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MULTI-FACTOR APPROACH FOR PRICING BASKET CREDIT LINKED NOTES UNDER ISSUER DEFAULT RISK

Po-Cheng Wu, Kainan University

ABSTRACT

This article proposes a multi-factor approach to incorporate issuer default risk into basket credit linked note (BCLN) pricing based on the Gaussian copula. The numerical analysis demonstrates that the issuer default risk increases the fair coupon rate. Contradicting the common belief that a positive default correlation between reference entities and an issuer increases the possibility of double losses and disfavors the BCLN holder, thereby driving up the BCLN coupon rate, analytical results reveal that a positively correlated issuer default mitigates this increase, while a negatively correlated issuer default increases the coupon rate further.

JEL: G01; G12; G13

KEYWORDS: Basket credit linked notes, issuer default risk, default correlation, factor copula, financial crisis

INTRODUCTION

Multi-name credit derivatives, which linked to a portfolio of underlyings subject to credit risk, recently have become popular. Basket credit linked note (BCLN) is one such product. BCLN is a note with a price or coupon linked to credit events of reference entities (obligations). The conventional form of BCLN is the k th-to-default BCLN. The BCLN holder (the protection seller) pays the notional principal to the BCLN issuer (the protection buyer) at the start of the contract and receives the coupon payments until either the k th default or the contract maturity, whichever occurs earlier. If the k th default occurs before contract maturity, the BCLN holder receives the recovered value of the reference entity from the BCLN issuer. Otherwise, the BCLN holder receives the notional principal back on contract maturity. In derivative markets, the issuer default risk is attracting considerable attention because of the recent financial turmoil and collapses of large financial institutions. If the BCLN issuer defaults, the BCLN holder will not receive the recovered value of the reference entity as the credit event happens, nor the notional amount at the contract maturity. The coupon payments also ceases due to the issuer default. Thus the issuer default results in a large loss. Therefore, it is important to incorporate issuer default risk in BCLN pricing to obtain a reasonable coupon rate.

This article focuses on how to incorporate issuer default risk into the BCLN pricing under the factor Gaussian copula framework. A new random variable corresponding to the issuer default time is introduced in this model. Numerical analysis reveals that issuer default risk increases the fair coupon rate and a negatively correlated issuer default increases the coupon rate further, while a positively correlated issuer default mitigates this increase. Moreover, considering the issuer default risk results in an asymmetric coupon rate curve and the asymmetry increases with the impact of the issuer default.

The remainder of this article is organized as follows. The Literature Review section reviews literature on the issuer default risk and the factor Gaussian copula model. The Methodology section describes the process for pricing a BCLN under the framework of factor Gaussian copula model and the proposed method for incorporating issuer default event into BCLN pricing. The Numerical Analysis and Simulation Results section summarizes the numerical analysis results and discusses the implication of the results.

Conclusions are finally drawn in the Concluding Comments section, along with recommendations for future research.

LITERATURE REVIEW

Two main approaches exist to model the default risk in the literature: the structural and reduced form models. The structural model was developed by Merton (1974), and defined default events as occurring when firm asset value falls below firm debt. The reduced form model, also known as the intensity model, was developed by Jarrow and Turnbull (1995). This model views the default event as an unexpected exogenous stochastic event and uses market data to estimate the default risk.

Hull and White (2000) provided a methodology for valuing credit default swap (CDS) without counterparty default risk when the payoff is contingent on the default of a single reference entity. Hull and White (2001) also developed a model of default correlations between different corporate or sovereign entities. The model of Hull and White is an extension of the structural model, sets a credit index variable for each reference entity, and selects correlated diffusion processes for the credit indexes. Their model defines default as the credit index falling below the predetermined default barriers. Monte Carlo simulation is used to calculate the vanilla CDS and basket default swap (BDS) spread given the possibility of seller default. Based on the reduced form model, Jarrow and Yu (2001) indicated that ignoring counterparty relationship causes the mispricing of the credit instruments.

Hui and Lo (2002) developed a model to price the single-name credit linked note (CLN) with issuer default risk using the framework of Merton's model. They demonstrated that the credit spreads of a CLN increase non-linearly with decreasing correlation between the reference entity and the issuer. Kim and Kim (2003) valued single-name CDS and BDS by considering counterparty default risk, as well as correlated market and credit risk. According to their results, the pricing error is substantial by ignoring the correlation between counterparty and reference credit. Leung and Kwok (2005) valued a single-name CDS with counterparty risk by using the reduced form model. According to their results, the swap premium becomes slightly lower when the protection seller has a higher correlation with the reference entity. Leung and Kwok (2009) analyzed the counterparty risk for multi-name CDS based on the Markov chain model with interactive default intensity. Their results indicated that the correlated risk between the protection seller and underlying entity can significantly impact swap rates under a high arrival rate of the external shock, which determines the default correlation and the increment of default intensities for various parties.

