THE VALUATION OF RESET OPTIONS WHEN UNDERLYING ASSETS ARE AUTOCORRELATED

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ABSTRACT

This paper introduces the autocorrelation effect of assets' returns into the valuation model of reset options. The MA(q) process, which is an extension of MA(1) process noted by Liao and Chen (2006), is applied to the valuation of reset options in this paper. Due to the impact of autocorrelation on the volatility of assets' returns, the probability of reset and the value of reset option are affected. Positive autocorrelation increases the value of a reset option by increasing the probability of reset. On the contrary, negative autocorrelation decreases the probability of a reset and reset premium. Moreover, the reset timing is affected by the autocorrelation characteristics. In the case of positive autocorrelation, the investors tend to reset earlier to prevent a possible loss. Positive autocorrelation is also significant for the hedging of reset options. This paper demonstrates that positive autocorrelation characteristics lessens the delta jump and gamma jump problem.

JEL: G12, G13

KEYWORDS: Reset Option, Autocorrelation; MA(q) process, Delta Jump; Gamma Jump

INTRODUCTION

To give investors more protection, increasing numbers of derivatives with embedded reset clauses have become available. These reset clauses can be exercised at any time during the life of the contract or only limited to some predetermined dates. The reset clauses commonly contain the date of maturity or the strike price. Options with reset rights on the maturity date commonly exist in crude oil offshore exploration and production contracts. This reset clause allows the holder to reset the maturity of the investment option and to look for better investment conditions. Crude oil price is the critical factor affecting the value of this kind of option. If crude oil price movements can be forecast by certain models, the value of the reset clause might be affected.

The model derived from Liao and Chen (2006) is an important contribution pricing of a vanilla option whose underlying asset has autocorrelation characteristics. However no existing studies consider the impact of this autocorrelation characteristic on the value of a reset option. Due to the path dependence of reset options, it is reasonable to expect that the impact of autocorrelation on the prices of reset options might be augmented. Many studies have demonstrated different valuation models for reset options with different reset conditions. The main objective of this paper is to apply a MA(q) process, which is an extension of MA(1) process mentioned by Liao and Chen (2006), to capture the effect of autocorrelation. We also discuss the impact of autocorrelation on a valuation model of a reset option.

The remainder of this paper is organized as follows. In Section 2, we introduce the autocorrelation effect and formulate modified valuation models for four kinds of options with reset rights embedded. The autocorrelation impact and the types of reset options are based on the previous studies of Gray and Whaley (1997, 1999), Cheng and Zhang (2000), Liao and Wang (2002), Liao and Chen (2006). In Section 3, we demonstrate the numerical analysis to compare the difference properties between the traditional reset option model and the proposed model. Finally, Section 4 provides some concluding comments.

LITERATURE REVIEW

Besides the phenomenon of mean reversion and extreme jump, many scholars argue that crude oil prices are highly autocorrelated [Deaton and Laroque (1992), Deaton and Laroque (1996), and Chambers and Bailey (1996)]. Just like for other commodities, such high autocorrelation is basically due to time dependencies in supply and demand shocks and performances of speculators. On the other hand, consider the options with reset rights on the strike price, which is the major consideration of this paper. Commonly, the underlying assets of this kind of reset option are financial assets. There is a growing consensus that many financial asset returns can be efficiently predicted. Many scholars argue that the lagged price autocorrelation of financial asset is one source of this predictability [Lo and MacKinlay (1988); Poterba and Summers (1988); Conrad and Kaul (1988); Mech (1993); Patro and Wu (2004); Bianco and Reno (2006)]. Especially in emerging markets, many studies attribute the pervasive phenomenon of autocorrelation to irrational trading strategies, such as the feedback trading strategy [McKenzie and Faff (2003); Faff, Hiller and McKenzie (2005)]. Whether the reset clauses are on the date of maturity or the strike price, they are designed to protect the holders from the uncertainty of underlying assets. However, once the predictability of the underlying assets is significant, it can be expected that decreasing uncertainty of underlying assets would affect the value of the reset right.

Existing reset option valuation models fail to consider autocorrelation characteristic of underlying assets. Like the Black-Scholes model, reset option valuation models are based on the assumption that the stock prices follows a geometric Brownian motion process, implying that stock returns are independent. Lo and Wang (1995) argue that predictability of asset returns makes the price of options based on the predictable underlying assets to be different from fair value under the assumption of independent stock returns. They introduce the predictability concept into the Black-Scholes model and argue that the effect on option prices critically depends on how predictability is specified in the drift. They find that if drift only depends on exogenous time-varying economic factors, an increase in predictability reduces the asset's prediction error variance and decreases option prices. If drift also depends on lagged prices, an increase in predictability can increase or decrease option values. Their conclusions are based on the assumptions that the conditional mean of asset returns doesn't depend on past prices or returns, conditional expectation of the prediction error is zero, and the unconditional variance of asset return is fixed.

Because of the convertible relationship between AR(∞) process and MA(1) process, Liao and Chen (2006) further use a first-order moving average process [MA(1) process] to extract the autocorrelation from the asset returns' first moment which introduces the predictability characteristics into the diffusion term of dynamic process of stock returns and improves the limitation of only capturing predictability in the drift term. Liao and Chen (2006) derive the valuation model of European options when underlying asset returns are autocorrelated, which is also a more flexible model than the Black-Scholes model. The major difference between Liao's model and the Black-Scholes model is volatility input. The total volatility input in Liao's model is the conditional standard deviation of continuous-compounded returns over the option's remaining life. The total volatility input in Black-Scholes model is indeed the diffusion coefficient of a geometric Brownian motion times the square root of an option's time to maturity. Liao and Chen find that the impact of autocorrelation introduced by the MA(1)-type process is significant to option values even when the autocorrelation between asset returns is weak.

The main objective of this paper is to apply a MA(q) process, which is an extension of MA(1) process mentioned by Liao and Chen (2006), to capture the effects of autocorrelation. We also discuss the impact of autocorrelation on a valuation model of a reset option. According to Liao and Chen (2006), if the asset's return is positively (negatively) correlated, the volatility input is greater (smaller) than that in traditional geometric Brownian motion. If the volatility is affected by the autocorrelation, the probability of triggering reset conditions which impact the value of reset options would also be affected.

After adjusting the valuation model of the reset option to account for autocorrelation effects, we find that when the underlying asset return is positively (negatively) autocorrelated, the value of the reset option derived from our model is higher (lower) than the value derived from other general reset valuation models. Furthermore, according to the study of Dai, Kwok, and Wu (2003), they state that the optimal reset policy of a reset option, which allows holders to discretionarily choose any timing to reset the strike price, depends on the time dependent behaviors of the expected discounted value of the at-the-money option received upon reset timing. In this paper, we further find that the autocorrelation characteristic of underlying asset returns also affects the optimal reset policy. When the autocorrelation is positive (negative), the holders of reset options tend to reset earlier (later). This paper also shows that if we consider the positive autocorrelation effect on the reset option, we can lessen the problem of hedging for the reset option, such as delta jump.

MODEL DEVELOPMENT

Dynamic Process of Autocorrelated Asset Return

Without loss of generality, this paper uses a stock as the underlying asset and assumes there is no dividend during the holding period. Assume that the current time is t_0, the maturity time is *T*, and the time to maturity is τ, where $\tau = T - t_0$ and $\tau > 0$. Instead of assuming that stock returns follow the geometric Brownian motion process, we not only follow the continuous MA(1)-type (first-order moving average process) dynamic process introduced by Liao and Chen (2006), but further extend the MA(1)-type dynamic process to the MA(*q*)-type dynamic process. We assume the dynamics process of the stock returns for all $t_0 \le t \le T$ as follows:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t + \sigma \sum_{\varphi=1}^{q} \beta_\varphi dW_{t-\varphi h} \tag{1}$$

where S_t is the stock price in the time t, dS_t / S_t is the instantaneous stock return, μ is the expected instantaneous rate of return, $\sigma > 0$ is the instantaneous standard deviation of return, $dt > 0$ is a small time interval, and $h > 0$ is a fixed, but arbitrary, small constant. q is the order of MA(*q*)-type dynamic process. W_t is a one-dimensional standard Brownian motion and $dW_{t-i}, i = 0,\ h,\ 2h, \ldots,\ qh$ are the increments of the standard Brownian motion at time $t-i$. For $\varphi = 1,\ 2,\ \ldots,\ q$, the coefficient β_φ represents the impact of the past shocks, which is assumed to satisfy $|\beta_\varphi| \le 1$.

