt this measure to investigate the relationship between the return and the liquidity risk.

Cao and Wei (2010) compared these three categories of liquidity measures and proposed five liquidity proxies for both the underlying asset and its derivative including the trading volume, the dollar trading volume, the bid-ask spread, the absolute ILLIQ and the percentage ILLIQ to show the evidence of commonality in options present in different liquidity measures.

LIQUIDITY MEASURES

We adopt the liquidity measures used in Cao and Wei (2010) as the proxies of the stock's liquidity and the option's liquidity. We employ the trading volume (VOL) and the dollar trading volume (DVOL) as the transaction-based measure and the bid-ask spread (PBA) as the order-based measure. We also include Amihud's ILLIQ as the price impact measure (AILLIQ, PILLIQ). Table 1 describes the definitions of the liquidity measures.

We briefly discuss the expected relationships among the different liquidity measures. The heavier the trading or dollar trading volume of an asset, the more liquid it is. Since the bid-ask spread represents the cost of a transaction, the lower bid-ask spread denotes higher liquidity. Furthermore, an asset with lower liquidity will have higher values of AILLIQ and PILLIQ. The underlying intuition behind a higher AILLIQ or PILLIQ is that the asset's price moves significantly in response to a small change in volume.

Table 1: Definitions of Liquidity Measures

	Option	Stock
PBA	$\sum_{j=1}^{N} VOL_j \times \frac{ask_j - bid_j}{(ask_j + bid_j)/2} / \sum_{j=1}^{N} VOL_j$	$\frac{ask - bid}{(ask + bid)/2}$
VOL	$\sum_{j=1}^{N} VOL_j$	VOL
DVOL	$\sum_{j=1}^{N} VOL_j \times (ask_j + bid_j)/2$	$VOL \times (ask + bid)/2$
AILLIQ	$\sum_{j=1}^{N} VOL_j \times \frac{\lvert (C_t^j - C_{t-1}^j) - \Delta_{t-1}^j (S_t - S_{t-1}) \rvert}{DVOL_t^j} / \sum_{j=1}^{N} VOL_j$	$\frac{\lvert S_t - S_{t-1} \rvert}{DVOL_t}$
PILLIQ	$\sum_{j=1}^{N} VOL_j \times \frac{\lvert (C_t^j - C_{t-1}^j) - \Delta_{t-1}^j (S_t - S_{t-1}) \rvert / C_{t-1}^j}{DVOL_t^j} / \sum_{j=1}^{N} VOL_j$	$\frac{\lvert S_t - S_{t-1} \rvert / S_{t-1}}{DVOL_t}$

This table is reported for the definitions of the stock liquidity and option liquidity. The Δ stands for the option's delta, C_t is the option price at time t, and the summation is over the distinct options that are traded during the day.

DATA AND METHODOLOGY

Data

We select five well-known companies with varying liquidity: Federal Express (FDX), General Electric (GE), International Business Machines (IBM), Intel Corporation (INTC), and Texas Instruments (TXN), for our analysis. The data are obtained from Ivy DB Option Metrics and TAQ for the period from January 1, 1996 through December 31, 2007. Specifically, we divided the entire sample period into two sub-periods: January 1, 1996 to March 31, 2001 (an upward trend of the stock price) and April 1, 2001 to December 31, 2007 (a downward trend of the stock price). For each stock, we take the average bid and ask quotes in the last five minutes of trading as the close bid and ask prices. The option prices are defined as the average of the bid and ask option prices. We follow Macbeth and Merville (1979) in adjusting the stock dividends and use the dividend-exclusive stock prices to value the options. Using exclusion filters, we exclude general arbitrage violations, options with a maturity of less than nine days or bid-ask spreads below zero, price quotes lower than 3/8, and zero trading volume options. After processing these filters, there are 43,259 options for FDX, 111,945 options for GE, 217,074 options for IBM, 206,877 options for INTC and 146,766 options for TXN.

METHODOLOGY

The purpose of this paper is to empirically examine whether the liquidity have potential to help resolve the strike-price biases of the Black-Scholes model. Thus, we first verify the quadratic relationship by estimating the coefficient of the degree of curvature in the absolute pricing errors for each day:

$$APE_{ijt} = \alpha_0 + \alpha_1 M_{ijt} + \alpha_2 M_{ijt}^2 + \varepsilon_{ijt}^{APE} \quad (1)$$

where APE_{ijt} is the absolute pricing error of option j on stock i on day t defined as the absolute difference between the market price and the model price, and M_{ijt} is the moneyness of option j on stock i on day t which is defined as the exercise price K_{ij} divided by the stock price S_{it}. In order to calculate the absolute pricing errors, following Macbeth and Merville (1979), we regress the implied volatility σ_{ijt} to the percentage moneyness $m_{ijt} = M_{ijt} - 1$ to obtain the implied volatility for an at-of-money (ATM) option for stock i for each day.