Pricing multi-name credit derivatives such as BCLN requires a joint distribution model of the reference entity default times. However, whether using the structural or reduced form models, valuing the multi-name credit derivative is computationally complex. Thus the copula function (Sklar, 1959), also known as the dependence function, which simplifies the estimation of the joint distribution, recently has been widely used to price the multi-name credit derivatives. Li (1999, 2000) first introduced the copula function to deal with the dependence structure in multi-name credit derivative pricing. Li assumed the default times of reference entities to be Poisson processed, and set the dependence structure as a Gaussian copula function. Finally, Li performed Monte Carlo simulation to obtain the default times. Mashal and Naldi (2003) applied Li's method to analyze how the default probabilities of the protection sellers and buyers affect BDS spread. While pricing the single-name CDS with counterparty risk based on the continuous-time Markov model, Walker (2006) indicated that using a time-dependent correlation coefficient can improve the market-standard Gaussian copula approach. By connecting defaults through a copula function, Brigo and Chourdakis (2009) found that when the counterparty risk is involved, both the default correlation and credit spread volatility impact the contingent CDS value.

However, the computational complexity of the Monte Carlo simulation with Gaussian copula increases with number of reference entities. The factor copula method, which makes the default event conditional on independent state variables, was introduced to deal with these problems. Andersen *et al*. (2003) found that one or two factors provide sufficient accuracy for the empirical correlation matrices one encounters in credit basket applications. Hull and White (2004) employed a multi-factor copula model to price the k th-to-default swap and collateralized debt obligation (CDO). Moreover, Laurent and Gregory (2005) used one factor Gaussian copula to simplify the dependence structure of reference entities, and applied this approach to price BDS and CDO. Wu (2010) developed three alternative approaches to price BCLN with issuer default risk using only one correlation parameter and showed that the impact of issuer default differs with changes in the correlation structure. On the other hand, acceleration techniques such as the importance sampling method and others are used to improve the simulation efficiency. Chiang *et al.* (2007) and Chen and Glasserman (2008) applied the Joshi-Kainth algorithm (Joshi and Kainth, 2004), and Bastide et al. (2007) used the Stein method (Stein, 1972) for the multi-name credit derivative pricing to reduce variance of the simulation results.

METHODOLOGY

Copula is a function which links the univariate marginal distributions $F_i(x_i),\ i = 1, 2, \ldots, N,$ to their full multivariate distribution $F(x_1, x_2, \ldots, x_N)$:

$$F(x_1, x_2, \ldots, x_N) = C(F_1(x_1), F_2(x_2), \ldots, F_N(x_N)) \tag{1}$$

where $F_i(x_i) \sim U(0, 1)$. The most widely used copula function is the Gaussian copula and its definition is as follows:

$$C^{Ga}(u_1, u_2, \ldots, u_n) = \Phi_R\left(\phi^{-1}(u_1), \phi^{-1}(u_2), \ldots, \phi^{-1}(u_n)\right) \tag{2}$$

where Φ_R denotes a multivariate cumulative normal (Gaussian) distribution, R represents the correlation coefficient matrix, and ϕ^{-1} is the inverse function of one dimensional cumulative normal distribution. Consider a credit portfolio which contains N reference entities, the default times of each reference entity are $\tau_1, \tau_2, \ldots, \tau_N$, respectively. According to the reduced form model, each reference entity default follows a Poisson process. The cumulative default probability before time t is:

$$F_i(t) = P(\tau_i \le t) = 1 - e^{-\lambda_i t}, \quad i = 1, 2, \ldots, N \tag{3}$$

where λ_i is the hazard rate of the reference entity i. Because $F_i(t) \sim U(0, 1)$, applying the Gaussian copula obtains the multivariate joint distribution of default times, as follows:

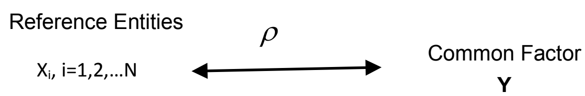
$$F(\tau_1, \tau_2, \ldots, \tau_N) = \Phi_R\left(\phi^{-1}(F_1(\tau_1)), \phi^{-1}(F_2(\tau_2)), \ldots, \phi^{-1}(F_N(\tau_N))\right) \tag{4}$$

Let X_i represent the normal random variable corresponding to the default time of the reference entity i. In the one factor model, the default time of reference entity i depends on a common factor Y and a firm specific risk factor ε_{X_i}. Y and ε_{X_i} are independent standard normal variables. Thus X_i can be created via Cholesky decomposition, as follows:

$$X_i = \rho_{X_iY} Y + \sqrt{1-\rho_{X_iY}{}^2}\,\varepsilon_{X_i}, \quad i = 1, 2, \ldots, N \tag{5}$$

where ρ_{X_iY} denotes the correlation coefficient between the reference entity X_i and the common factor Y. One factor Gaussian copula model with constant pairwise correlations has become the standard market model. In the standard market model, all ρ_{X_iY} in Eqn. (5) are equal to ρ, then the constant pairwise correlation $\rho_{X_iX_j}$ ($i \neq j$) will be ρ^2. The idea of one factor Gaussian copula is shown in Figure 1.

