IMPLIED INDEX AND OPTION PRICING ERRORS: EVIDENCE FROM THE TAIWAN OPTION MARKET

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ABSTRACT

This study examines both restricted and unrestricted Black-Sholes models, according to Longstaff (1995). Using the Taiwan index options for each day from January 2005 to December 2008, the unrestricted model simultaneously solves the implied index value and implied volatility whereas the restricted model only solves the implied volatility. Next, this study compares the pricing performance of restricted and unrestricted Black-Scholes models. The empirical results show he implied index value is almost higher than the actual index value. Moneyness has a significant negative impact on the index pricing error for calls but negative impact for puts. Open interest has a significantly negative impact on the index pricing error for calls. Volatility for calls has no significant effect on the index pricing error. The path-dependent effect on index pricing error increases with index returns. The unrestricted model has significantly less option pricing bias for calls than the restricted model. The option pricing error for calls in the restricted model has much larger negative bias near the middle maturity. The R-square in the restricted model is always much larger than the unrestricted model for both calls and puts. Finally, the option pricing errors are significantly affected by moneyness and time to expiration for all cases; this fact is consistent with Longstaff (1995). Additionally, based on the criterion of adjusted R-square, this study investigated the optimal explanatory variables of index pricing error.

JEL: G12; G13; G14

KEYWORDS: Index pricing error, option pricing error, Black-Scholes, implied volatility, implied index

INTRODUCTION

Under the assumptions that agents are risk neutral and rational, Samuelson (1965) proved that futures prices must be martingales with respect to the information set. According to the theory about the Martingale property and no arbitrage opportunity, Black and Scholes (1973) and Merton (1973) developed the option pricing framework. Option pricing theory can be viewed as a main pillar of modern finance theory. Harrison and Kreps (1979) showed that violations of a martingale restriction on the expected future stock price under a risk-neutral density represent arbitrage opportunities in frictionless security markets. Similarly, under the no-arbitrage condition, the value of a European option is given by its expected future payoff under a risk-neutral probability measure discounted at the riskless interest rate. Restated, in the no-arbitrage framework, the underlying asset price implied by the option pricing model must be equal to its actual market price. This means that the cost of a synthetic position via the option market is equal to the price of the underlying asset. However, because of market frictions and the unrealistic assumption in option pricing models (e.g. Black-Scholes (simply B-S)), the implied price need not equal its actual market price.

Longstaff (1995) showed that the implied index value is nearly higher than the actual index value. In this paper, we attempt to verify the results of Longstaff (1995). Additionally, following Longstaff (1995), this study examines both the restricted and unrestricted B-S models. Using the Taiwan Index Options (TXO) data for each day, the unrestricted model simultaneously solves the implied index value and implied volatility whereas the restricted model only solves the implied volatility. Naturally, the implied index need not be equal to the actual index. Having an extra parameter, however, the unrestricted model should fit the actual option prices better. There are the following reasons why TXO is examined to verify the results of Longstaff. First, American options are used as sample in Longstaff (1995) , while the B-S model is based on European options. TXO with European-style contracts therefore is more in line with the B-S assumption. Additionally, when dividends are announced, the American call option prices are usually higher than European options. On the other hand, regardless of whether there are dividends, American put option prices are always higher than European options. Hence, Longstaff only examined call options. Due to TXO with European-style contracts, we can examine both the call and put options. Moreover, unlike many index markets, there is a price limit for Taiwan market, accordingly, the result in this study may not necessarily be the same as that of previous research.

This study is organized as follows. The next section briefly discusses the relevant literature. The third section introduces implied volatility and implied index. The fourth section presents the definition and impact factors of pricing errors, including the index pricing error and option pricing error. The fifth section analyzes the empirical results, including the difference between the implied index and actual index; the pricing performance comparison of restricted and unrestricted Black-Scholes models; and the regression of option pricing errors on the moneyness and time to expiration. Finally, the sixth section concludes.