Liao and Chen (2006) proved there exists a probability measure *Q* for the MA(1) process defined in Eq. (1). Following their study, the MA(*q*) process under the martingale probability measure *Q* can be represented as follows.

$$\frac{dS_t^Q}{S_t^Q} = rdt + \sigma dW_t^Q + \sigma \sum_{\varphi=1}^{q} I_A \cdot \beta_\varphi dW_{t-\varphi h}^Q \tag{2}$$

where $I_A = 1_{\{t_0 + qh \le t \le T\}}$. Note that when $t_0 \le T \le t_0 + qh$, the dynamic process reduces to a geometric Brownian motion. Accordingly, the Black-Scholes formula is a special case of the MA(*q*)-type option model with maturity shorter than *h*. On the other hand, if $T \ge t_0 + qh$, the dynamic process of the stock return is not identical to a geometric Brownian motion. However, we can view the dynamic process of the stock return to be driven by $(q+1)$ one-dimensional Brownian motions $W_{1,t-t_0}^Q$, $W_{2,t-t_0}^Q, \ldots,$ $W_{q+1,t-t_0}^Q$, where we make the assumption that $\{W^Q_{1,t-t_0},\ W^Q_{2,t-t_0},\ldots,\ W^Q_{q+1,t-t_0}\} \equiv \{W^Q_{t-t_0},\ I_A \cdot W^Q_{(t-h)-t_0},\ I_A \cdot W^Q_{(t-2h)-t_0},\ldots,\ I_A \cdot W^Q_{(t-qh)-t_0}\}$, given the following properties:

(i) For $t \in [t_0 + qh,\ T]$, $W^Q_{1,(t-h)-t_0} \equiv W^Q_{2,t-t_0}$, $W^Q_{2,(t-h)-t_0} \equiv W^Q_{3,t-t_0},\ldots,\ W^Q_{q,(t-h)-t_0} \equiv W^Q_{q+1,t-t_0}$, and $dW^Q_{1,t-h} = W^Q_{(t-h)-t_0+dt} - W^Q_{(t-h)-t_0} = dW^Q_{2,t},\ dW^Q_{2,t-h} = dW^Q_{3,t},\ldots,\ dW^Q_{q,t-h} = dW^Q_{q+1,t}$.

(ii) $dW^Q_{1,t},\ dW^Q_{2,t},\ldots,\ dW^Q_{q+1,t}$ and are independent, which also means the covariance $E(dW^Q_{1,t} \cdot dW^Q_{2,t} \cdot \ldots \cdot dW^Q_{q+1,t}) = 0$.

Based on the definition of $W^Q_{1,t-t_0},\ W^Q_{2,t-t_0},\ldots,\ W^Q_{q+1,t-t_0}$, the dynamic process of the stock return in Eq. (2) can be further represented as:

$$\frac{dS_t^Q}{S_t^Q} = rdt + \sigma d\left(W_1^Q + \sum_{\varphi=1}^{q} \beta_\varphi W^Q_{1+\varphi}\right)_t \tag{3}$$

Following the MA(q)-type dynamic process in Eq. (3), the Itô integral equation of stock price is:

$$S_t^Q = S_{t_0}^Q \exp\{r(t-t_0) - \frac{1}{2}\sigma^2[\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(t-\varphi h - t_0) + (t-t_0)] + \sigma\sum_{\varphi=1}^{q}(1+\beta_\varphi)W^Q_{(t-\varphi h)-t_0} + \sigma W^Q_{t-(t-qh)}\} \tag{4}$$

where $S^Q_{t_0}$ is the stock price in the time t_0 under the martingale probability measure Q, the quadratic variation of $\left(W_1^Q + \sum_{\varphi=1}^{q} \beta_\varphi W^Q_{1+\varphi}\right)_t$ equals $\int_{t_0}^{t}(1+\sum_{\varphi=1}^{q} I_{B(u)} \cdot \beta_\varphi)^2 du$, and $I_{B(u)} = 1_{\{t_0 \le u \le t-qh\}}$ is an indication variable. Furthermore, according to Girsanov's theorem, we can transform the martingale probability measure Q into probability measure R. The dynamic process of a one-dimension R-Brownian motion W_z^R can be defined as:

$$dW_z^R = \begin{cases} dW_z^Q - \sigma(1+\sum_{\varphi=1}^{q}\beta_\varphi)dz, & \forall z \in [t_0,\ T-qh] \\ dW_z^Q - \sigma dz, & \forall z \in [T-qh,\ T] \end{cases}$$

where $z = t,\ t-\varphi h$, for $\varphi = 1,\ 2,\ \ldots,\ q$. And, the solution of the stock price at time t under probability measure R can be represented by using Itô lemma as:

$$S_t^R = S_{t_0}^R \exp\{r(t-t_0) + \frac{1}{2}\sigma^2[\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(t-\varphi h - t_0) + (t-t_0)] + \sigma\sum_{\varphi=1}^{q}(1+\beta_\varphi)W^R_{(t-\varphi h)-t_0} + \sigma W^R_{t-(t-qh)}\}. \tag{5}$$

The Valuation of Standard Reset Option with Single Reset Right under the Ma(Q) Process

The reset option we mention in this subsection is a standard European-type reset option with single reset right. This type of reset option gives its holder a right to reset their strike price on only one pre-specified reset date. In the remainder of this subsection we take a standard European-type reset put with single reset right as the example of valuation. The holder of the standard European-type reset put with single reset has the right to reset the strike price to the prevailing stock price when the stock price exceeds the original strike price on the pre-specified reset date. The terminal payoff of the reset put is:

$$\begin{cases} S_t - S_T & \text{if } S_t > K,\ S_T \le S_t \quad \text{(reset on time } t\text{)} \\ K - S_T & \text{if } S_t \le K,\ S_T \le K \quad \text{(not reset on time } t\text{)} \\ 0 & \text{if } (S_t > K,\ S_T \ge S_t)\ or\ (S_t \le K,\ S_T \ge K) \end{cases}$$

where time t is the pre-specified reset date, S_t is the prevailing stock price on time t, K is the original strike price, and S_T is the stock price at the maturity. Therefore, the expected terminal value of the reset put is the sum of the expected conditional terminal payoffs, weighted by their probability of occurring, and the value of the standard European-type reset put under the martingale probability measure Q is represented as follows.

$$\begin{aligned} RP_{t_0} &= e^{-r(T-t_0)} E^Q(S_t - S_T \mid S_t > K, S_T < S_t) \cdot P_r^Q(S_t > K, S_T < S_t) \\ &\quad + e^{-r(T-t_0)} E^Q(K - S_T \mid S_t \le K, S_T < K) \cdot P_r^Q(S_t \le K, S_T < K) \end{aligned} . \tag{6}$$

And we define

$$\begin{aligned} P_1 &= e^{-r(T-t_0)} E^Q(S_t - S_T \mid S_t > K, S_T < S_t) \cdot P_r^Q(S_t > K, S_T < S_t) \\ P_2 &= e^{-r(T-t_0)} E^Q(K - S_T \mid S_t \le K, S_T < K) \cdot P_r^Q(S_t \le K, S_T < K) \end{aligned} \tag{7}$$

where RP_{t_0} is the value of MA(q)-type standard European-type reset put at time t_0. Assume the underlying asset returns follow the MA(q)-type dynamic process described as Eq. (3), then the value of reset part of standard European-type reset put, P_1, can be represented as:

$$P_1 = S_{t_0} \cdot e^{-r(T-t)} \cdot N(d'_{1+}) \cdot N(-b'_{1-}) - S_{t_0} \cdot N(d'_{1+}) \cdot N\left(-b'_{1+}\right). \tag{8}$$

And, in the similar way, the value of non-reset part of standard European-type reset put, P_2, can be represented as:

$$P_2 = e^{-r(T-t_0)} \cdot K \cdot N_2\left(-d'_{1-}, -d'_{2+}, \rho\right) - S_{t_0} \cdot N_2\left(-d'_{1+}, -d'_{2-}, \rho\right). \tag{9}$$

Therefore, the value of MA(q)-type standard European-type reset put can be represented as:

$$
\begin{aligned}
RP_{t_0} &= P_1 + P_2 \\
&= S_{t_0} \cdot e^{-r(T-t)} \cdot N(d'_{1+}) \cdot N(-b'_{1-}) - S_{t_0} \cdot N(d'_{1+}) \cdot N\left(-b'_{1+}\right) \\
&\quad + e^{-r(T-t_0)} \cdot K \cdot N_2\left(-d'_{1-}, -d'_{2+}, \rho\right) - S_{t_0} \cdot N_2\left(-d'_{1+}, -d'_{2-}, \rho\right)
\end{aligned} \tag{10}
$$

where t is the reset date of the reset put, r is the risk-less interest rate, $N(a)$ is a cumulative univariate normal distribution function with upper integral limit a. $N_2(c,d,\rho)$ is a cumulative bivariate normal distribution function of c and d with covariance ρ and, the other parameters as follows.