Figure 1: One Factor Gaussian Copula.



This figure shows the relationship between the reference entity variables X_i and the common factor Y in the one factor Gaussian copula. ρ denotes the correlation coefficient between the reference entity X_i and the common factor Y.

Let $X_1 = \phi^{-1}(F_1(\tau_1))$, $X_2 = \phi^{-1}(F_2(\tau_2))$, $\ldots$, $X_N = \phi^{-1}(F_N(\tau_N))$, by mapping τ_i and X_i, we can simulate the default time of the reference entity i using the following equation:

$$\tau_i = F_i^{-1}(\phi(X_i)) = \frac{-\ln(1-\phi(X_i))}{\lambda_i}, \quad i = 1, 2, \ldots, N \tag{6}$$

The conventional form of BCLN is the k th-to-default BCLN. Consider a k th-to-default BCLN involving N reference entities which the notional principal of each reference entity is one dollar. The coupon rate is c. The coupon (the notional principal multiplied by the coupon rate) is paid annually, and the payment dates are $t_i, i = 1, 2, \ldots, T$. The maturity date of the BCLN is t_T. Furthermore, τ_k is the k th default time, and $\tau_1 < \tau_2 < \cdots < \tau_N$. δ_k is the recovery rate of the k th default reference entity. Thus δ_k denotes the redemption proceeds (the notional principal multiplied by the recovery rate) which the issuer pays to the BCLN holder on the k th default. The discount rate is $r\%$. Finally, Q denotes the risk-neutral probability measure, and $I(\cdot)$ is an indicator function. The value of a k th-to-default BCLN can be represented as follows:

$$BCLN = E^Q\left[c \times \sum_{i=1}^{T} e^{-rt_i} I(t_i < \tau_k) + \delta_k \times e^{-r\tau_k} \times I(\tau_k \le t_T) + e^{-rt_T} \times I(\tau_k > t_T)\right] \tag{7}$$

Let the above equation equals one, the equation can be rewritten as:

$$
\begin{aligned}
& c \times E^{Q}\left[\sum_{i=1}^{T} e^{-rt_i} I(t_i < \tau_k)\right] \\
& = E^{Q}\left[1 - \delta_k \times e^{-r\tau_k} \times I(\tau_k \le t_T) - e^{-rt_T} \times I(\tau_k > t_T)\right]
\end{aligned} \tag{8}
$$

Rearranging Eqn. (8) can yield the fair coupon rate at the start of the BCLN as follows:

$$
c = \frac{E^{Q}\left[1 - \delta_k \times e^{-r\tau_k} \times I(\tau_k \le t_T) - e^{-rt_T} \times I(\tau_k > t_T)\right]}{E^{Q}\left[\sum_{i=1}^{T} e^{-rt_i} I(t_i < \tau_k)\right]} \tag{9}
$$

By using W runs of Monte Carlo simulation to price the BCLN, the fair value of the coupon rate c is:

$$
c = \frac{\sum_{s=1}^{W}\left[1 - \delta_k^s \times e^{-r\tau_k^s} \times I(\tau_k^s \le t_T) - e^{-rt_T} \times I(\tau_k^s > t_T)\right]}{\sum_{s=1}^{W}\left[\sum_{i=1}^{T} e^{-rt_i} I(t_i < \tau_k^s)\right]} \tag{10}
$$

where δ_k^s denotes the recovery rate of the k th default reference entity at the s th simulation, and τ_k^s represents the k th default time at the s th simulation.