LITERATURE REVIEW

Previous studies have examined the option pricing biases in the B-S model. For example, Black and Scholes (1973) found call options are over-valued/under-valued if the stock return volatility is high/low. MacBeth and Mervile (1980) used the stock options traded on CBOE, and found that the CEV (constant elasticity of variance) model fits market prices of call options significantly better than the Black-Scholes model. Using all reported trades and quotes on the 30 most active CBOE option classes from August 23, 1976 through August 31, Rubinstein (1985) performed the nonparametric tests of alternative option pricing models. The result showed that short-maturity out-of-the-money calls are priced significantly higher relative to other calls than the Black-Scholes model, and striking price biases relative to the Black-Scholes model are also statistically significant but have reversed themselves after long periods of time.

Although implied volatility is widely believed to be informationally superior to historical volatility since it is the “market's” forecast of future volatility, Canina and Figlewski (1993) examined the S&P 100 index options traded in 1983 and found implied volatility is a poor forecast of subsequent realized volatility. They verified that implied volatility has virtually no correlation with future volatility, and it does not incorporate the information contained in recent observed volatility. Using S&P 100 index options data, Longstaff (1995) verified that the implied cost of the index is significantly higher in the options market than in the stock market, and is directly related to measures of transaction costs and liquidity. Also, the Black-Scholes model has strong bid-ask spread, trading volume, and open interest biases. The result indicates that option pricing models that relax the martingale restriction perform significantly better. Strong and Xu (1999) investigated the Longstaff's martingale restriction on S&P 500 index options over the period 1990 through 1994. Assuming the S&P index follows a lognormal distribution, the result reveals that the implied index value from options consistently overestimates the market value. However, when adopting a generalized distribution allowing for nonnormal third and fourth moments, the martingale restriction is economically insignificantly rejected.

Turvey and Komar (2006) extracted the implied market price of risk for options on live cattle futures as a test of the Martingale restriction. The results generally reject the Martingale restriction and the risk neutral hypothesis. Corrado (2007) focused on a method to impose a martingale restriction in an option pricing model developed from a Gram–Charlier and Edgeworth density expansion, and found that a martingale restriction “invisible” in the option price. By adopting the weekly observations of both put and call $/£ currency options on corresponding futures contracts for the period December 1989 to November 2000 in the empirical analysis, Busch (2008) employed the normal inverse Gaussian distribution for estimating the option implied risk neutral density, and found that the martingale restriction is in most cases satisfied. Ville-Goyets et al. (2008) empirically investigated the martingale hypothesis for agricultural futures prices, by using a nonparametric approach where the expected return depends nonparametrically on a linear index. The empirical result indicated that the Samuelson’s (1965) hypothesis is statistically rejected. Restated, the results indicate that the estimated index contains statistically significant information regarding the expected futures returns.

METHODOLOGY: IMPLIED VOLATILITY AND IMPLIED INDEX

Black and Scholes (1973) assumed that stock prices follow a lognormal distribution. If the stock price is substituted by the futures price, Black-Scholes formulas for call and put options are as follows.

$$\begin{cases} Call = e^{-rT}[F_0\,N(d_1) - K\,N(d_2)] \\ Put = e^{-rT}\left[K\,N(-d_2) - F_0\,N(-d_1)\right] \\ d_1 = \dfrac{\ln(F_0/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}},\ \ d_2 = d_1 - \sigma\sqrt{T} \end{cases} \tag{1}$$

where F_0 is the futures price at time 0, r is the risk-free interest rate, σ is the volatility of the futures return, K is the strike price, T is the expiration date, and $N(\cdot)$ is the cumulative distribution function of a standard normal variable. The theoretical option price $P = P(F_0, r, \sigma, K, T)$ based on Equation (1) implies that excluding the futures index, risk-free interest rate, strike price and expiration date, the option price is related to the magnitude of the volatility. Accordingly, by inverting the Black-Scholes formula, we can obtain the implied volatility corresponding to the restricted model. Additionally, following Longstaff (1995), we invert the B-S model to estimate both the implied index value and the implied volatility corresponding to the unrestricted model. Mathematically,

$$\text{Restricted Model: } \min_{\sigma} \sum_{i=1}^{n} (\hat{P}_i - P_i)^2 \tag{2}$$

$$\text{Unrestricted Model: } \min_{\sigma,\, F} \sum_{i=1}^{n} (\hat{P}_i - P_i)^2 \tag{3}$$

where n represents the number of option observations, P_i and $\hat{P}_i$ represent the actual option price and fitted option price of the *i*-th option. In Equation (2), $\hat{P}_i$ is obtained by using actual index as the value for F_0 in the B-S formula; but in Equation (3), $\hat{P}_i$ is obtained by using implied index as the value for F_0.