The valuation model of standard European-type reset put mentioned in the study of Gray and Whaley (1997, 1999) can be viewed as the special case of our valuation model when $\beta_\varphi = 0$ and $h = 0$, which means the underlying asset return is independent.

<u>The Valuation of a Standard Reset Option with Multiple Reset Rights Under the Ma(Q) Process</u>

In this subsection, we extend the valuation model of standard European-type reset options to a more generalized formula, which has multiple reset times, only one of which the holders can choose. According to the valuation model for this type of reset mentioned by Cheng and Zhang (2000), we assume a standard European-type reset put with n reset times $0 < t_1 < t_2 < \ldots < t_n < T$, and we define $t_0 = 0$, $T = t_{n+1}$. The holder of this reset put has the right to reset the strike price to the prevailing stock price when the stock price exceeds the original strike price on the pre-specified reset dates. The terminal payoff of the reset put is:

$$
\begin{cases}
S_{t_i} - S_T & \text{if } S_{t_i} = Max[K, S_{t_1}, S_{t_2}, \ldots, S_{t_n}],\ S_T \le S_{t_i} \qquad \text{(reset on time } t_i\text{)} \\
K - S_T & \text{if } K = Max[K, S_{t_1}, S_{t_2}, \ldots, S_{t_n}],\ S_T \le K \qquad \text{(not reset on time } t_i\text{)} \\
0 & \text{if } (S_{t_i} = Max[K, S_{t_1}, S_{t_2}, \ldots, S_{t_n}] > K,\ S_T \ge S_{t_i}) \text{ or } (K = Max[K, S_{t_1}, S_{t_2}, \ldots, S_{t_n}],\ S_T \ge K)
\end{cases}
$$

And, the value of the MA(q)-type standard European-type reset put with n reset times at the time t_0 is represented as follows.

$$
NRP_{t_0} = e^{-r(T-t_0)} E\{Max[K, S(t_1), S(t_2), \ldots, S(t_n), S(T)]\}^+ = NP_1 + NP_2 \tag{11}
$$

where $NP_1 = e^{-r(T-t_0)} \sum_{i=1}^{n} \{E[S(t_i) - S(T)] \cdot I_{\{S(t_i) = Max[K, S(t_1), S(t_2), \ldots, S(t_n), S(t_{n+1})]\}}\}$ is the value of reset part of reset put if the holder reset on any of the pre-specified reset time t_i, and $NP_2 = e^{-rT} \{E[K - S(T)] \cdot I_{\{K = Max[S(t_1), S(t_2), \ldots, S(t_n), S(t_{n+1})]\}}\}$ is the value of non-reset part of reset put. $S(t_1), \ldots, S(t_n)$ are stock prices at the reset times $t_1,\ t_2,\ \ldots,\ t_n$.

$$d'_{1\pm} = \frac{\ln(\frac{S_{t_0}}{K}) + r(t-t_0) \pm \frac{1}{2}\sigma^2[\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(t-\varphi h - t_0) + (t-t_0)]}{\sigma\sqrt{\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(t-\varphi h - t_0) + (t-t_0)}},$$

$$b'_{1\pm} = \frac{r(T-t) \pm \frac{1}{2}\sigma^2[\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(T-\varphi h - t) + (T-t)]}{\sigma\sqrt{\sum_{\varphi=1}^{q}(1+\beta_\varphi)^2(T-\varphi h - t) + q(T-t)}},$$

$$d'_{2\pm} = \frac{\ln(\frac{S_{t_0}}{K}) + r(T-t_0) \mp \frac{1}{2}\sigma^2[\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(T-\varphi h - t_0) + (T-t_0)]}{\sigma\sqrt{\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(T-\varphi h - t_0) + (T-t_0)}},$$

$$\rho = \frac{t-qh-t_0}{\sqrt{\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(t-\varphi h - t_0) + (T-t_0)} \cdot \sqrt{\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(T-\varphi h - t_0) + (T-t_0)}}.$$

Again, assume the underlying asset returns follow the MA(q)-type dynamic process described as Eq. (3), then

$$\begin{aligned} NRP_{t_0} = \sum_{i=1}^{n} \{ & S_{t_0} e^{r(t_i - t_0)} \cdot N(C_0, C_1, C_2, \ldots, C_{i-1}; \Sigma_i) \cdot [e^{-r(T-(t_i - t_0))} N(\hat{\hat{\hat{C}}}_{i+1}, C_{i+2}, \ldots, C_{n+1}; \hat{\Sigma}_i) \\ & - e^{-rt_i} \cdot \overline{N}(C_{i+1}, C_{i+1}, \ldots, C_{n-1}; \overline{\Sigma}_i)]\} + Ke^{-rT} \cdot N(D_{1+}, D_{2+}, \ldots, D_{(n+1)+}; \tilde{\Sigma}_i) \\ & - S_{t_0} \cdot e^{-rt_0} \cdot N(D_{1-}, D_{2-}, \ldots, D_{(n+1)-}; \dot{\Sigma}_i) \end{aligned} \qquad (12)$$

where $N(C_0, \ldots, C_{i-1}, \Sigma_i)$, $N(\hat{\hat{C}}_{i+1}, \ldots, C_{n+1}, \hat{\Sigma}_i)$, $\overline{N}(C_{i+1}, \ldots, C_{n+1}, \overline{\Sigma}_i)$, $N(D_{1+}, \ldots, D_{(n+1)+}, \tilde{\Sigma}_i)$, and $N(D_{1-}, \ldots, D_{(n+1)-}; \dot{\Sigma}_i)$ are the cumulative multivariate normal distribution functions with covariance Σ_i, $\overline{\Sigma}_i$, $\hat{\Sigma}_i$, $\tilde{\Sigma}_i$, $\dot{\Sigma}_i$ respectively. And, the other parameters as follows.

$$C_0 = \frac{\ln(\frac{S_{t_0}}{K}) + r(t_i - t_0) + \frac{1}{2}\sigma^2[\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(t_i - \varphi h - t_0) + (t_i - t_0)]}{\sigma\sqrt{\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(t_i - \varphi h - t_0) + (t_i - t_0)}},$$

$$C_k = \frac{\{r + \frac{1}{2}\sigma^2 \cdot [1 + \sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)]\} \cdot (t_i - t_k)}{\sigma\sqrt{[1 + \sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)] \cdot (t_i - t_k)}}, \qquad \forall k = 1, 2, \ldots, i-1,$$

$$\hat{C}_k = \frac{\{r - \frac{1}{2}\sigma^2 \cdot [1 + \sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)]\} \cdot (t_i - t_k)}{\sigma\sqrt{[1 + \sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)] \cdot (t_k - t_i)}}, \qquad \forall k = i+1, i+2, \ldots, (n+1),$$

$$D_{i\pm} = \frac{-\ln(\frac{S_{t_0}}{K}) - r(t_i - t_0) \pm \frac{1}{2}\sigma^2[\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(t_i - \varphi h - t_0) + (t - t_0)]}{\sigma\sqrt{\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(t_i - \varphi h - t_0) + (t - t_0)}}, \qquad \forall i = 1, 2, \ldots, (n+1).$$

Also, the valuation model in this subsection provides more flexibility than the model derived by Cheng and Zhang (2000). The model of Cheng and Zhang (2000) is the special case of the model developed here when $\beta_\varphi = 0$ and $h = 0$, which means the underlying asset return is independent.