In order to incorporate the issuer default into the BCLN pricing, this article introduces a new normal random variable Z corresponding to the issuer default time. The default time of reference entity i now depends on the two common factors Y and Z, and a firm specific risk factor ε_{X_i}. Y, Z and ε_{X_i} are independent of each other. Thus X_i, which is the normal random variable corresponding to the default time of the reference entity i is obtained as follows:

$$
X_i = \rho_{X_iY} Y + \rho_{X_iZ} Z + \sqrt{1 - \rho_{X_iY}^2 - \rho_{X_iZ}^2}\,\varepsilon_{X_i} \tag{11}
$$

where ρ_{X_iZ} denotes the correlation coefficient between X_i and Z. To ensure that only the real number is applied in the above equation and use only one correlation coefficient ρ, this article proposes an improved model in which

$$
\rho_{X_iY} = \sqrt{\alpha}\rho \tag{12}
$$
$$
\rho_{X_iZ} = \sqrt{1-\alpha}\rho \tag{13}
$$

then

$$\sqrt{1-\rho_{X_iY}^2-\rho_{X_iZ}^2}=\sqrt{1-\rho^2} \tag{14}$$

Thus Eqn. (11) is modified as follows:

$$X_i=\sqrt{\alpha}\rho Y+\sqrt{1-\alpha}\rho Z+\sqrt{1-\rho^2}\varepsilon_{X_i} \tag{15}$$

The parameter α decides the proportional influences of the common factor Y and the issuer default Z. In the extreme case, when $\alpha=1$, the common factor Y and the firm specific risk factor ε_{X_i} together fully determine the default time of the reference entity as follows:

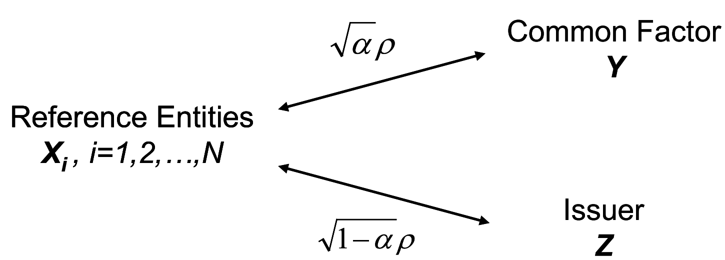
$$X_i=\rho Y+\sqrt{1-\rho^2}\varepsilon_{X_i} \tag{16}$$

On the other hand, when $\alpha=0$, the default time is solely determined by another common factor Z, which represents the issuer default event, and the firm specific risk factor ε_{X_i}:

$$X_i=\rho Z+\sqrt{1-\rho^2}\varepsilon_{X_i} \tag{17}$$

The relationships between X_i, Y and Z are shown in Figure 2.

Figure 2: Relationships between X_i, Y and Z in the Proposed Model.



This figure shows the relationship between the reference entity variables X_i, *the issuer default variable* Z *and the common factor* Y *in the proposed model. The parameter* α *decides the proportional influences of the common factor* Y *and the issuer default variable* Z.

In situations involving issuer default risk, it is necessary to consider whether the issuer default occurs before or after the kth default. This article defines $\hat{\tau}$ as the issuer default time and $\hat{\delta}$ as the issuer recovery rate. The BCLN holder gets back the recovered value of the reference obligation if the kth default occurs before both the issuer default time $\hat{\tau}$ and maturity date t_T. If the issuer default occurs before the kth default and maturity date, the issuer will not provide the BCLN holder with the redemption proceeds and stop the coupon payments. In this situation, the notional principal multiplied by the issuer recovery rate is returned to the BCLN holder. To obtain all of the notional principal back, both the kth default time and the issuer default time must be later than the contract maturity date. Thus, the value of a kth-to-default BCLN with issuer default risk is modified as follows:

$$BCLN = E^Q \begin{bmatrix} c \times \sum_{i=1}^{T} e^{-rt_i} I\left(t_i < \min(\tau_k, \hat{\tau})\right) + \delta_k \times e^{-r\tau_k} \times I\left(\tau_k < \min(\hat{\tau}, t_T)\right) \\ + \hat{\delta} \times e^{-r\hat{\tau}} \times I\left(\hat{\tau} < \min(\tau_k, t_T)\right) + e^{-rt_T} \times I\left(t_T < \min(\tau_k, \hat{\tau})\right) \end{bmatrix} \tag{18}$$

Therefore, the fair value of the coupon rate c with issuer default risk is:

$$c = \frac{\sum_{s=1}^{W} \begin{bmatrix} 1 - \delta_k^s \times e^{-r\tau_k^s} \times I\left(\tau_k^s < \min(\hat{\tau}^s, t_T)\right) \\ - \hat{\delta} \times e^{-r\hat{\tau}^s} \times I\left(\hat{\tau}^s < \min(\tau_k^s, t_T)\right) - e^{-rt_T} \times I\left(t_T < \min(\tau_k^s, \hat{\tau}^s)\right) \end{bmatrix}}{\sum_{s=1}^{W} \left[\sum_{i=1}^{T} e^{-rt_i} I\left(t_i < \min(\tau_k^s, \hat{\tau}^s)\right) \right]} \tag{19}$$

where δ_k^s and τ_k^s are defined as in Eqn. (10), and $\hat{\tau}^s$ represents the issuer default time at the s th simulation.