Pricing Errors

Referring to Longstaff (1995), we regress the index pricing error on moneyness, time to expiration, market liquidity, and volatility.

$$(\hat{F} - F)/F = \alpha_0 + \beta_1 M + \beta_2 T + \beta_3 AR + \beta_4 AR_{-1} + \beta_5 AR_{-2} \tag{4}$$

$$+ \beta_6 OI + \beta_7 Vol + \beta_8 N + \beta_9 R + \beta_{10} R_{-1} + \beta_{11} R_{-2} + \varepsilon$$

where $(\hat{F}-F)/F$ is index pricing error defined as the percentage difference between the implied index $\hat{F}$ and actual index *F*; *M* represents moneyness which equals *F/K* for call and *K/F* for put; *T* represents time to expiration, and ε is disturbance; AR, AR_{-1} and AR_{-2} are used as the volatility proxies, respectively denoting the current and first two lagged absolute daily returns on the index; *OI*, *Vol* and *N* are used as the liquidity proxies, respectively representing the open interest, total trading volume and number of options used to compute the implied index value for that day; R, R_{-1} and R_{-2} are employed to capture the path-dependent effects on option pricing, respectively denoting the current and first two lagged daily returns on the index. Additionally, this study will regress the option pricing errors from the restricted and unrestricted models on the moneyness and time to expiration as follows.

$$\text{Option Pricing Error} = \alpha_0 + \beta_1 \text{M} + \beta_2 \text{T} + \varepsilon \quad (5)$$

where the option pricing errors include the four indicators: $\hat{P}-P$, $|\hat{P}-P|$, $(\hat{P}-P)/P$ and $|(\hat{P}-P)/P|$.

Data

The daily call and put option prices are obtained from Taiwan Economic Journal (TEJ) with the sample period from January 2005 to December 2008. The risk-free interest rate is one-year time savings deposit interest rate by the First Bank of Taiwan. Options with expiration dates in the nearby months are actively traded and are able to reflect the most information. However, the implied volatility will change abnormally in the week before expiration. Accordingly, nearby-month options whose expiration dates are longer than 8 days are selected.

The trading hours of TXO is from 8:45 am to 1:45 pm while the trading hours in the spot market is from 9:00 am to 1:30 pm. To avoid the non-synchronous trading problem, this study estimates the daily implied volatility and implied index by using the five-minute window, 13:25 to 13:30 pm, in which options are the most actively traded. For each day, the number of contracts with no more than three is excluded. Options that violate the upper or lower boundary conditions are also eliminated, where the conditions are $(F-K)e^{-rT} \le Call \le F$ and $(K-F)e^{-rT} \le Put \le Ke^{-rT}$. The resulting data includes 239,044 call and 193,301 put options.

EMPIRICAL RESULTS

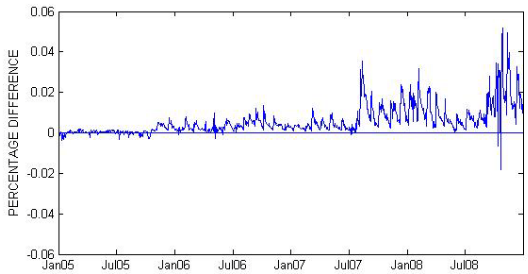
Index Pricing Error

Empirically, the Matlab grid search method is carried out to estimate the parameters. Figure 1 plots the time series of index pricing error for call and put options. As shown, like Longstaff (1995), on average, the index pricing error is positive. That is, the implied index value is almost higher than the actual index value. We can view an option as a levered position in the underlying asset, so, the cost of purchasing stock via the options market is more expensive than in the stock market.

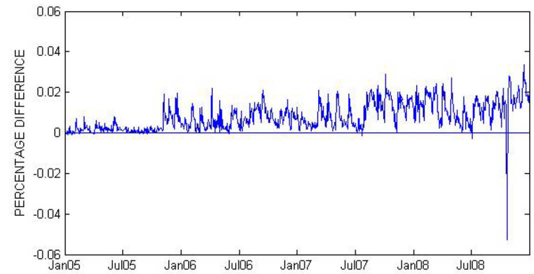
For the total sample period from 2005 to 2008, the regression result of Equation (4) are shown in Table 1. Additionally, for examining the robustness of time horizon, this study divides the total sample period into two subsamples in which the regression results of index pricing error are listed in Tables 2 and 3, respectively. The sample period for Table 2 is 2005 to 2006 and the sample period for Table 3 is 2007 to 2008. It is worth mentioning the "financial tsunami" during 2008. Accordingly, we suspect a structural change during that period of time.