<u>The Valuation of Reset Option with *M* Reset Level and Continuous Time Under the Ma(Q) Process</u>

In practice, a European-type reset option with *m* reset level is a more common instrument than a European-type standard reset option we mentioned before. In this subsection we apply the MA(*q*) process to the valuation of a European-type reset option with *m* reset levels which has been derived by Liao and Wang (2002). For example, a European-type reset call with *m* reset levels $D_1, D_2, \ldots, D_m$ has a set of reset strike prices $K_1, K_2, \ldots K_m$ and the original strike price $K = K_0$. The trigger condition of reset is that, if the minimum stock price during the period of $[0,\ T_0]$ or what we call the monitoring window falls in the pre-specified reset intervals, the strike price can be reset to corresponding strike price. The terminal payoff of this type of reset call can be represented as:

$$\begin{cases} S_T - K^*, \text{ where } K^* = K_i & \text{if } D_i \geq \underset{t \leq s \leq T_0}{Min}\ S(s) > D_{i+1},\ S_T \geq K_i,\ for\ i = 0, \ldots, m \\ 0 & \text{if } D_i \geq \underset{t \leq s \leq T_0}{Min}\ S(s) > D_{i+1},\ S_T \leq K_i,\ for\ i = 0, \ldots, m \end{cases}$$

Based on the concept of partial barrier options and the martingale method, Liao and Wang (2002) mentions that the European-type reset call with *m* reset levels can be viewed as the combination strategy of vanilla call and down-and-out call, which can be inferred as:

$$\begin{aligned} C_t^{\mathrm{Re}set} &= e^{-r(T-t)} Max\left[S(T) - K^*\right]^+ \\ &= e^{-r(T-t)} \sum_{i=0}^{m} [S(T) - K_i]^+ \{I[\underset{t \leq s \leq T_0}{Min}\ S(s) \geq D_{i+1}] - I[\underset{t \leq s \leq T_0}{Min}\ S(s) \geq D_i]\} \\ &= S(t)N[d_+(K_m, \tau)] - K_m e^{-r\tau} N[d_-(K_m, \tau)] + \sum_{i=1}^{m}(DOC_t^{i-1,i} - DOC_t^{i,i}) \end{aligned} \qquad (13)$$

where $C_t^{\mathrm{Re}set}$ is the value of MA(*q*)-type reset call with *m* reset levels at the time t, $S(t)$ is the stock price at the time t. $DOC_t^{j,i}$ refers to the down-and-out call with strike price K_j and barrier level D_i, and $\tau = (T - t)$ is time to maturity. $N(a)$ is a cumulative univariate normal distribution function with upper integral limit *a*. Therefore, it is straightforward to see that the European-type reset call with *m* reset levels and continuous reset date with reset period $\lambda = (T_0 - t)$ less than time to maturity τ can be replicated with the following trading strategies: (1) Purchase one unit of European call option with strike price K_m; (2) Purchase one unit of European down-and-out call option with strike price , K_{i-1}, barrier D_i, $i = 0,...,m$, for each *i*; (3) Short sell one unit of European down-and-out call option with strike price K_i, barrier D_i, $i = 0,...,m$, for each *i*. The valuation model of a European call option with strike price K_m under MA(1) process has been derived by Liao and Chen (2006). If we extend to the MA(*q*) process, the call price can be represented as:

$$C_t = S(t)N[d_+(K_m,\tau)] - K_m e^{-r\tau} N[d_-(K_m,\tau)]$$

where

$$d_\pm(K_m,\tau) = \frac{\ln(\frac{S_{t_0}}{K_m}) + r\tau \pm \frac{1}{2}\sigma^2[\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(\tau - \varphi h) + \tau]}{\sigma\sqrt{\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(\tau - \varphi h) + \tau}}.$$

Also, based on the MA(*q*) process and the martingale method, the valuation model of a MA(*q*)-type European down-and-out call option with strike price K_j and barrier D_i, where $j = i-1 \text{ or } i$ and $i = 0,...,m$, can be represented as follows.

$$\begin{aligned} DOC_t^{j,i} &= S(t)N_2[d_+(K_j,\tau), d_+(D_i,\lambda), \rho'] - K_j e^{-r\tau} N_2[d_-(K_j,\tau), d_-(D_i,\lambda), \rho'] \\ &- \{S(t)(\frac{D_i}{S(t)})^{1+\frac{2r}{\sigma^2}} N_2[g_+(D_i,K_j), h_+(D_i), \rho'] - K_j e^{-r\tau}(\frac{D_i}{S(t)})^{-1+\frac{2r}{\sigma^2}} N_2[g_-(D_i,K_j), h_-(D_i), \rho']\} \end{aligned} \tag{14}$$

where $N_2(c,d,\rho')$ is a cumulative bivariate normal distribution function of *c* and *d* with covariance ρ'. And, the other parameters as follows.

$$d_\pm(D,\lambda) = \frac{\ln(\frac{S(t)}{D}) + r\lambda \pm \frac{1}{2}\sigma^2\left[\sum_{\varphi=1}^{q}\left(2\beta_\varphi + \beta_\varphi^2\right)\left(\lambda - \varphi h\right) + \lambda\right]}{\sigma\sqrt{\sum_{\varphi=1}^{q}\left(2\beta_\varphi + \beta_\varphi^2\right)\left(\lambda - \varphi h\right) + \lambda}},$$

$$g_\pm(D,K) = \frac{\ln(\frac{D^2}{K \cdot S(t)}) + r\tau \pm \frac{1}{2}\sigma^2[\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(\tau - \varphi h) + \tau]}{\sigma\sqrt{\sum_{\varphi=1}^{q}(2\beta_\varphi + \beta_\varphi^2)(\tau - \varphi h) + \tau}},$$

$$h_{\pm}(D)=\frac{\ln(\frac{S(t)}{D})+r\lambda\pm\frac{1}{2}\sigma^{2}[\sum_{\varphi=1}^{q}(2\beta_{\varphi}+\beta_{\varphi}^{2})(\lambda-\varphi h)+\lambda]}{\sigma\sqrt{\sum_{\varphi=1}^{q}(2\beta_{\varphi}+\beta_{\varphi}^{2})(\lambda-\varphi h)+\lambda}},$$

$$\rho'=\frac{\lambda-qh}{\sqrt{\sum_{\varphi=1}^{q}(2\beta_{\varphi}+\beta_{\varphi}^{2})(\tau-\varphi h)+\tau}\cdot\sqrt{\sum_{\varphi=1}^{q}(2\beta_{\varphi}+\beta_{\varphi}^{2})(\lambda-\varphi h)+\lambda}}.$$

Straightforwardly, when $\beta_{\varphi}=0$ and $h=0$, which means the underlying asset return is independent, the valuation model in this subsection can be reduced to the valuation model of a European-type reset call with *m* reset levels and continuous reset date derived by Liao and Chen (2006).

NUMERICAL ANALYSES OF MA(1)-TYPE RESET OPTIONS

Effects on the Value of Reset Options

To capture the effect of autocorrelated underlying asset returns on the value of reset options, we make some numerical analysis to compare the difference between a MA(*q*)-type reset option and a non-autocorrelated reset option. For the simplification, we consider the MA(*q*)-type reset option with order $q=1$. First, assume there are four MA(1)-type standard reset puts, and each reset put has a single reset date on $t=0.5$ year from the initiation of contract. Assume these four MA(1)-type standard reset puts have the same original strike price $K_0=\$40$, risk-free rate $r=5\%$, volatility of the underlying asset $\sigma=40\%$, time to maturity $T-t_0=1$ year, and autocorrelation persistence period $h=1/12$ year. But these four MA(1)-type standard reset puts have different autocorrelation coefficients $\beta=0.75,\ 0.5,\ -0.1,\ -0.2$. Fig.1 demonstrates the value comparison between the non-autocorrelate $(\beta=0)$ standard reset put and four MA(1)-type standard reset puts. Different from a vanilla put, Fig. 1 shows that the reset characteristics makes the non-autocorrelated standard reset put and the MA(1)-type standard reset puts have the U-shaped behavior. From the Eq. (10), the MA(1)-type standard reset put with single reset right, we know that the value of the put (non-reset part) with the initial strike price, P_2, decreases when the stock price rises. But, the increasing stock price also increases the probability of reset and makes value of reset part, P_1, increases. This reset premium makes the reset put have the U-shaped behavior. To discuss the impact of autocorrelation of underlying asset returns on the reset put, the Eq. (10) can be further represented as follows.

$$\begin{aligned}RP_{t_0}&=P_1+P_2\\&=S_{t_0}\cdot e^{-r(T-t)}\cdot\{N(d_{2+})+[N(d'_{1+})-N(d_{2+})]\}\cdot\{N(-b_{2-})+[N(-b'_{1-})-N(-b_{2-})]\}\\&\quad-S_{t_0}\cdot\{N(d_{2+})+[N(d'_{1+})-N(d_{2+})]\}\cdot\{N(-b_{2+})+[N(-b'_{1+})-N(-b_{2+})]\}\\&\quad+e^{-r(T-t_0)}\cdot K\cdot\{N_2(-d_{2-},-d_{3+},\rho^*)+[N_2(-d'_{1-},-d'_{2+},\rho)-N_2(-d_{2-},-d_{3+},\rho^*)]\}\\&\quad-S_{t_0}\cdot\{N_2(-d_{2+},-d_{3-},\rho^*)+[N_2(-d'_{1+},-d'_{2-},\rho)-N_2(-d_{2+},-d_{3-},\rho^*)]\}\end{aligned}\tag{17}$$

where

$$d_{2\pm}=\frac{\ln(\frac{S_{t_0}}{K})\pm(r+\frac{1}{2}\sigma^2)(t-t_0)}{\sigma\sqrt{t-t_0}}\quad,\qquad b_{2\pm}=\frac{\left(r\pm\frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}},\qquad d_{3\pm}=\frac{\ln(\frac{S_{t_0}}{K})+\left(r\mp\frac{1}{2}\sigma^2\right)\lambda}{\sigma\sqrt{\lambda}},$$

$$\rho^*=\sqrt{\frac{t-t_0}{T-t_0}}.$$

From the Eq. (17), we know that when the underlying asset return is not autocorrelated, $N(d_{2+})$, $N(-b_{2-})$ and $N(-b_{2+})$ represent the probability that the reset put is reset and in the money at the maturity. $N_2(-d_{2-},-d_{3+},\rho^*)$ and $N_2(-d_{2+},-d_{3-},\rho^*)$ represent the probability that the reset put is not reset and become a vanilla in-the-money put at the maturity. After introducing the autocorrelation effect into the reset put, the probability of reset is affected. Even if the reset put is not reset, the autocorrelation effect still affects the probability that the reset put is in the money at the maturity.

