

# WHY DO BANKS DEFAULT WHEN ASSET QUALITY IS HIGH?

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## ABSTRACT

*Short-term financing, e.g., asset-backed commercial paper (ABCP) or repurchase agreements (repo), was prevalent prior to the 2007-2008 financial crises. Banks funded by short-term debts, however, are exposed to rollover risk as the banks are unable to raise sufficient funds to finance their long-term assets. Under such circumstance, banks' equity holders need to absorb the rollover loss. Both deteriorating collateral assets' fundamentals and market illiquidity are important drivers of the rollover risk. In this paper, we develop a structural default model based on Leland (1994), in which default is an endogenously determined decision made by equity holders, to analyze the joint effect of market liquidity and interest rate sensitive fundamentals of collateral assets' on the survival times of banks relying on day-to-day short-term finance. The proposed model provides an explanation of the empirical observed phenomenon that banks default even when the quality of their fundamentals is still high.*

JEL C41; C36; G17; G21; G33; G32

**KEYWORDS:** Asset-backed commercial paper (ABCP), Repurchase agreements (repo), Rollover risk, Collateral assets' fundamental, Market illiquidity, Structural default model.

## INTRODUCTION

The financial crisis of the years 2007-2008 has been a particular turbulent one for the US banking and financial system. The first bank failure occurred when two Bear Stearns hedge funds invested in sub-prime assets filed for bankruptcy on July 31, 2007. A year later, on September 14<sup>th</sup>, 2008, the Lehman Brothers declared its bankruptcy which triggered a series of banks and insurance companies announce their failures. Unlike the banking panics of the 19<sup>th</sup> century in which depositors *en masse* withdrew cash in exchange of demand and savings deposits, the financial crisis of the years 2007-2008 is a systemic banking run driven by the withdrawal of short-term debts, e.g., asset-backed commercial paper (ABCP) or repurchase agreements (repo), with tenors no more than 270 days (Brunnermeier, 2009; Krishnamurth, 2010; Gorton et. al., 2008, 2009).

Such short-term financing was prevalent prior to the 2007-2008 financial crises. Often these short-term debts are collateralized by securities backed by assets like real estate, autos and other commercial assets. One of the determinant economic factors of a debt's capacity is its collateral assets' fundamentals. However, collateral assets with high quality fundamentals do not guarantee a bank's ability to raise new funds when the market liquidity deteriorates (Acharya, Gale, and Yorulmazer, 2009; He and Xiong, 2010b). The failure of Bear Stearns in mid-March 2008 provides such a counter-example. After two Bear Stearns hedge funds filed for bankruptcy on July 31, 2007, the calculation of the net asset values of the other three investment funds was suspended as it is no longer possible to value certain assets fairly regardless of their quality or credit rating (Acharya, Gale, and Yorulmazer, 2009). The same phenomenon is observed in the repo market during the 2007-2008 financial crises (Gorton, 2009).

The implications of the above observations are consistent with the widely held views that both deteriorating fundamentals and market illiquidity are important drivers of bank failures. In this paper, we develop a structural default model based on Leland (1994) to analyze the joint effect of market

liquidity and collateral assets' fundamentals that are interest rate sensitive. In the proposed model, default is an endogenously determined decision when the bank does not raise sufficient funds to repay a fraction of its maturing debt's principle (Huang and Huang, 2002). Monte Carlo simulations are performed for 24 scenarios of different parameters values on long-term interest rate, the volatility of the collateral assets' fundamental, and the shift and shape parameters that control for the information structure and the likelihood of the occurrence of unusual economic events in the market. The simulation shows that survival curve of banks with the smallest volatility for its collateral asset's fundamental is approximating to that of the highest volatility when the long-term interest rate is lower. The result provides an explanation of the empirical observed phenomenon that banks default even when the quality of their fundamentals is still high.

The paper is organized as follows. Literature reviews are given in Section 2. Section 3 develops a structural default model for banks that rely on short-term debt in a stochastic interest rate environment. In the proposed structural default model, an interest rate sensitive fundamental of collateral assets and a stochastic purely jump debt capacity ratio that accounts for market liquidity are incorporated. A simulation study is performed in Section. Section 5 concludes.