Figure 1: Percentage Difference between the Implied Index and the Actual Index

Sample: Call options



Sample: Put options



This figure plots the time series of index pricing error for call and put options. The call and put option prices are obtained from Taiwan Economic Journal (TEJ).

We observe that the main results are robust for the total sample period and two subsamples, as follows. Moneyness has a significant negative impact on the index pricing error for calls; this fact is consistent with Longstaff (1995). However, moneyness has a significant positive impact on the index pricing error for puts. Next, the coefficient for the open interest is significantly negative for calls, which agrees with the result from Longstaff (1995). In terms of volatility for calls, different from Longstaff (1995), the current and first two lagged absolute daily returns on the index have no significant effect on the index pricing error. Moreover, contrary to Longstaff (1995), the path-dependent effect on index pricing error increases with index returns.

Finally, the best fitting regression model provided in Tables 1 through 3 have the largest adjusted R-square among these explanatory variables in Equation (4). The optimal explanatory variables of index pricing error exclude the volatility proxies for calls during the total sample period as well as the period of 2005 to 2006. Besides, the optimal explanatory variables additionally exclude T and R_{-2} during the period of 2007 to 2008.

For puts, however, the optimal explanatory variables of index pricing errors somewhat differ from the total sample period and the two subsamples. For instance, the optimal explanatory variables for the total sample period include M, T, AR_{-2}, OI and N. The optimal explanatory variables for the sample period of 2005 to 2006 include M, OI, Vol, N, R, R_{-1} and R_{-2}. The optimal explanatory variables for the sample period of 2007 to 2008 include M, T, AR_{-2}, OI, Vol and N.

<u>Option Pricing Errors</u>

Figure 2 shows that option pricing error from restricted model decreases with the moneyness for calls; this fact is opposite to Longstaff (1995). Like Longstaff (1995), the unrestricted model has significantly less option pricing bias for calls than the restricted model. Figure 3 displays the option pricing error for calls in the restricted model has much larger negative bias near the middle maturity. Tables 4 and 5 display the regression results of option pricing errors on the moneyness and the time to expiration.

Table 1: Regression Results of Index Pricing Error from 2005 to 2008

	Call options				Put options			
Model	**Equation (4)**		**Best fitting**		**Equation (4)**		**Best fitting**	
$Int \times 10^{-2}$	27.97	(13.03)	28.42	(15.64)	-21.32	(-12.53)	-21.39	(-12.67)
$M \times 10^{-2}$	-27.59	(-13.02)	-28.04	(-15.64)	21.15	(12.44)	21.26	(12.85)
$T \times 10^{-5}$	-6.57	(-2.16)	-6.88	(-2.50)	8.85	(3.43)	8.77	(3.40)
$AR \times 10^{-5}$	8.31	(0.38)			26.18	(1.04)		
$AR_{-1} \times 10^{-5}$	9.62	(0.42)			21.32	(0.91)		
$AR_{-2} \times 10^{-5}$	14.68	(0.80)			40.25	(2.76)	43.51	(2.59)
$OI \times 10^{-9}$	-7.00	(-6.07)	-6.92	(-6.15)	-2.81	(-1.60)	-3.88	(-2.08)
$Vol \times 10^{-9}$	-3.41	(-1.41)	-3.08	(-1.46)	-1.63	(-0.49)		
$N \times 10^{-5}$	-12.03	(-1.45)	-10.03	(-1.11)	10.52	(1.42)	11.48	(1.66)
$R \times 10^{-5}$	117.32	(5.55)	119.32	(5.99)	-0.60	(-0.05)		
$R_{-1} \times 10^{-5}$	52.92	(4.39)	53.46	(4.34)	1.99	(0.21)		
$R_{-2} \times 10^{-5}$	23.61	(1.84)	22.90	(1.77)	-0.95	(-0.10)		
R^2	0.7286		0.7282		0.6775		0.6756	
Adj R^2	0.7256		0.7260		0.6739		0.6740	

This table shows the regression result of Equation (4). Newey-West heteroscedasticity and autocorrelation robust t-statistics are shown in parentheses. Int is the regression intercept.