Compared to the non-autocorrelated standard reset put, Figure 1 demonstrates that when the autocorrelation of underlying asset returns is positive (negative) which means $\beta>0$ ($\beta<0$), the values of the MA(1)-type standard reset puts have higher (lower) values than that of the non-autocorrelated standard reset put. Because the positive (negative) shock introduced by MA(1)-type process increases (decreases) the volatility of underlying asset returns, which also increases (decreases) the probability of reset. Due to the increasing (decreasing) probability of reset, a MA(1)-type standard reset put has more (less) reset premium than the non-autocorrelated standard reset put. As the degree of positive (negative) autocorrelation increases, the increasing (decreasing) probability of reset makes the difference between the MA(1)-type standard reset put and the non-autocorrelated standard reset put further increase.

Figure 1: Value Comparison between Non-autocorrelated Standard Reset Put and MA(1)-type Standard Reset Puts when Time to Maturity is One Year

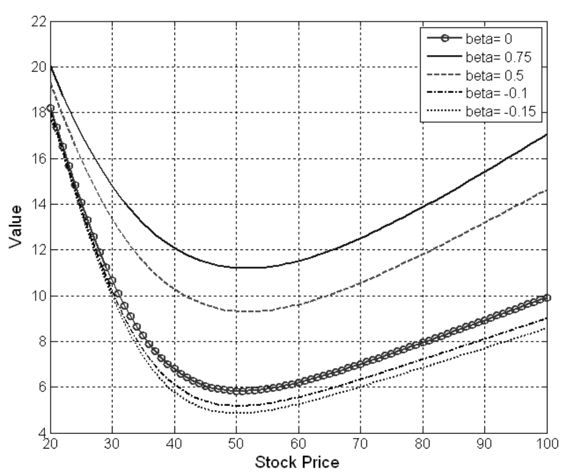

Figure 1 shows the value of different standard reset puts with $\beta=0.75,\ 0.5,\ 0, -0.1, -0.15$ *under different stock price. Assume reset date is* $t=0.5$ *year from buying the option, original strike price is* $K_0=\$40$, *risk-free rate is* $r=5\%$, *volatility of the underlying asset is* $\sigma=40\%$, *time to maturity is* $T-t_0=1$ *year, and autocorrelation persistence period is* $h=1/12$ *year. We find that the reset option with positive autocorrelation has a higher (lower) value than the non-autocorrelated reset option.*

Compared to the MA(1) process, if the past shocks are in the same direction (all $\beta_\varphi > 0$ or all $\beta_\varphi < 0$, where $\varphi = 1,\ 2,\ \ldots,\ q$), the impact of the positive or negative autocorrelation captured by the MA(*q*) process on the reset put is augmented. However, if the past shocks are in different directions (some $\beta_\varphi > 0$, and some $\beta_\varphi < 0$), the impact of autocorrelation would be uncertain. Fig. 2 demonstrates this phenomenon between the MA(1)-type reset put and the MA(2)-type reset put. When the β_1 and β_2 are all positive (negative), the probability of reset would be much higher (lower), and the value of reset put would increase (decrease) more. However, if the signs of β_1 and β_2 are different, the interaction of β_1 and β_2 would have a uncertain impact on the reset put, which means the value of reset put might be increase or decrease.

Figure 1: Value Comparison among Non-autocorrelated Standard Reset Put, MA(1)-type Standard Reset Puts, and MA(2)-type Standard Reset Puts when Time to Maturity is One Year

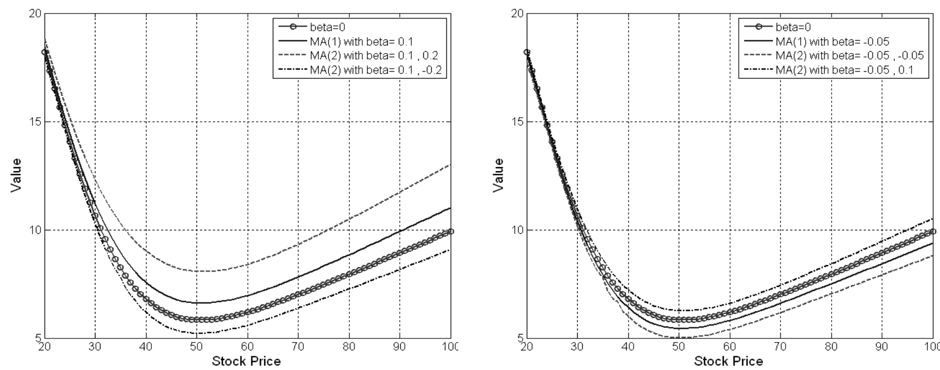

Figure 2 shows the values of non-autocorrelated reset put ($\beta = 0$), MA(1)-type reset puts ($\beta_1 = 0.1,\ -0.05$), and MA(2)-reset puts ($(\beta_1,\ \beta_2) = (0.1,\ 0.2),\ (0.1,\ -0.2),\ (-0.05,\ -0.05), (-0.05,\ 0.1)$) under different stock price. Assume reset date is $t = 0.5$ year from buying the option, original strike price is $K_0 = \$40$, risk-free rate is $r = 5\%$, volatility of the underlying asset is $\sigma = 40\%$, time to maturity is $T - t_0 = 1$ years, and autocorrelation persistence period is $h = 1/365$ year. Compared to the MA(1)-type reset put, the value of the MA(2)-type reset put becomes much larger (smaller) when both β_1 and β_1 are positive (negative). However, if the signs of β_1 and β_1 are different, the impact of autocorrelation would be uncertain.

Moreover, we find that the effect of autocorrelation characteristics on other type of reset options, such as a reset option with *m* reset levels, is consistent with the effect on standard reset options. Consider five reset calls with three reset levels which are the common practical cases. Assume the original strike price is $K_0 = \$100$, risk-free rate is $r = 5\%$, three reset levels are $(D_1, D_2, D_3) = (80, 70, 60)$, time to maturity is $T - t_0 = 1$ year, autocorrelation persistence period is $h = 1/365$ year, and the autocorrelation is $\beta = -0.2,\ -0.1,\ 0,\ 0.25,\ 0.4$ respectively. Table 1 and Table 2 demonstrate the value comparison of these five reset calls when the reset period is $T_0 - t_0 = 1/12$ year and $T_0 - t_0 = 3/12$ year respectively, given different stock prices $S(t_0) = \$80,\ \$100,\ \$115$, volatility $\sigma = 30\%,\ 50\%$, and corresponding reset strike prices (K_1, K_2, K_3).

From Table 1 and Table 2, we first find that if the autocorrelation of underlying asset return is positive, the value of the MA(1)-type reset call with three reset levels is higher than that of the non-autocorrelated reset call with the same reset levels. If the autocorrelation of underlying asset return is negative, the value of the MA(1)-type reset call with three reset levels is lower than that of the standard reset call with the same reset levels. The difference in value increases (decreases) as the autocorrelation becomes more positive (negative). Second, under the same reset levels $(D_1, D_2, D_3) = (80, 70, 60)$, the reset call has a higher value with lower reset strike prices (K_1, K_2, K_3). Third, in cases of higher stock price than reset levels and the lower volatility of stock returns, the value of a non-autocorrelated reset call reduces to the vanilla call, and the value of a MA(1)-type reset call reduces to the MA(1)-type vanilla call. Also, we find that when the duration of the reset period increase (from one month to three month), both the value of the MA(1)-type reset call and the non-autocorrelated reset call increase.