As expected, the estimated α_2 are positive for all companies. These provide evidence to show that there is a quadratic relationship between the absolute pricing errors and the degree of moneyness. To examine whether the asset's liquidity could help resolve the strike-price biases, we employ the idea of Pena *et al.* (1999) to directly examine the relationship between the degree of curvature α_2, the stock's liquidity and the option's liquidity. The degree of curvature in the absolute pricing errors increases at higher levels of α_2.

Geske and Roll (1979) found that there are striking price biases, time-to-expiration biases and variance biases in the pricing errors of the Black-Scholes model. Therefore, we include the individual stock's volatility and the average time-to-expiration for stock i on day t as the explanatory variables for α_2. Since the individual stock's volatility is related to the option's liquidity and the stock's liquidity, we employ the market volatility on day t as a good instrument for the individual stock's volatility and run the following fixed effects panel regression:

$$\alpha_{2,it} = \beta_0 + \beta_1 OL_{it} + \beta_2 V_t + \beta_3 mat_{it} + \beta_4 r_t + \varepsilon_{it}^{\alpha} \quad (2)$$

$$\alpha_{2,it} = \beta_0 + \beta_1 SL_{it} + \beta_2 V_t + \beta_3 mat_{it} + \beta_4 r_t + \varepsilon_{it}^{\alpha} \quad (3)$$

where OL_{it} (SL_{it}) are the option's (stock's) liquidity measures for stock i on day t, V_t is the market volatility on day t, computed as the annualized standard deviation of the $S\&P500$ from the previous three months, mat_{it} is the average time-to-expiration for stock i on day t, and r_t is the average risk-free interest rate on day t. To facilitate the comparison of the two types of liquidity, the unit for the stock's trading volume is 10^9 and for the option's trading volume it is 10^3.

Furthermore, we empirically examine whether the stock's liquidity and the option's liquidity play a simultaneous role in explaining the curvature of the strike-price biases. In practice, the option's liquidity is related to its underlying asset's liquidity. Therefore, before examining whether the stock's liquidity and the option's liquidity need to be considered into option pricing model simultaneously, we must clarify the relationship between the stock's liquidity and the option's liquidity among different liquidity measures. Thus, we run the following panel regression with fixed effects:

$$OL_{it} = \mu_0 + \mu_1 SL_{it} + \varepsilon_{it}^{OL} \tag{4}$$

If there exists for the covariation between the stock's liquidity and the option's liquidity, we would employ the technology using in the Cao and Wei (2010) and use the orthogonalized option's liquidity to control for the potential relationship between the stock's liquidity and the option's liquidity. We run the following panel regression with fixed effects:

$$\alpha_{2,it} = \gamma_0 + \gamma_1 SL_{it} + \gamma_2 \varepsilon_{it}^{OL} + \gamma_3 V_t + \gamma_4 mat_{it} + \gamma_5 r_t + \varepsilon_{it}^{\alpha} \tag{5}$$

where ε_{it}^{OL} is the residual form of the regression that regresses the option's liquidity on the stock's liquidity.

EMPIRICAL RESULTS

Table 2 reports the results of the panel regression for both the option's liquidity and the stock's liquidity as well as for both the entire sample period and two sub-periods. Regardless of sample period, the trading volume measures, VOL and DVOL, the degree of curvature in the absolute pricing errors is negative and significantly related to the option's liquidity and the stock's liquidity in most cases. While for AILLIQ and PILLIQ, the estimated coefficient of the asset's liquidity is positive and significant for both the option's liquidity and the stock's liquidity, except the sub-period prior to the 2001. The price impact measure of liquidity seems to be weakly related to the degree of curvature in the absolute pricing errors for the sub-period during January 1996-March 2001. As for PBA, the estimated coefficient of the stock's liquidity and option's liquidity is positive but the estimated coefficient of the stock's liquidity is negative for the sub-period during January 1996-March 2001.

On average, these results mean that, the lower the liquidity of the option or the lower the liquidity of the stock, the more likely it is that the degree of curvature of the strike-price biases will tend to increase, regardless of the liquidity measure used. It is also the case that the market volatility, the average time-to-expiration and the average risk-free interest rate are also the determinants of the degree of curvature in the absolute pricing errors. This means that despite the fact that the stochastic volatility can provide the improvement of pricing performance associated with the Black-Scholes model, incorporating the asset's liquidity into the option pricing model is necessary. We further compare the relative explanatory power of the option's liquidity and the stock's liquidity. Inspection of the R^2 reveals that the stock's liquidity has an excellent ability in explaining the strike-price biases compared with the option's liquidity in terms of the trading volume, the dollar trading volume, the absolute ILLIQ and the percentage ILLIQ, especially after the sample period April 2001.