Table 2: Regression Results of Index Pricing Error from 2005 to 2006

	Call options		Put options	
Model	**Equation (4)**	**Best fitting**	**Equation (4)**	**Best fitting**
$Int \times 10^{-2}$	12.80 (6.79)	12.72 (7.26)	-23.13 (-9.41)	-23.77 (-11.76)
$M \times 10^{-2}$	-12.24 (-6.58)	-12.17 (-7.04)	23.09 (9.29)	23.77 (11.86)
$T \times 10^{-5}$	-3.78 (-2.24)	-3.74 (-2.29)	1.85 (0.76)	
$AR \times 10^{-5}$	-4.20 (-0.24)		7.61 (0.28)	
$AR_{-1} \times 10^{-5}$	-9.80 (-0.77)		-14.36 (-0.63)	
$AR_{-2} \times 10^{-5}$	4.20 (0.32)		10.12 (0.51)	
$OI \times 10^{-9}$	-7.60 (-8.74)	-7.61 (-8.75)	-2.53 (-1.51)	-2.64 (-1.62)
$Vol \times 10^{-9}$	2.84 (1.75)	2.70 (1.96)	8.33 (2.78)	8.70 (3.13)
$N \times 10^{-5}$	-23.70 (-2.93)	-24.36 (-3.05)	-18.52 (-1.36)	-21.67 (-1.75)
$R \times 10^{-5}$	80.43 (7.15)	80.01 (7.38)	38.78 (2.16)	35.24 (2.03)
$R_{-1} \times 10^{-5}$	47.92 (5.91)	48.58 (6.15)	54.94 (4.08)	52.87 (3.91)
$R_{-2} \times 10^{-5}$	39.89 (3.86)	40.47 (3.87)	37.89 (2.26)	36.65 (2.26)
R^2	0.4949	0.4941	0.6125	0.6108
Adj R^2	0.4833	0.4858	0.6037	0.6052

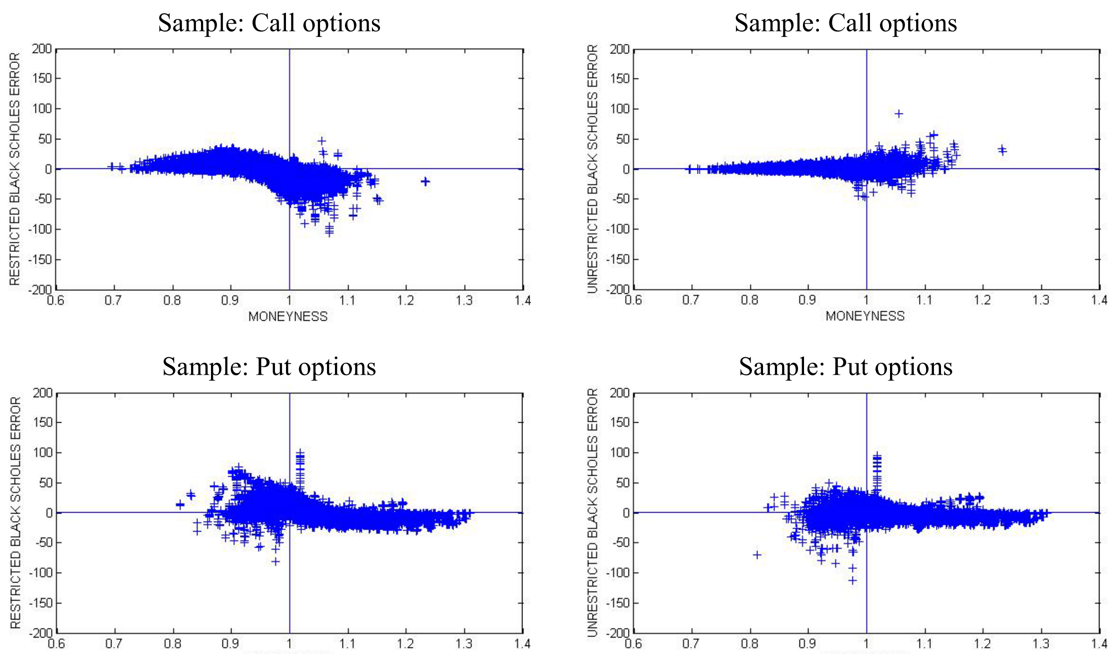
This table shows the regression result of Equation (4). Newey-West heteroscedasticity and autocorrelation robust t-statistics are shown in parentheses. Int is the regression intercept.