Table 1: Comparison between Non-autocorrelated Reset Calls and MA(1)-type Reset Calls, with Three Reset Levels and One Month Reset Period

σ	$S(t_0)$	(K_1, K_2, K_3)	Degree of Autocorrelation				
			$\beta = -0.2$	$\beta = -0.1$	$\beta = 0$	$\beta = 0.25$	$\beta = 0.4$
30%	85	(80,70,60)	7.0372	8.4260	9.7863	13.0876	15.0047
		(85,75,65)	6.1443	7.4288	8.6996	11.8306	13.6751
		(90,80,70)	5.4349	6.6145	7.7947	10.7486	12.5146
	100	(80,70,60)	11.9738	13.1254	14.2978	17.3279	19.2051
		(85,75,65)	11.9705	13.1157	14.2761	17.2473	19.0726
		(90,80,70)	11.9680	13.1079	14.2579	17.1783	18.9577
	115	(80,70,60)	22.9370	23.8748	24.8644	27.4917	29.1423
		(85,75,65)	22.9370	23.8748	24.8643	27.4902	29.1370
		(90,80,70)	22.9370	23.8748	24.8643	27.4888	29.1324
50%	85	(80,70,60)	14.3718	16.4713	18.5189	23.4302	26.2379
		(85,75,65)	13.0191	15.0578	17.0588	21.9058	24.7027
		(90,80,70)	11.8451	13.8130	15.7592	20.5241	23.3008
	100	(80,70,60)	18.4484	20.5733	22.7227	28.1105	31.2999
		(85,75,65)	18.3210	20.3795	22.4563	27.6632	30.7569
		(90,80,70)	18.2114	20.2093	22.2196	27.2584	30.2617
	115	(80,70,60)	28.4268	30.2935	32.2105	37.1745	40.2297
		(85,75,65)	28.4223	30.2811	32.1838	37.0819	40.0820
		(90,80,70)	28.4185	30.2703	32.1601	36.9982	39.9474

Table 1 shows the value of different reset calls with different stock price $S(t_0) = \$85,\ \$100,\ \$115$*, different reset strike price* $(K_1, K_2, K_3) = (80,70,60),\ (85,75,65),\ (90,80,70)$*, and different volatility* $\sigma = 30\%,\ 50\%$*, given* $\beta = -0.2,\ -0.1,\ 0, 0.25,\ 0.4$*. Assume original strike price is* $K_0 = \$40$*, risk-free rate is* $r = 5\%$*, time to maturity is* $T - t_0 = 1$ *year, reset period is* $T_0 - t_0 = 1/12$ *year, and autocorrelation persistence period is* $h = 1/365$ *year.*

Table 2: Comparison between Non-autocorrelated Reset Calls and MA(1)-type Reset Calls, with Three Reset Levels and Three Months Reset Period

σ	$S(t_0)$	(K_1, K_2, K_3)	Degree of Autocorrelation				
			$\beta = -0.2$	$\beta = -0.1$	$\beta = 0$	$\beta = 0.25$	$\beta = 0.4$
30%	85	(80,70,60)	8.8218	10.3992	11.8476	15.3117	17.2997
		(85,75,65)	7.2630	8.7791	10.1582	13.4891	15.4285
		(90,80,70)	6.0653	7.4622	8.7552	11.9249	13.8003
	100	(80,70,60)	12.3346	13.7600	15.1891	18.8338	21.0373
		(85,75,65)	12.1819	13.5321	14.8773	18.3020	20.3826
		(90,80,70)	12.0757	13.3509	14.6216	17.8512	19.8191
	115	(80,70,60)	22.9514	23.9209	24.9671	27.8582	29.7426
		(85,75,65)	22.9452	23.9046	24.9330	27.7425	29.5578
		(90,80,70)	22.9409	23.8918	24.9051	27.6452	29.4000
50%	85	(80,70,60)	16.7147	18.9395	21.0248	25.9087	28.6384
		(85,75,65)	14.7320	16.9865	19.0633	23.9577	26.7194
		(90,80,70)	13.0382	15.2718	17.3220	22.1952	24.9729
	100	(80,70,60)	20.2613	22.8048	25.2422	31.1031	34.4358
		(85,75,65)	19.5642	22.0078	24.3382	29.9863	33.2357
		(90,80,70)	18.9879	21.3129	23.5391	28.9809	32.1469
	115	(80,70,60)	28.9805	31.1902	33.4455	39.2145	42.6657
		(85,75,65)	28.7919	30.9217	33.0859	38.6256	41.9590
		(90,80,70)	28.6400	30.6888	32.7688	38.0968	41.3193

Table 2 shows the value of different reset calls with different stock price $S(t_0) = \$85,\ \$100,\ \$115$*, different reset strike price* $(K_1, K_2, K_3) = (80,70,60),\ (85,75,65),\ (90,80,70)$*, and different volatility* $\sigma = 30\%,\ 50\%$*, given* $\beta = -0.2,\ -0.1,\ 0, 0.25,\ 0.4$*. Assume original strike price is* $K_0 = \$40$*, risk-free rate is* $r = 5\%$*, time to maturity is* $T - t_0 = 1$ *year, reset period is* $T_0 - t_0 = 3/12$ *year, and autocorrelation persistence period is* $h = 1/365$ *year.*

Effects on the Reset Timing of Reset Options

Recall that a reset option may allow a holder to choose reset time discretionarily, and whether the option value is maximized depends on the timing that the holder elects to exercise their reset right. Figure 3 demonstrates the optimal reset timing for a non-aotocorrelated reset put $(\beta = 0)$ and four MA(1)-type reset puts $(\beta = 0.75,\ 0.5,\ -0.04)$. Assume the initial stock price is $S_0 = \$40$, original strike price is $K_0 = \$40$, risk-free rate is $r = 5\%$, volatility of the underlying asset is $\sigma = 40\%$, time to maturity is $T - t_0 = 2$ years, and autocorrelation persistence period is $h = 1/12$ year.

Figure 2: Optimal Reset Timing Comparison between the Non-autocorrelated Standard Reset Put and MA(1)-type Standard Reset Puts

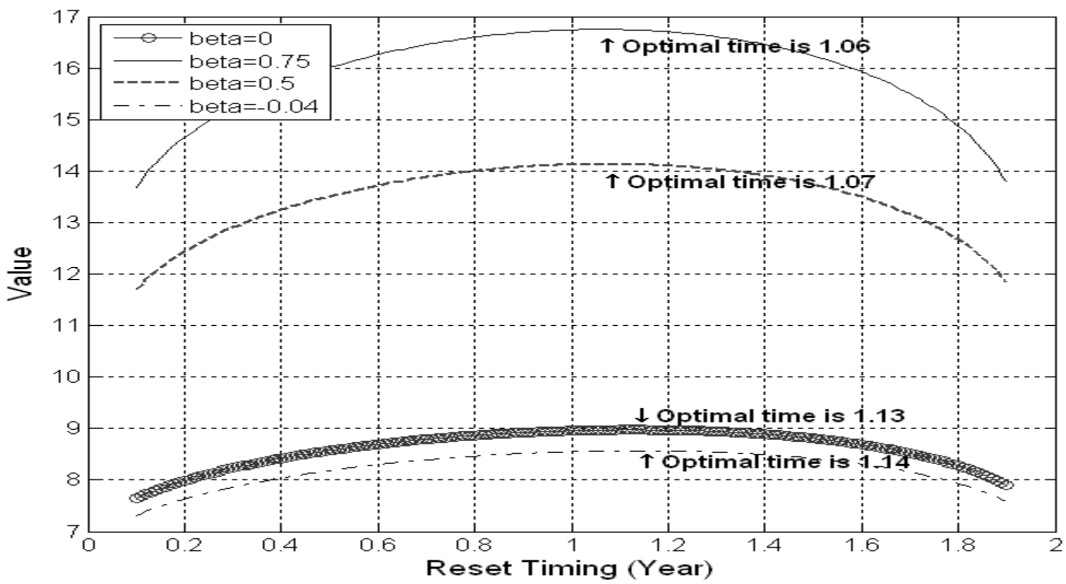

Figure 3 shows the optimal reset timing of different shout puts with $\beta = 0.75,\ 0.5,\ 0,\ -0.04$ *under different stock price. Assume initial stock price is* $S_0 = \$40$*, original strike price is* $K_0 = \$40$*, risk-free rate is* $r = 5\%$*, volatility of the underlying asset is* $\sigma = 40\%$*, and autocorrelation persistence period is* $h = 1/12$ *year. We find that the positive (negative) autocorrelation makes the optimal reset timing earlier (later).*

Figure 3 shows that, for a non-aotocorrelated reset put, the optimal reset timing is 1.13 years from the initial time of the put. However, if the underlying asset return has positive autocorrelation, the optimal reset timing for a MA(1)-type reset put is advanced. The optimal reset timings of MA(1)-type reset puts with $\beta = 0.75$ and $\beta = 0.5$ are 1.06 years and 1.07 years respectively. If the autocorrelation is positive, this positive shock makes the volatility of underlying asset returns increase. Holders of the positive MA(1)-type reset puts have more uncertainty of the underlying assets price movement. Therefore, rational holders of the MA(1)-type reset puts tend to reset earlier to protect themselves from possible loss. On the other hand, if the underlying asset return has negative autocorrelation, we find that the optimal reset timing for a MA(1)-type reset put is postponed. The optimal reset timings of MA(1)-type reset puts with $\beta = -0.04$ is 1.14 years. Because the negative autocorrelation decreases the volatility of underlying asset returns, holders of the MA(1)-type reset puts have more certainty that the underlying asset price would not change too much. Therefore, to wait for more profit, holders tend to delay their reset timing.