We next empirically examine whether the stock's liquidity and the option's liquidity play a simultaneous role in explaining the curvature of the strike-price biases. We first clarify the relationship between the stock's liquidity and the option's liquidity. As expected, the estimated coefficients of the stock's liquidity are both positive and significant in relation to the option's liquidity measure in terms of VOL, DVOL, AILLIQ and PILLIQ. As for PBA, the estimated coefficient of the stock's liquidity is negative and significant. These mean that when an underlying asset has low liquidity, its derivative will have imperfect liquidity in terms of transaction-based measure and the price impact measure. These findings suggest that the liquidity of the derivative is partially attributed to illiquidity present in the underlying asset among the different liquidity measures.

Table 2: Comparison stock's liquidity effect with option's liquidity effect

	Option's Liquidity					Stock's Liquidity				
	OL_{it}	V_t	mat_{it}	r_t	R^2	SL_{it}	V_t	mat_{it}	r_t	R^2
Panel A: Jan 1996 – Dec 2007										
VOL	-0.014**	-11.393**	-5.224**	0.388**	0.259	-27.761**	-11.831**	-4.555**	0.350**	0.262
DVOL	-0.001**	-11.419**	-5.589**	0.407**	0.254	-0.158**	-11.664**	-5.523**	0.403**	0.254
PBA	9.251**	-11.384**	-5.366**	0.395**	0.257	0.278	-11.562**	-5.585**	0.401**	0.254
AILLIQ	0.001**	-11.672**	-5.560**	0.396**	0.254	0.004**	-11.500**	-5.562**	0.394**	0.260
PILLIQ	0.005**	-11.597**	-5.575**	0.396**	0.255	0.660**	-11.577**	-5.607**	0.400**	0.255
Panel B: Jan 1996 – Mar 2001										
VOL	-0.018**	-19.706**	-6.648**	-0.597**	0.266	-59.452**	-17.710**	-6.145**	-0.536**	0.274
DVOL	-0.0003	-20.530**	-7.317**	-0.620**	0.261	-0.068	-20.418**	-7.340**	-0.613**	0.261
PBA	7.373**	-19.563**	-7.209**	-0.457**	0.263	-9.508**	-20.131**	-7.008**	-0.593**	0.262
AILLIQ	-0.002	-20.614**	-7.385**	-0.626**	0.261	0.026*	-20.364**	-7.206**	-0.611**	0.262
PILLIQ	0.008	-20.632**	-7.332**	-0.630**	0.261	-1.091	-20.825**	-7.294**	-0.637**	0.261
Panel C: Apr 2001– Dec 2007										
VOL	-0.004**	-1.203*	-1.685**	0.694**	0.410	-2.412	-1.363*	-1.658**	0.689**	0.410
DVOL	-0.001**	-1.129	-1.700**	0.690**	0.410	0.144**	-1.341*	-1.682**	0.677**	0.411
PBA	1.420	-1.308*	-1.722**	0.691**	0.410	0.725**	-1.053	-1.626**	0.684**	0.412
AILLIQ	0.001*	-1.392**	-1.683**	0.684**	0.410	0.004**	-1.382**	-1.633**	0.669**	0.420
PILLIQ	0.004**	-1.364*	-1.676**	0.680**	0.411	0.822**	-1.433**	-1.671**	0.674**	0.414

*The table is reported for the results of the panel regression model that takes both the stock's liquidity and the option's liquidity into account. * and ** denote rejection at the 10% and 5% significance levels, respectively.*

Table 3 reports the results of the panel regression on investigating whether the stock's liquidity and the option's liquidity need to be considered into option pricing model simultaneously. In terms of the trading volume, VOL and DVOL, the estimated coefficients of the stock's liquidity and the residual of the option's liquidity are negative and significant in most cases, but for the sub-period during January 1996-March 2001, only the stock's liquidity is significant. As for the PBA, the explanatory power of the stock's liquidity and the option's liquidity are not consistent for different sample period. For AILLIQ and PILLIQ, the stock's liquidity is positive and significant. This means that, for AILLIQ and PILLIQ, the stock's liquidity have more explanatory power in terms of explaining the pattern of strike-price biases than the option's liquidity. The market volatility, the average time-to-expiration and the average risk-free interest rate also seem to be determinants associated with the pattern of strike-price biases. It particular,

inspection of the R^2 reveals that taking both these two types of liquidity risk into account could effectively increase the explanatory power in explaining the degree of curvature in the absolute pricing errors compared to using the stock's liquidity and the option's liquidity, respectively. The results also suggest that the option pricing model with stochastic volatility will not be correctly specified as long as we do not consider both these two types of liquidity risk. In summary, all these empirical results provide evidence to conclude that taking the underlying asset's liquidity and its derivative's liquidity into account is a move in the right direction for the further development of a more general pricing model.