Table 3: Regression Results of Index Pricing Error from 2007 to 2008

	Call options				Put options			
Model	Equation (4)		Best fitting		Equation (4)		Best fitting	
$Int \times 10^{-2}$	27.39	(10.41)	28.48	(17.22)	-18.25	(-7.74)	-18.13	(-7.11)
$M \times 10^{-2}$	-27.12	(-10.50)	-28.14	(-16.64)	18.30	(7.89)	18.22	(7.48)
$T \times 10^{-5}$	2.94	(0.57)			16.88	(4.18)	16.91	(4.15)
$AR \times 10^{-5}$	16.45	(0.60)			11.64	(0.38)		
$AR_{-1} \times 10^{-5}$	13.40	(0.49)			8.65	(0.33)		
$AR_{-2} \times 10^{-5}$	18.20	(0.80)			25.92	(1.59)	27.78	(1.71)
$OI \times 10^{-9}$	-4.50	(-1.71)	-5.08	(-2.00)	-2.61	(-0.77)	-3.63	(-1.08)
$Vol \times 10^{-9}$	-11.72	(-2.26)	-10.32	(-2.39)	-10.26	(-1.72)	-9.09	(-2.46)
$N \times 10^{-5}$	-11.14	(-1.15)	-10.86	(-1.21)	9.02	(1.17)	8.81	(1.15)
$R \times 10^{-5}$	107.59	(4.04)	111.48	(4.74)	3.25	(0.21)		
$R_{-1} \times 10^{-5}$	43.39	(2.96)	44.10	(2.99)	-4.88	(-0.51)		
$R_{-2} \times 10^{-5}$	9.29	(0.60)			-5.94	(-0.59)		
R^2	0.7024		0.7005		0.6319		0.6311	
Adj R^2	0.6956		0.6968		0.6236		0.6266	

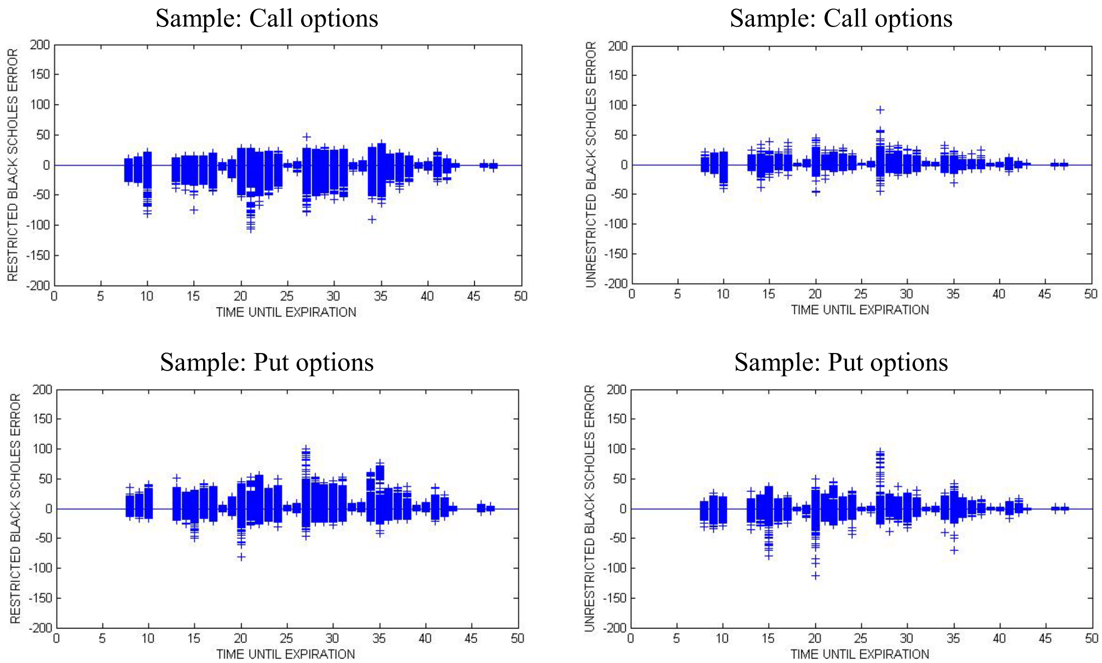
This table shows the regression result of Equation (4). Newey-West heteroscedasticity and autocorrelation robust t-statistics are shown in parentheses. Int is the regression intercept.