Figure 3: Delta Jumps of Three Reset Calls with Different Degrees of Positive Autocorrelation under Volatility 30%

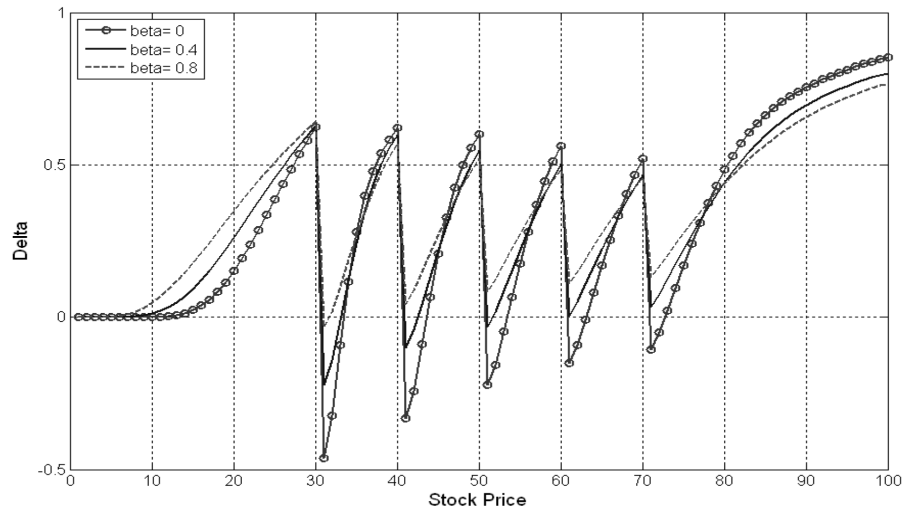

Figure 4 shows the delta jumps of three reset calls with different degree of positive autocorrelation $\beta = 0,\ 0.4,\ 0.8$ *, same five reset levels* $(D_1, D_2, D_3, D_4, D_5) = (70, 60, 50, 40, 30)$ *. Assume the original strike price is* $K_0 = \$80$ *, risk-free rate is* $r = 5\%$ *, volatility is* $\sigma = 30\%$ *, time to maturity is* $T - t_0 = 1$ *year, reset period is* $T_0 - t_0 = 1/12$ *year, autocorrelation persistence period is* $h = 1/365$ *year, and reset strike prices are* $(K_1, K_2, K_3, K_4, K_5) = (70, 60, 50, 40, 30)$ *. We find that the positive autocorrelation decrease the delta jumps problems.*

Effects on the Delta Jump and the Gamma Jump of Reset Options

Following the study of Liao and Chen (2006) and introducing MA(1)-process, the delta of the reset call with *m* reset levels can be represented as:

$$\frac{\partial C_t^{\text{Re}set}}{\partial S(t)} = N[d_+(K_m, \tau)] + \sum_{i=1}^{m} \left(\frac{\partial DOC_t^{i-1,i}}{\partial S(t)} - \frac{\partial DOC_t^{i,i}}{\partial S(t)}\right). \tag{20}$$

It is well known that the *delta jump* problem makes hedging difficult for reset options. When the holders of the reset option reset their strike prices, the reset option changes from an out-of-money option into an in-of-money option, which causes a significant change of delta (delta jump). We can also explain the phenomenon of delta jump from Eq. (20). Because the strike prices of $DOC_t^{i-1,i}$ and $DOC_t^{i,i}$ are different, $\left(\frac{\partial DOC_t^{i-1,i}}{\partial S(t)} - \frac{\partial DOC_t^{i,i}}{\partial S(t)}\right)$ would not equal zero. Therefore, if the stock price touches any of the reset barriers, the delta jump happens. For example, consider three reset calls with the same contract terms, except for the different degrees of autocorrelation $\beta = 0,\ 0.4,\ 0.8$. Assume these three reset calls have the same initial original strike price $K_0 = \$80$, five reset levels $(D_1, D_2, D_3, D_4, D_5) = (70, 60, 50, 40, 30)$, risk-free rate $r = 5\%$, five corresponding reset strike prices $\left(K_1, K_2, K_3, K_4, K_5\right) = (70, 60, 50, 40, 30)$, time to maturity $T - t_0 = 1$ year, and reset period $T_0 - t_0 = 1/12$ year. Figure 4 and Table 3 demonstrate the comparisons of delta jump between the non-autocorrelated reset call and two positive autocorrelation reset calls under volatility $\sigma = 30\%$. Figure 5 and Table 4 demonstrate the comparisons of delta jump between the non-autocorrelated reset call and two negative autocorrelation reset calls under a higher volatility $\sigma = 30\%$.

Table 3: Delta Gap Comparison of Three Reset Calls with Different Degree of Positive Autocorrelation under Volatility 30%

	Stock Price				
	30	40	50	60	70
$\beta = 0$	1.0869	0.9522	0.8237	0.7144	0.6277
$\beta = 0.4$	0.8498	0.6980	0.5818	0.4981	0.4361
$\beta = 0.8$	0.6737	0.5297	0.4383	0.3745	0.3268

Table 3 shows that, under the lower volatility $\sigma = 30\%$, as the degree of autocorrelation increases, delta gaps of MA(1)-type reset calls with positive autocorrelation become more lower than that of the non-autocorrelated reset call.

Figure 4: Delta Jumps of Three Reset Calls with Different Degrees of Negative Autocorrelation under Volatility 30%

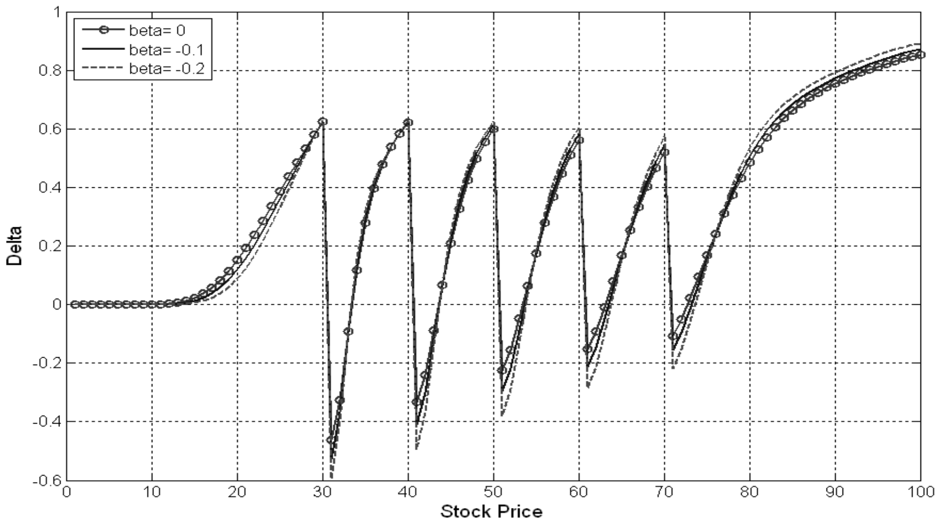

Figure 5 shows the delta jump of three reset calls with different degree of negative autocorrelation $\beta = 0$, -0.1, -0.2, same five reset levels $(D_1, D_2, D_3, D_4, D_5) = (70, 60, 50, 40, 30)$. Assume the original strike price is $K_0 = \$80$, risk-free rate is $r = 5\%$, volatility is $\sigma = 30\%$, time to maturity is $T - t_0 = 1$ year, reset period is $T_0 - t_0 = 1/12$ year, autocorrelation persistence period is $h = 1/365$ year, and reset strike prices are $(K_1, K_2, K_3, K_4, K_5) = (70, 60, 50, 40, 30)$. We find that the negative autocorrelation makes the delta jumps more serious than the non-autocorrelated reset option.