NUMERICAL ANALYSIS AND SIMULATION RESULTS

This article adopts a five-year BCLN with three reference entities as an example of numerical analysis. All three reference entities have notional principal one dollar, hazard rate 5% and recovery rate 30%. Furthermore, the coupon is paid annually, the hazard rate and recovery rate of the issuer is 1% and 30%, respectively. Sixty-thousand runs of Monte Carlo simulation are executed to calculate the coupon rates and the results are shown in Figure 3.

According to the proposed model, the correlation between the reference entities X_i and X_j, i.e. $\rho_{X_i X_j}$, is derived as follows:

$$\begin{aligned} \rho_{X_i X_j} &= corr\left(\sqrt{\alpha}\rho Y + \sqrt{1-\alpha}\rho Z + \sqrt{1-\rho^2}\varepsilon_{X_i}, \sqrt{\alpha}\rho Y + \sqrt{1-\alpha}\rho Z + \sqrt{1-\rho^2}\varepsilon_{X_j}\right) \\ &= \alpha\rho^2 + (1-\alpha)\rho^2 \\ &= \rho^2 \end{aligned} \tag{20}$$

ρ^2 is positively correlated to $|\rho|$. As shown in Figure 3(a), the coupon rate of the first-to-default is negatively correlated with $|\rho|$, because the probability of the first-to-default (k =1) occurring increases as $|\rho|$ decreases. Conversely, as shown in Figure 3(b) and (c), the coupon rate of the second (k =2) and third-to-default (k =3) is positively correlated with $|\rho|$, because the probability of the joint default increases as $|\rho|$ increases.

Figure 3: Comparisons for the k th-to-default BCLN Coupon Rates without and with Issuer Default Risk: (a) k =1; (b) k =2; (c) k =3.