Figure 2: Option Pricing Error against the Moneyness of Options



The unrestricted model simultaneously solves the implied index value and implied volatility whereas the restricted model only solves the implied volatility. Option pricing error represents $\hat{P} - P$. *M is moneyness which equals F/K for call and K/F for put.*

Figure 3: Option Pricing Error against the Time to Expiration of Options



The unrestricted model simultaneously solves the implied index value and implied volatility whereas the restricted model only solves the implied volatility. Option pricing error represents $\hat{P} - P$.

Table 4: Regression of Option Pricing Errors on Moneyness (*F/K*) and Time to Expiration

Pricing Error	Est. Model	Int	M	T	R^2
Panel A: Call options					
$\hat{P} - P$	Restricted	100.81 (346.1)	-102.01 (-348.2)	-0.0702 (-50.03)	0.3373
	Unrestricted	5.84 (46.43)	-5.81 (-45.94)	-0.0041 (-6.72)	0.0088
$\lvert \hat{P} - P \rvert$	Restricted	11.18 (40.97)	-8.88 (-32.34)	0.0883 (67.11)	0.0273
	Unrestricted	-2.96 (-30.43)	4.34 (44.44)	0.0142 (30.32)	0.0101
$(\hat{P} - P)/P$	Restricted	2.49 (261.9)	-2.46 (-257.9)	-0.0018 (-39.58)	0.2182
	Unrestricted	0.0388 (7.56)	-0.0442 (-8.58)	0.0004 (16.02)	0.0017
$\lvert (\hat{P} - P)/P \rvert$	Restricted	2.76 (328.9)	-2.70 (-319.7)	-0.0024 (-59.14)	0.2996
	Unrestricted	0.9548 (219.7)	-0.91 (-209.0)	-0.0015 (-73.26)	0.1574
Panel B: Put options					
$\hat{P} - P$	Restricted	112.42 (391.1)	-111.26 (-398.1)	0.0685 (45.92)	0.4505
	Unrestricted	44.51 (218.9)	-44.48 (-225.0)	0.0363 (34.44)	0.2078
$\lvert \hat{P} - P \rvert$	Restricted	-27.43 (-100.8)	30.68 (116.0)	0.0424 (30.01)	0.0736
	Unrestricted	-15.17 (-84.72)	17.39 (99.9)	-0.0072 (-7.78)	0.0492
$(\hat{P} - P)/P$	Restricted	3.02 (444.3)	-3.10 (-469.9)	0.0047 (134.3)	0.5405
	Unrestricted	1.87 (316.7)	-1.93 (-336.1)	0.0036 (118.4)	0.3819
$\lvert (\hat{P} - P)/P \rvert$	Restricted	-2.64 (-394.5)	2.752 (423.1)	-0.0045 (-129.4)	0.4899
	Unrestricted	-1.73 (-305.5)	1.81 (328.4)	-0.0036 (-122.7)	0.3734

M is moneyness which equals F/K for call and K/F for put. The regression result is based on the ordinary least squares method. The t-statistics are shown in parentheses.

In Table 4, the moneyness is equal to *F/K* for calls and *K/F* for puts. In Table 5, the moneyness is equal to *F-K* for calls and *K-F* for puts. Whatever the explained variables are $\hat{P}-P$, $(\hat{P}-P)/P$, $|\hat{P}-P|$ and $|(\hat{P}-P)/P|$, Tables 4 and 5 show that the R-square in the restricted model is always much larger than the unrestricted model for both calls and puts. This fact infers that the option pricing errors for the unrestricted model are less affected by moneyness and time to expiration. It is worth mention that the option pricing errors are significantly affected by moneyness and time to expiration for all cases.