## LITERATURE REVIEW

### Short Term Debts

According to Diamond and Rajan (2000), banks are best to finance their illiquid long-term investments that are less likely to produce cash flows in the short run with short-term rather than long-term debts. Like banks, asset-backed commercial paper (ABCP) or repo programs issue liquid short term debt that is highly-rated, collateralized to finance illiquid and long-term assets. These short-term debt markets grew dramatically in recent years. The ABCP market nearly doubles in size between 2004 and 2007. At the end of July 2007, just before the widespread turmoil, the total ABCP outstanding is \$1060 billion (Moody's, 2007). In a study of repo markets by Hordahl and King (2008), it was estimated that the top US investment banks funded roughly half of their assets using repo markets before the 2008 financial crisis.

Like traditional banks, ABCP or repo programs are subject to the risk of fundamentals-driven runs, whereby investors quickly flee from potentially insolvent and poorly supported programs (Diamond and Dybvig, 1983). On the other hand, as the demand deposits in the traditional banking create information-insensitive debts (Gorton and Pennacchi, 1990), banks may also be vulnerable to runs not based on fundamentals. Similarly, runs in the ABCP or repo programs maybe linked to non-program specific variables, such as broader financial market strains and market-wide proxies for liquidity risks. With an average haircut zero in 2007 to nearly 50% at the peak of the financial crisis in late 2008, the concerns about the market liquidity of the securitized collateral assets had led to the insolvency of the US banking system (Gorton et. al., 2010a).

However, unlike the traditional banking in which depositors are protected by deposit insurance provided by the Federal Reserve, the ABCPs or repos do not have explicit deposit insurance provided by the government. As a bankruptcy-remote special purpose vehicle (SPV), or conduit, is created in an ABCP or repo program to issue short-term debts to finance assets, for an ABCP program, the committed back-stop liquidity lines are provided by sponsored commercial banks (Fitching Rating, 2001; Covitz et al., 2009). Similar to an ABCP program, the collateral assets, often securitized bonds, are the liabilities of a SPV, and creditors receive these securitized bonds as collateral for protection (Gorton et. al., 2010a, 2010b). This has exposed banks relying on short-term financing such as ABCPs or repos programs to even larger rollover risk that triggers the financial crisis in 2007-2008.

### Rollover Risk Associated With Short-Term Debts

When a debt matures, the bank issues a new debt with the same face value and maturity to replace the maturing debt at the new debt's capacity, i.e., the maximum amount of funds that can be obtained based on the debt's collateral assets, which can be higher or lower than the principal of the maturing debt. When insufficient funds can be raised to pay off maturing creditors, banks' equity holders need to absorb the rollover loss. A shorter debt maturity can exacerbate a bank's rollover risk as equity holders are forced to quickly absorb the losses incurred by the bank's debt financing. When the bank defaults, the maturing creditors need to seize and liquidate the collateral assets in an illiquid secondary market at fire sale prices. This in turn has exposed banks to even significant funding liquidity risk and eventually contagious bank failures (Diamond and Rajan, 2000, 2001).

Acharya, Gale, and Yorulmazer (2009) provide a theoretical model with two different information structures, namely, optimistic versus pessimistic, for the market liquidity. The model explains sudden freezes in secured short-term debt markets even when the assets are subject to very limited credit risk. Diamond and Rajan (2005) show that runs by depositors on insolvent banks can have contagious effect on the whole banking system. However, they do not analyze the coordination problem between depositors. In contrast to the static bank-run model, He and Xiong (2010b) derived an equilibrium bank run model, in which the creditors coordinate their asynchronous rollover decisions based on the publicly observable time-varying fundamental of collateral assets. In He and Xiong (2010b), a uniquely determined threshold of the fundamental under which a maturing creditor chooses to run on the short-term debt is derived. In another paper, He and Xiong (2010a) analyze the interaction between debt market liquidity and credit risk due to the intrinsic conflict of interest between debt and equity holders, which causes an earlier default by the equity holders. The model captures the phenomenon of the 2007-2008 financial crisis that even in the absence of any fundamental deterioration, pre-emptive runs by creditors on a solvent bank occur.