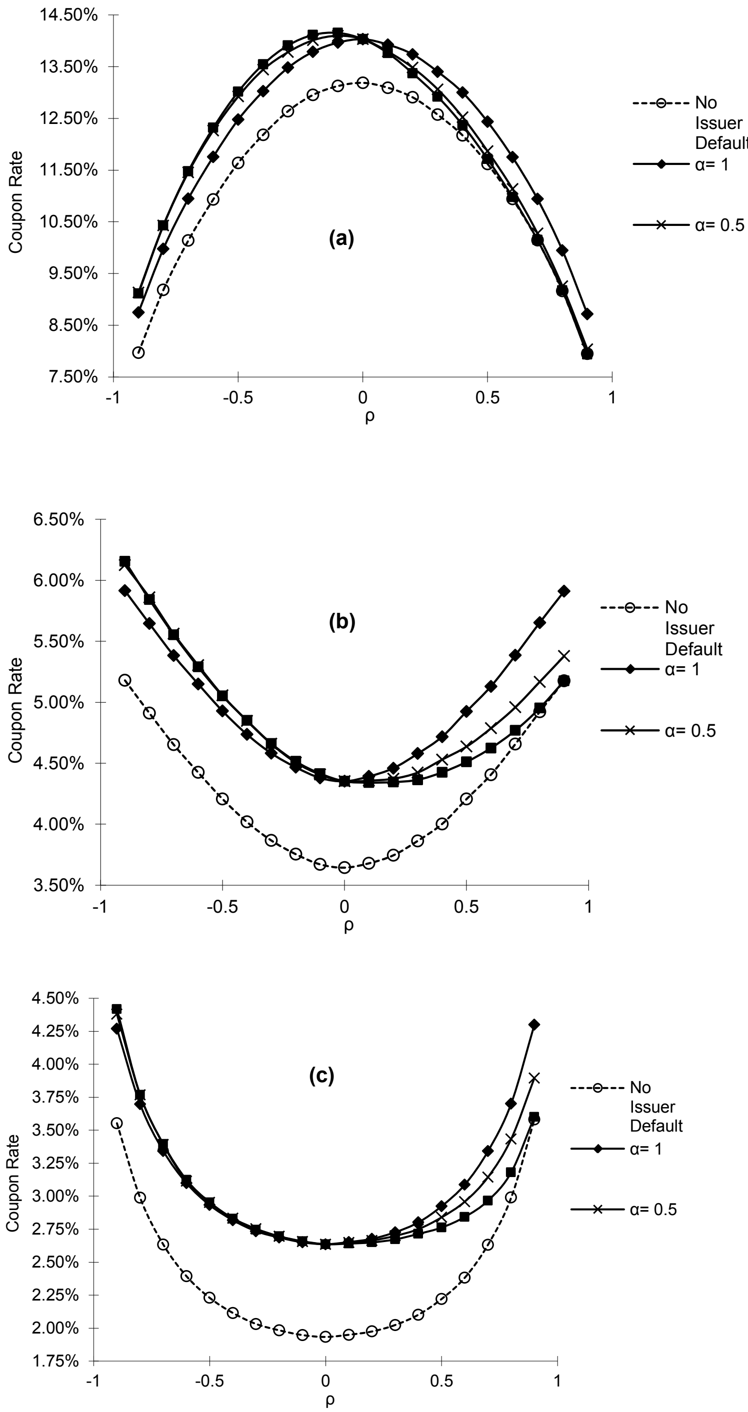


Figure 3(a) shows that the coupon rate of the first-to-default is negatively correlated with $|\rho|$, because the probability of the first-to-default (k =1) occurring increases as $|\rho|$ decreases. Conversely, as shown in Figure 3(b) and (c), the coupon rate of the second (k =2) and third-to-default (k =3) is positively correlated with $|\rho|$, because the probability of the joint default increases as $|\rho|$ increases.

Figure 3 indicates that coupon rates with issuer default risk exceed those without issuer default risk, while Table 1 to Table 3 present the relevant data. The maximum increments in coupon rates due to the existence of issuer default exceed 100bps in the first-to-default of Table 1, 90bps in the second-to-default of Table 2, and 80bps in the third-to-default in Table 3. The coupon rates raise mostly when ρ is negative and $\alpha = 0$. The issuer default risk appears to affect the three contracts similarly.

The coupon rate curves of different α intersect at the point $\rho = 0$. When ρ is negative, i.e. the correlation $\rho_{X_i Z} (= \sqrt{1-\alpha}\rho)$ between the reference entities and the issuer default is negative, the coupon rate increases as α approaches zero (the reference entities default time is only determined by Z and ε_{X_i}). On the other hand, when ρ is positive, i.e. $\rho_{X_i Z}$ is positive, the coupon rate decreases as α approaches zero. This contradicts the common belief that the positive default correlation increases the possibility of double defaults of the reference entities and the issuer, thus disfavoring the BCLN holder and driving up the BCLN coupon rate.

Table 1: First-to-default BCLN Coupon Rates without and with Issuer Default Risk.

ρ	No Issuer Default	α = 0	α = 0.5	α = 1
-0.9	7.967%	9.113%	9.131%	8.747%
-0.8	9.181%	10.430%	10.430%	9.978%
-0.7	10.138%	11.477%	11.456%	10.947%
-0.6	10.933%	12.319%	12.265%	11.757%
-0.5	11.639%	13.015%	12.923%	12.476%
-0.4	12.183%	13.545%	13.443%	13.025%
-0.3	12.641%	13.912%	13.782%	13.481%
-0.2	12.954%	14.115%	14.012%	13.789%
-0.1	13.126%	14.154%	14.100%	13.967%
0	13.188%	14.032%	14.032%	14.032%
0.1	13.094%	13.765%	13.794%	13.928%
0.2	12.907%	13.370%	13.481%	13.739%
0.3	12.572%	12.921%	13.062%	13.403%
0.4	12.170%	12.372%	12.523%	13.000%
0.5	11.619%	11.713%	11.865%	12.434%
0.6	10.942%	10.977%	11.137%	11.750%
0.7	10.146%	10.148%	10.276%	10.941%
0.8	9.162%	9.174%	9.255%	9.947%
0.9	7.947%	7.931%	8.034%	8.716%

This table shows the first-to-default (k =1) BCLN coupon rates without and with issuer default risk under various correlation coefficients ρ and adjustment parameters α.

Table 2: Second-to-default BCLN Coupon Rates without and with Issuer Default Risk.

ρ	**No Issuer Default**	α **= 0**	α **= 0.5**	α **= 1**
-0.9	5.180%	6.156%	6.124%	5.915%
-0.8	4.911%	5.843%	5.860%	5.645%
-0.7	4.652%	5.553%	5.561%	5.382%
-0.6	4.424%	5.290%	5.303%	5.150%
-0.5	4.207%	5.051%	5.056%	4.928%
-0.4	4.018%	4.851%	4.848%	4.737%
-0.3	3.866%	4.661%	4.658%	4.581%
-0.2	3.754%	4.515%	4.506%	4.467%
-0.1	3.671%	4.415%	4.407%	4.380%
0	3.644%	4.352%	4.352%	4.352%
0.1	3.679%	4.341%	4.357%	4.392%
0.2	3.745%	4.345%	4.373%	4.459%
0.3	3.864%	4.364%	4.424%	4.579%
0.4	4.002%	4.425%	4.530%	4.717%
0.5	4.206%	4.511%	4.636%	4.925%
0.6	4.406%	4.624%	4.786%	5.130%
0.7	4.657%	4.769%	4.959%	5.385%
0.8	4.922%	4.952%	5.168%	5.653%
0.9	5.175%	5.175%	5.378%	5.910%

This table shows the second-to-default (k =2) BCLN coupon rates without and with issuer default risk under various correlation coefficients ρ and adjustment parameters α.