Table 3: The Effect of Stock's Liquidity and Option's Liquidity

	Option's Liquidity and Stock' Liquidity					
	SL_{it}	ε_{it}^{OL}	V_t	mat_{it}	r_t	R^2
Panel A: Jan 1996 – Dec 2007						
VOL	-27.297**	-0.007**	-11.647**	-4.605**	0.356**	0.263
DVOL	-0.158**	-0.001*	-11.476**	-5.545**	0.409**	0.255
PBA	0.277	9.462**	-11.192**	-5.378**	0.400**	0.257
AILLIQ	0.004**	-0.0004	-11.499**	-5.568**	0.395**	0.260
PILLIQ	0.660**	0.003**	-11.546**	-5.601**	0.398**	0.256
Panel B: Jan 1996 – Mar 2001						
VOL	-59.453**	-0.0003	-17.715**	-6.141**	-0.536**	0.274
DVOL	-0.067	-0.0003	-20.466**	-7.319**	-0.616**	0.261
PBA	-10.457**	6.748**	-19.206**	-6.928**	-0.440**	0.264
AILLIQ	0.026*	-0.002	-20.334**	-7.248**	-0.608**	0.262
PILLIQ	-1.091	0.008	-20.812**	-7.282**	-0.638**	0.261
Panel C: Apr 2001– Dec 2007						
VOL	-2.310	-0.005**	-1.205*	-1.736**	0.691**	0.411
DVOL	0.146**	-0.001**	-0.993	-1.687**	0.679**	0.411
PBA	0.725**	1.458	-0.967	-1.654**	0.688**	0.412
AILLIQ	0.004**	-0.001**	-1.379**	-1.638**	0.670**	0.421
PILLIQ	0.822**	0.001	-1.425**	-1.668**	0.673**	0.414

*The table is reported for the results of the panel regression for the option's liquidity and the stock's liquidity, respectively. * and ** denote rejection at the 10% and 5% significance levels, respectively.*

CONCLUSION

The purpose of this paper is to examine empirically the effects of the underlying asset's liquidity and its derivative's liquidity on the curvature of the strike-price biases associated with the Black-Scholes model. For this purpose, we employ the liquidity measures used in Cao and Wei (2010) including the trading volume, the dollar trading volume, the bid-ask spread, the absolute ILLIQ and the percentage ILLIQ as the proxy of the option's liquidity and the stock's liquidity.

Empirically, we select five well-known companies with varying liquidity for the period from January 1, 1996 through December 31, 2007 as our empirical data. We employ the idea of Pena *et al*. (1999) to examine directly the relationship between the degree of curvature α_2, the stock's liquidity and the option's liquidity. In general, the results indicate the lower the liquidity of underlying asset or the lower the liquidity of its derivative, the more likely it is that the degree of curvature of the strike-price biases will tend to increase among different liquidity measures. On the other hand, inspection of the R^2 reveals that the stock's liquidity is able to more accurately explain the curvature of the strike-price biases compared with the option's liquidity in terms of the trading volume, the dollar trading volume, the absolute ILLIQ and the percentage ILLIQ, especially for the period after April 2001. The empirical results also show that the stock's liquidity, the option's liquidity and the stock's volatility are all the determinants of the curvature of the strike-price biases. This means that the option pricing model with stochastic volatility will not be correctly specified as long as we do not take the asset's liquidity into account. Furthermore, we provide evidence to show that when pricing derivatives, the liquidity of the underlying asset and its derivative are both important and necessary for further development of a more general pricing model.

This paper offers a direct insight into the effects of the stock's liquidity and its derivative's liquidity on the performance of the Black-Scholes model. We choose five individual stocks for the empirical analysis, and these are more likely to be liquid stocks. Since the number of individual stock option for illiquid stock is less than the liquid stock, and it is not easy to examine the empirical issue addressed in this paper for small firm due to the limitation of empirical data. However, it is important to clarify whether the asset's liquidity is the determinants of the performance of pricing model and the empirical results of this paper document the important of asset's liquidity to option pricing. Furthermore, the future research can examine the effects of underlying asset's liquidity and its derivative's liquidity on pricing performance using data from options on future, and currency options.

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BIOGRAPHY

Shih-Ping Feng can be contacted at: College of Management, National Taiwan University, No. 1, Section 4, Roosevelt Road, Taipei, Taiwan. Email: d92724016@ntu.edu.tw