### Structural Default Models

For modeling credit risk, two classes of models exist: structural and reduced form. Structural models originated with Merton (1974), and reduced form models originated with Jarrow and Turnbull (1992, 1995) and Duffie and Singleton (1999). This paper considers structural models only, which can be classified as exogenously versus endogenously determined default models in literature. The exogenously determined default model assumes that bankruptcy is triggered when the firm's asset value falls to its debt's principle value, where an exogenously determined default barrier is usually assumed in this type of structural model. The pioneer works of Merton (1974), Black and Cox (1976), Longstaff and Schwartz (1995), and Briys and Varenne (1997) all are cases of the first type structural model. The second type of structural model assumes that bankruptcy is an optimal decision made by equity holders to surrender control to bond holders to maximize the value of equity, and therefore the optimal default barrier is endogenously determined (Leland, 1994; Leland and Toft, 1996).

In either type of the aforementioned structural models, the determinant of a default event is the firm's asset value  $V$ : default occurs if the process  $V$  falls below to the default barrier for the first time or at maturity. For banks rely heavily on day-to-day short-term debts, as the intrinsic conflict between debt and equity holders arises when equity holders bear the rollover losses while maturing debt holders get paid in full, the equity holders may choose to default earlier (He and Xiong, 2010b). Therefore, we adopt Leland's (1994) endogenously determined default model, in which a short-term debt is continuously rolled over unless terminated because the bank cannot raise sufficient funds to repay a fraction  $\beta$  ( $0 < \beta < 1$ ) of the maturing debt's principle (Huang and Huang, 2002).

Structure Default Model for Banks

Our model differs from that of Leland’s (1994) in two respects. First, an interest rate sensitive fundamental in a stochastic interest rate environment following a mean-reversion diffusion process of Vasicek (1977) is assumed. Second, market liquidity is considered. Instead of a series of Poisson liquidity shocks that drive the debt redemption rate (He and Xiong, 2010b), we use a purely jump VG process to model the dynamics of the debt’s capacity ratios, where deterioration of debt market liquidity in terms of jumps of to lower levels.

Stochastic Fundamentals: Diffusion-Based Processes

Suppose the bank holds a collateral asset which matures at time  $T$ . Instead of a constant interest rate environment (He and Xiong, 2010a), here we assume a stochastic risk-free interest rate  $r(t)$  obeying the mean reverting process by Vasicek(1977) as

$$dr = \theta_r(\pi - r)dt + \sigma_r dZ_r \tag{1}$$

where  $\pi$  is a central tendency parameter,  $\theta_r$  is the reverting rate, and  $Z_r$  is a standard Weiner processes. An interest-rate-sensitive fundamental  $R(t)$  obeying a geometric Brownian motion under the risk neutral measure  $Q$

$$dR(t)/R(t) = r(t)dt + \sigma_1 dZ_r + \sigma_2 dZ_R \tag{2}$$

is assumed, where the standard Weiner process  $Z_R$  is independent of  $Z_r$ . We adopt the standard argument in efficient markets that the collateral asset’s market value is the expected present value of cash flows at maturity (Acharya, Gale, and Yorulmazer, 2009). As the interest rate  $r(t)$  is stochastic, the time- $t$  asset value  $V(t)$  is

$$V(t) = E_t \left[ \exp \left( - \int_t^T r(s) ds \right) f(R(T)) \right] \tag{3}$$

where  $f(R(T))$  is the collateral asset’s cash flow at maturity  $T$ . Here we consider an option-like cash flow employed by He and Xiong (2010a), in which the cash flow  $f(R(T))$  equals the fundamental  $R(T)$  if  $R(T)$  is above a threshold  $R^*$ , otherwise the cash flow  $f(R(T))$  is zero. The closed form solution of the market value  $V(t)$  is derived in the following Lemma.