Table 3: Third-to-default BCLN Coupon Rates without and with Issuer Default Risk.

ρ	**No Issuer Default**	α **= 0**	α **= 0.5**	α **= 1**
-0.9	3.553%	4.418%	4.380%	4.269%
-0.8	2.988%	3.768%	3.768%	3.697%
-0.7	2.634%	3.395%	3.391%	3.342%
-0.6	2.393%	3.124%	3.114%	3.099%
-0.5	2.230%	2.953%	2.953%	2.933%
-0.4	2.115%	2.829%	2.832%	2.818%
-0.3	2.032%	2.745%	2.753%	2.735%
-0.2	1.983%	2.695%	2.695%	2.686%
-0.1	1.948%	2.659%	2.656%	2.651%
0	1.934%	2.636%	2.636%	2.636%
0.1	1.950%	2.641%	2.648%	2.653%
0.2	1.975%	2.650%	2.665%	2.677%
0.3	2.024%	2.673%	2.699%	2.727%
0.4	2.102%	2.714%	2.751%	2.802%
0.5	2.221%	2.762%	2.840%	2.924%
0.6	2.383%	2.843%	2.959%	3.088%
0.7	2.632%	2.966%	3.145%	3.341%
0.8	2.990%	3.181%	3.434%	3.702%
0.9	3.582%	3.601%	3.896%	4.300%

This table shows the third-to-default (k =3) BCLN coupon rates without and with issuer default risk under various correlation coefficients ρ and adjustment parameters α.

Figure 4: Different Time Sequences of the Issuer Default Time $\hat{\tau}$, the k th Reference Entity Default Time τ_k, and the BCLN Contract Maturity t_T.

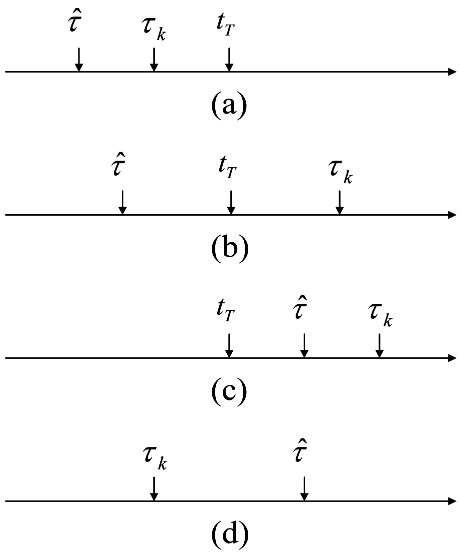


Figure 4(a) shows that the k th default occurs when the contract is still valid and the issuer defaults before the k th default. Figure 4(b) shows that the issuer defaults before contract maturity and the k th default occurs after contract maturity. Figure 4(c) shows that the k th and the issuer defaults both occur after the maturity. Figure 4(d) shows that the issuer default occurs after the k th default.

Following a careful survey, issuer default influences BCLN in three ways. In the first scenario, as shown in Figure 4(a), the k th default occurs when the BCLN contract is still valid and the issuer defaults before the k th default. The notional principal multiplied by the issuer recovery rate is returned to the BCLN holder and the subsequent coupon income ceases after the issuer default. The difference in the present value of the cash flow received by the BCLN holder between the cases with and without issuer default, i.e. Eqn. (18) minus Eqn. (7), is:

$$-c \times \sum_{i=1}^{T} e^{-rt_i} I(\hat{\tau} < t_i < \tau_k) + \left(\hat{\delta} \times e^{-r\hat{\tau}} - \delta_k \times e^{-r\tau_k}\right) \tag{21}$$

The former term of Eqn. (21) is due to the cessation of the coupon income (named the coupon cessation effect) and the last term is the loss of principal capital resulting from the issuer default (named the principal loss effect). The coupon cessation effect always disfavors the BCLN holder. Whether the principal loss effect disfavors the BCLN holder depends on whether the last term is negative or positive.