From Figure 4, we find that, whether for the non-autocorrelated reset call or MA(1)-type reset calls, the delta jumps happen whenever the stock price touches the reset barriers. However, the degree of delta jumps of MA(1)-type reset calls with positive autocorrelation is lower than that of the non-autocorrelated reset call. We measure this degree of delta jump by *delta gap* which is the difference of delta before reset timing and after reset timing. Table 3 shows that, under the lower volatility $\sigma = 30\%$, as the degree of autocorrelation increases, delta gaps of MA(1)-type reset calls with positive autocorrelation become more lower than that of the non-autocorrelated reset call. For example, when the reset barrier is \$30, the higher degree of positive autocorrelation makes the delta gap decrease from 1.0869 to 0.6737. When the stock price decreases becomming closer to the reset barrier, due to the fact that the strike price can be adjusted to a new higher level if the stock the stock price actually touches the barrier, the reset call is more valuatable. Moreover, because the MA(1)-type reset call with positive autocorrelation causes a higher probability of reset than the non-autocorrelated reset call, the value of MA(1)-type reset call is even higher than that of the non-autocorrelated reset call near the reset barrier. And, the associated delta of MA(1)-type reset call with positive autocorrelation is highr than that of the non-autocorrelated reset call near the reset barrier.

Table 4: Delta Gap Comparison of Three Reset Calls with Different Degrees of Negative Autocorrelation under Volatility 30%

	Stock Price				
	30	40	50	60	70
$\beta = 0$	1.0869	0.9522	0.8237	0.7144	0.6277
$\beta = -0.1$	1.1542	1.0328	0.9074	0.7945	0.7007
$\beta = -0.2$	1.2219	1.1221	1.0033	0.8898	0.7900

Table 4 shows the comparisons of delta jumps between the non-autocorrelated reset call and two MA(1)-type reset calls with negative autocorrelation, i.e. $\beta = -0.1$ *and* $\beta = -0.2$. *For the MA(1)-type reset calls with negative autocorrelation, because the negative autocorrelation of asset return makes the probability of reset near the barriers decrease, the value and the delta of the MA(1)-type reset calls with negative autocorrelation are less than that of the non-autocorrelated reset call. Therefore, the negative autocorrelation causes the delta gap to be larger.*

Besides the hedging difficulty of delta jump, gamma jump is another problem for hedgeing. Recall that gamma measures the rate of change in delta as the underlying stock price changes. In other words, gamma can be viewd as a measure of how poorly a dynamic delta hedge would perform when it is not rebalanced in response to a change in the asset price. Figure 6 demonstrates that gamma jumps happen whenever the stock price touches reset barriers. When the stock price decreases becomming closeser to the reset barrier, gamma tends to increase because the foreseen delta jump would make the delta change dramatically. This means that the performance of a dynamic hedge is quite poor near every reset barrier. However, the positive (negative) autocorrelation actually decreases (increases) the problem of gamma jump. The advantage of positive autocorrelation effect on the hedging of reset options is the most significant contribution of this paper.

Figure 5: Gamma Jumps of Three Reset Calls with Different Degrees of Autocorrelation under Volatility 30%

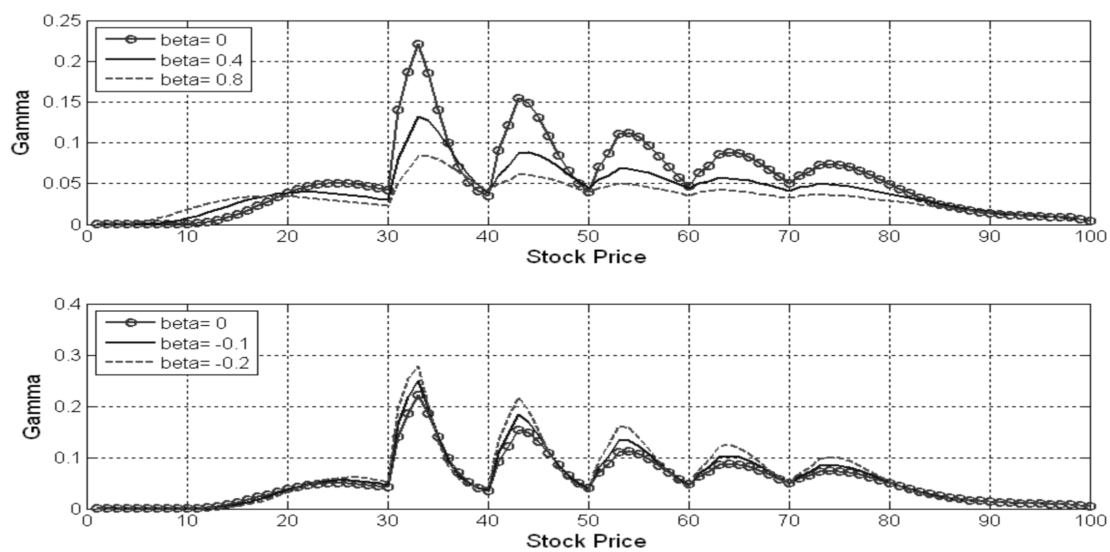

Figure 6 shows the gamma jumps of reset calls with different degrees of positive autocorrelation $\beta = 0,\ 0.4,\ 0.8$ *and negative autocorrelation* $\beta = 0,\ -0.1,\ -0.2$, *same five reset levels (*D_1, D_2, D_3, D_4, D_5*)=(70,60,50,40,30). Assume the original strike price is* $K_0 = \$80$, *risk-free rate is* $r = 5\%$, *volatility is* $\sigma = 30\%$, *time to maturity is* $T - t_0 = 1$ *year, reset period is* $T_0 - t_0 = 1/12$ *year, autocorrelation persistence period is* $h = 1/365$ *year, and reset strike prices are (*K_1, K_2, K_3, K_4, K_5*)=(70,60,50,40,30). We find that the positive autocorrelation decreases the gamma jumps and the negative autocorrelation increases the gamma jumps*

CONCLUSIONS

The main contribution of this paper is to demonstrate the impact of underlying assets' autocorrelation on reset options. The autocorrelation of asset returns is a pervasive phenomenon in the financial field. This autocorrelation characteristic affects not only the dynamic process of asset prices, but some characteristics of the reset option. We apply the MA(q) process. This process is an extension of the MA(1) process mentioned by Liao and Chen (2006) who extracted the autocorrelation of financial asset returns from the asset returns' first moment through the form of a first-order moving average process.

We develop modified models for different types of reset options with reset clause on the strike prices. For a MA(1)-type reset option, we find that the positive (negative) autocorrelation of underlying asset returns makes the volatility of underlying assets, reset probability, and the value of reset option increase (decrease). Furthermore, we find that the positive autocorrelation of underlying asset returns makes the holder of the reset option tend to reset earlier, which prevents a possible loss. To the contrary, the negative autocorrelation of underlying assets makes the holder of the reset option tend to reset later because the small volatility of the underlying asset weakens the advantage of reset. On the other hand, the effect of underlying assets' autocorrelation on hedging of the reset option is also an important contribution in this paper. When the holder of the reset option resets their strike price, the reset option changes from an out-of-money option into an in-of-money option, which causes significant changes of delta (delta jump) and gamma (gamma jump). Although the problem of delta jump and gamma jump still exist in the reset option, we find that the positive autocorrelation of underlying assets actually lessen the degree of delta jump and gamma jump.

This paper has some limitations. First, the paper only considers autocorrelation characteristics of asset returns from the asset returns' first moment through the form of a first-order moving average process. Further research, might consider other different dynamic pricing processes to capture the autocorrelation, such as ARCH or GARCH models. Instead of autocorrelation, one can also capture the predictability of underlying asset returns from other exogenous variables. One can discuss the effect of different types of asset predictability on the reset option. Second, besides considering the single stock price as the reset trigger, further research can consider the reset option using average prices as a reset trigger. This type of reset option is very common in practice. If average price is used as the reset trigger, the impact of predictability of underlying asset return might be augmented. One can discuss the effect of the predictability of asset returns on this type of reset option. Third, this paper only focuses on the reset option with reset clause on the strike price. Further research might consider other reset options with different reset clauses, such as the reset clause on the maturity date. This type of reset option gives the holder a right to postpone the maturity date of the reset option. One can discuss the effect of predictability of asset returns on this type of reset option. Last, the reset conditions in this paper are based on the underlying asset. Further research might consider other reset options whose reset conditions are related to the other particular events, such as the credit quality of a firm. For example, a reset option may be designed to allow the holders to reset the strike price if the credit rating of firm decreases to certain level. One can discuss the impact on this type of reset option if the underlying asset returns or credit quality are predictable.

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