Lemma 1: The time- $t$  price of the collateral asset  $V(t)$  is

$$V(t) = P(t, T) F(t) \Phi(w) \tag{4}$$

where  $F(t) = R(t)/P(t, T)$ ,  $P(t, T)$  is the time- $t$  price of a default-free zero-coupon bond that matures at time  $T$ , and the constant

$$w = \frac{\log(F(t)/R^*) + b^2/2}{b} \tag{5}$$

$$b^2 = \int_t^T (\sigma_1 - S(v, T))^2 dv + \sigma_2^2 (T - t) \tag{6}$$

*Proof.* See the Appendix.

To calculate the time- $t$  price of the collateral asset  $V(t)$ , the time- $t$  price  $P(t, T)$  of a default-free zero-coupon bond is required. According to Vasicek (1977), the time- $t$  price  $P(t, T) = \exp\{B(t, T) - U(t, T)r(t)\}$ , where

$$U(t, T) = \frac{1 - e^{-\theta_r(T-t)}}{\theta_r}$$

$$B(t, T) = \{U(t, T) - (T-t)\} \left( \pi - \frac{\sigma_r^2}{2\theta_r^2} \right) - \frac{\sigma_r^2}{4\theta_r} U^2(t, T)$$

Stochastic Haircut: Pure Jump Process

To characterize a bank that rolls over its short-term debt several times before the maturity  $T$  of its long-term collateral asset, let a short-term debt be rolled over at discrete times  $0, \Delta t, 2\Delta t, \dots, (K-1)\Delta t$ , respectively, where  $T = K\Delta t$ . At each time point  $t = j\Delta t, j \geq 0$ , the underlying asset's liquidity is measured in terms of the debt capacity ratio  $\alpha_t$ , where  $0 \leq \alpha_t \leq 1$ . For simplicity, it is assumed that the debt's capacity ratios  $\{\alpha_t: t \geq 0\}$  are independent of the risk-free interest rate  $r(t)$  and the asset's fundamental  $R(t)$ .

In a plot of collateral assets' haircut index, which is one minus the debt capacity ratio  $\alpha_t$ , of the repo market from 2007 to 2008, a series of jumps of random magnitudes at random times corresponding to a stream of economic events such as financial crisis, terrorist attacks, ... , etc., are exhibited (Gorton *et al.*, 2010a). To describe the dynamics of the jump behavior in the haircut index, or, the debt capacity ratios, a variance-gamma (VG) process is used. Being a purely jump Levy process, the VG process is a popular model in finance for processes with random jump behavior (Madan and Seneta, 1990; Madan and Milne, 1991; Madan, 1998; Geman, 2001). To ensure positivity as in assets' price modeling, we consider the logarithm of a debt's capacity ratio, i.e.,  $\{\log(\alpha_t): t \geq 0\}$ . Specifically, suppose the process  $\{\log(\alpha_t): t \geq 0\}$  follows a VG process with drift and volatility  $\theta$  and  $\sigma_g$ , respectively. It can be shown that the ratio of the logarithms at times  $(i-1)\Delta t$  and  $i\Delta t$  is

$$\log\left(\frac{\alpha_{i\Delta t}}{\alpha_{(i-1)\Delta t}}\right) = \phi + \theta_g g + \varepsilon_i \tag{7}$$

where the innovation  $\varepsilon_i = \sigma_g Z_g(g)$ , the random scale  $g$  is gamma distributed with shape parameter  $\nu$  and scale parameter  $\gamma = 1/\nu$ , respectively. Conditional on the random scale  $g$  is, the innovation  $\varepsilon_i | g \sim \mathcal{N}(0, \sigma_g^2 g)$  is normally distributed. The parameter

$$\phi = \nu \Delta t \times \ln\left(1 - \frac{\theta_g}{\nu} - \frac{1}{2\nu} \sigma_g^2\right) \tag{8}$$

The derivation of Eqs. (7)-(8) is given in the Appendix.