In the second scenario, as shown in Figure 4(b), the issuer defaults before contract maturity and the k th default occurs after contract maturity. The difference in the present value of the cash flow between the cases with and without issuer default is:

$$-c \times \sum_{i=1}^{T} e^{-rt_i} I(\hat{\tau} < t_i < t_T) + \left(\hat{\delta} \times e^{-r\hat{\tau}} - 1 \times e^{-rt_T}\right) \tag{22}$$

Again the former term of Eqn. (22) results from the coupon cessation effect, and the last term results from the principal loss effect. The coupon cessation effect always disfavors the BCLN holder. However, in the second scenario the principal loss effect generally disfavors the BCLN holder because that $\hat{\delta}$ is much less than one.

Finally, the scenarios that the kth and the issuer defaults both occur after the maturity, as shown in Figure 4(c), and the issuer default occurs after the kth default, as shown in Figure 4(d), are classified into the third way. In both scenarios, the cash flow is identical to that without issuer default and neither effect exists. The existence of issuer default risk does not alter the cash flow of the BCLN holder. Therefore, from the above discussion, the difference in the coupon rates between the cases with and without issuer default depends on the combined effect of the coupon cessation and the principal loss effects in the above four scenarios.

Figure 3 shows that all the coupon rates exceed those without issuer default. While the recovery rates of the issuer and the kth default entity are the same as the numerical example presented here, the combined effect disfavors the BCLN holder, resulting in a higher coupon rate than in the case without issuer default. When $\alpha = 1$, the default time of reference entities is decided by the common factor Y and firm specific risk factor ε_{X_i}, and the issuer default Z is independent of the reference entity defaults. Unlike the symmetric coupon rate curve with $\alpha = 1$, the other two coupon rate curves, with $\alpha = 0.5$ and $\alpha = 0$, are asymmetric. For the $\rho < 0$ side, the coupon rates considering correlated issuer default risk, i.e. $\alpha = 0.5$ and $\alpha = 0$, exceed those with independent issuer default risk, i.e. $\alpha = 1$. However, for the $\rho > 0$ side, the coupon rates considering correlated issuer default risk are lower than those with independent issuer default risk. The reason for this phenomenon is explained as follows.

When ρ is negative, the correlation between the reference entities and the issuer default, i.e. ρ_{X_iZ} ($=\sqrt{1-\alpha}\rho$), is also negative. As α approaches zero, resulting a more negative ρ_{X_iZ}, the default times of the issuer and the kth default become increasingly dispersed and the time interval between the issuer and the kth default increases. Therefore, the coupon cessation effect increases as α approaches zero, and the disfavorable situation increases the BCLN coupon rate. On the other hand, a positive ρ results in a positive ρ_{X_iZ}. As α approaches zero, the default times of the issuer and the kth default become increasingly concentrative and the time interval between the issuer and the kth defaults decreases. The coupon cessation effect decreases as α approaches zero, and the favorable situation reduces the BCLN coupon rate. In summary, when the default correlation between the reference entities and the issuer is negative, the BCLN coupon rate increases as the impact of the issuer default increases. While the default correlation is positive, the BCLN coupon rate decreases as the impact of the issuer default increases.

As α approaches zero, the impact of issuer default increases and so too does the coupon rate asymmetry. This demonstrates an increasing impact of a positive or negative default correlation between the reference entities and the issuer. In the extreme case $\alpha = 0$, the coupon rate curve is the most asymmetric.

CONCLUDING COMMENTS

Obtaining the most reasonable BCLN coupon rate requires considering issuer default risk. This article proposes a framework to incorporate issuer default risk into BCLN pricing based on a multi-factor Gaussian copula, and uses a parameter α to adjust the weights of the original common factor and issuer default factor. The proposed model is simulated to price a five-year BCLN with three reference entities. The analysis results demonstrate that issuer default risk increases the fair coupon rate. Furthermore, analytical results also reveal that a negative default correlation between the reference entities and the issuer increases the coupon rate as the impact of issuer default increases. However, a positive default correlation results in a decreasing coupon rate as the impact of issuer default increases. This is contradicting the common belief that a positive default correlation between the reference entities and the issuer increases the possibility of double losses, thus driving up the BCLN coupon rate. The proposed framework is easy to implement, and the issuer default risk is effectively reflected in the fair BCLN coupon rate.

This work adopts two parameters, i.e. the correlation coefficient and the adjustment parameter, to model the interaction between the reference entity defaults, issuer default and the common factor. The issuer default is independent of the common factor in the setting, thus the proposed model does not consider the correlation between the issuer default and the common factor. The author recommends that future research extend the proposed model to include this correlation.

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BIOGRAPHY

Dr. Po-Cheng Wu is an Assistant Professor of Finance at Kainan University, He can be contacted at: Department of Banking and Finance, Kainan University, No.1, Kainan Road, Luzhu Shiang, Taoyuan, Taiwan, R.O.C. Email: pcwu@mail.knu.edu.tw