Table 5: Regression of Option Pricing Errors on Moneyness (F-K) and Time to Expiration

Pricing Error	Est. Model	Int	M	T	R^2
Panel A: Call options					
$\hat{P}-P$	Restricted	-1.27 (-42.20)	-0.0142 (-363.50)	-0.0749 (-54.13)	0.3568
	Unrestricted	0.02 (1.16)	-0.0009 (-53.21)	-0.0051 (-6.72)	0.0117
$\|\hat{P}-P\|$	Restricted	2.28 (79.75)	-0.0014 (-38.72)	0.0864 (65.69)	0.0292
	Unrestricted	-1.37 (-134.47)	0.0004 (31.96)	0.0131 (27.83)	0.0062
$(\hat{P}-P)/P$	Restricted	0.02 (261.9)	-0.0003 (-258.81)	-0.0019 (-40.67)	0.2193
	Unrestricted	-0.01 (9.62)	-0.0000 (-3.65)	0.0004 (17.00)	0.0014
$\|(\hat{P}-P)/P\|$	Restricted	0.06 (69.00)	-0.0004 (-319.32)	-0.0024 (-60.19)	0.2991
	Unrestricted	0.04 (89.05)	-0.0001 (-225.27)	-0.0016 (-77.73)	0.1780
Panel B: Put options					
$\hat{P}-P$	Restricted	1.14 (34.58)	0.0172 (405.76)	0.0769 (51.93)	0.4600
	Unrestricted	-0.10 (-4.08)	0.006 (192.74)	0.0358 (32.93)	0.1615
$\|\hat{P}-P\|$	Restricted	3.32 (104.95)	-0.0043 (-104.76)	0.0422 (29.65)	0.0623
	Unrestricted	2.34 (111.87)	-0.0019 (-68.77)	-0.0048 (-5.14)	0.0239
$(\hat{P}-P)/P$	Restricted	-0.09 (-109.17)	0.0005 (455.74)	0.0049 (136.99)	0.5255
	Unrestricted	-0.06 (88.72)	0.0003 (304.68)	0.0037 (115.87)	0.3384
$\|(\hat{P}-P)/P\|$	Restricted	0.12 (146.44)	-0.0004 (407.09)	-0.0046 (-131.10)	0.4710
	Unrestricted	0.08 (120.11)	-0.0003 (-297.99)	-0.003 (-120.06)	0.3310

M is moneyness which equals F-K for call and K-F for put. The regression result is based on the ordinary least squares method. The t-statistics are shown in parentheses.

CONCLUSIONS

This study tests the pricing error between the implied index and the actual index in Taiwan index options market. The paper analyzes the performance of the restricted and unrestricted Black-Sholes model in describing option prices. Using the Taiwan index options for each day from January 2005 to December 2008, the unrestricted model simultaneously solves the implied index value and implied volatility whereas the restricted model only solves the implied volatility. Next, this study compares the pricing performance of restricted and unrestricted Black-Scholes models. Finally, the regression of the option pricing error from the restricted and unrestricted models on moneyness and time to expiration is performed. The main results are summarized as follows.

First, the implied index value is almost higher than the actual index value; this implies that the cost of purchasing stock via the options market is more expensive than in the stock market. Second, moneyness has a significant negative impact on the index pricing error for calls but negative impact for puts. Third, the open interest is significantly negative impact on the index pricing error for calls. Fourth, in terms of volatility for calls, the current and first two lagged absolute daily returns on the index have no significant effect on the index pricing error. Fifth, the path-dependent effect on index pricing error increases with index returns. Six, based on the criterion of adjusted R-square, the optimal explanatory variables of index pricing error exclude the volatility proxies for calls. The optimal explanatory variables for the total sample period include *M*, *T*, AR_{-2}, *OI* and *N* for puts. Seventh, the option pricing error from restricted model decreases with the moneyness for calls. The unrestricted model has significantly less option pricing bias for calls than the restricted model. The option pricing error for calls in restricted model has much larger negative bias near the middle maturity. Eighth, the R-square in the restricted model is always much larger than the unrestricted model for both calls and puts. Finally, the option pricing errors are significantly affected by moneyness and time to expiration for all cases; this fact is consistent with Longstaff (1995).

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ACKNOWLEDGEMENT

The authors wish to thank the editor and the anonymous reviewers for their excellent comments, resulting in a significant improvement in the quality of this paper.

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