Since the drift parameter  $\theta_g$  controls for the skewness of a VG process in that negative  $\theta_g$  gives rise to negative skewness, while positive  $\theta_g$  gives rise to positive skewness. Therefore, negative drift parameter  $\theta_g$  represents a pessimistic information structure, while non-negative drift parameter  $\theta_g$  represents an

optimistic information structure for the liquidity in the market. The shape parameter  $\nu$  is used to control for the arrival rate of unusual economic events that affect market liquidity due to the fact that smaller shape parameter  $\nu$  raises the likelihood of larger jumps in the random scale  $g$ , and therefore the likelihood of larger jumps of the debt capacity ratios given in (7).

Defining The Default Event And Survival Probability

To define the default event, we consider a parallel of the specification by Leland (1994) in which a continuously renewable short-term debt will be rolled over if and only if the firm’s market asset value is sufficient to repay the debt’s principle. Suppose the short-term debt has been successfully rolled over till time  $t=j\Delta t, j\geq 1$ . At time  $t'=(j+1)\Delta t$ , the maturing debt’s principle, or the maturing debt’s capacity,  $C(t)$  is proportional to the asset’s market value  $V(t)$  and its debt’s capacity ratio  $\alpha_t$  in the form

$$C(t)=\alpha_t V(t) \tag{9}$$

Here we adopt the default barrier proposed by Huang and Huang (2002) that a bank defaults if the fund raised at time  $t'$  cannot repay a fraction  $\beta$  of the maturing debt’s principle  $C(t)$ . As bank’s equity holders need to bear the rollover losses, thus the decision of the level of fraction  $\beta$  is made endogenously by equity holders. Specifically, default occurs at time  $t'$  if  $C(t')<\beta C(t)$ , i.e.,

$$\alpha_t V(t')<\beta \alpha_t V(t) \tag{10}$$

The bank’s default time  $\tau$  can now be formulated as

$$\tau=\inf\{t': \alpha_t V(t')<\beta \alpha_t V(t), \text{ where } t'=(j+1)\Delta t \text{ and } t=j\Delta t, j\geq 1\} \tag{11}$$

Thus the probability that the bank survives after time  $t$  is given by  $S(t)=\Pr\{\tau>t\}$ . In the following, Monte Carlo simulation technique will be employed to explore the three independent determinant factors, namely, the interest-rate sensitive fundamental and the debt’s capacity ratio, on the survival probability curve  $S(t)$

**SIMULATION STUDY**

Suppose a long-term collateral asset matures at  $T=10$  years, and short-term debts are rolled over at discrete times  $0, \Delta t, 2\Delta t, \dots, (K-1)\Delta t$ , respectively, where  $\Delta t=(1/360)$  year and  $K=3600$ . In the following, a simulation study is performed to illustrate the roles the four determinant parameters play in the distributions of a bank’s survival times. The four determinant parameters are: (1) The long term equilibrium interest rate (low and high central tendency parameter  $\pi=0.05, 0.07$ ); (2) The volatility of the asset’s fundamental (low, medium, and high volatility  $\sigma_g=0.1, 0.2, \text{ and } 0.5$ ); (3) The information structure of the market liquidity: Optimistic versus pessimistic (shift parameter  $\theta_g=0.00, -0.05$ ); (4) The occurrence of unusual economic events (low and high shape parameter  $\nu=0.1 \text{ and } 1$ ). There are a total of 24 various scenarios considered. Throughout the 24 various scenarios, the mean-reverting rate  $\theta_r$  is set to 0.5, the volatilities of interest rate, asset’s fundamental, and debt capacity ratio are set to  $\sigma_r=0.01, \sigma_1=0.1, \text{ and } \sigma_g=0.02$ , respectively. The fraction  $\beta$  of default barrier in (10) is set to 0.9. For each of the 24 scenarios, Monte Carlo simulation of  $N=10,000$  runs are performed.

In Table 1, summary means and standard deviations of bank’s survival times of 24 various scenarios are given. Figure 1 shows a realization of downward debt capacity ratio corresponding to a pessimistic information structure ( $\theta_g=-0.05$ ), while Figure 2 shows a realization of an upward debt capacity ratio

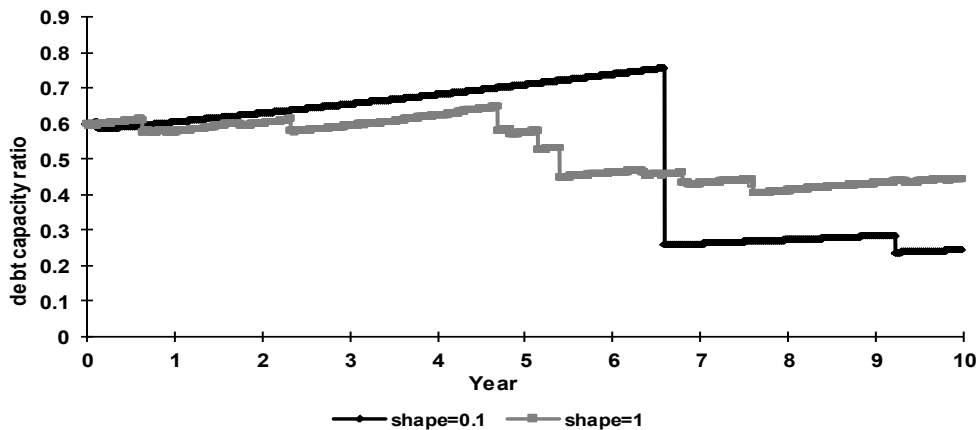
corresponding to an optimistic information structure ( $\theta_g=0.00$ ). In either of the plots, the smaller shape parameter, i.e.,  $\nu=0.1$ , shows larger jump sizes due to unusual economic events.

Table 1: Summary of Survival Times of 24 Scenarios

	$\pi$	$\sigma_2$	$\theta_g$	$\nu$	Mean	S.t.dev.
1	0.05	0.10	-0.05	0.10	1.1739	1.3861
2	0.05	0.10	-0.05	1.00	1.4048	1.6932
3	0.05	0.10	0.00	0.10	1.5322	1.9348
4	0.05	0.10	0.00	1.00	1.5774	1.9832
5	0.05	0.20	-0.05	0.10	2.2921	1.9850
6	0.05	0.20	-0.05	1.00	2.5515	2.1947
7	0.05	0.20	0.00	0.10	2.9696	2.4473
8	0.05	0.20	0.00	1.00	3.0804	2.5373
9	0.05	0.50	-0.05	0.10	1.0237	1.0970
10	0.05	0.50	-0.05	1.00	1.0749	1.1669
11	0.05	0.50	0.00	0.10	1.1519	1.3082
12	0.05	0.50	0.00	1.00	1.1814	1.2988
13	0.07	0.10	-0.05	0.10	2.6565	2.2843
14	0.07	0.10	-0.05	1.00	3.1196	2.6325
15	0.07	0.10	0.00	0.10	3.6562	2.8269
16	0.07	0.10	0.00	1.00	3.8018	2.9106
17	0.07	0.20	-0.05	0.10	3.0513	2.3569
18	0.07	0.20	-0.05	1.00	3.6450	2.5697
19	0.07	0.20	0.00	0.10	4.2689	2.7721
20	0.07	0.20	0.00	1.00	5.0024	3.0466
21	0.07	0.50	-0.05	0.10	1.1965	1.1893
22	0.07	0.50	-0.05	1.00	1.2919	1.3429
23	0.07	0.50	0.00	0.10	1.3576	1.4670
24	0.07	0.50	0.00	1.00	1.4616	1.5495

Note  $\pi$  is the equilibrium interest rate;  $\sigma_2$  is the volatility of the collateral asset's fundamental;  $\theta_g$  and  $\nu$  are the shift and shape parameters representing the information structure and the likelihood of unusual events, respectively, for the debt capacity ratio process

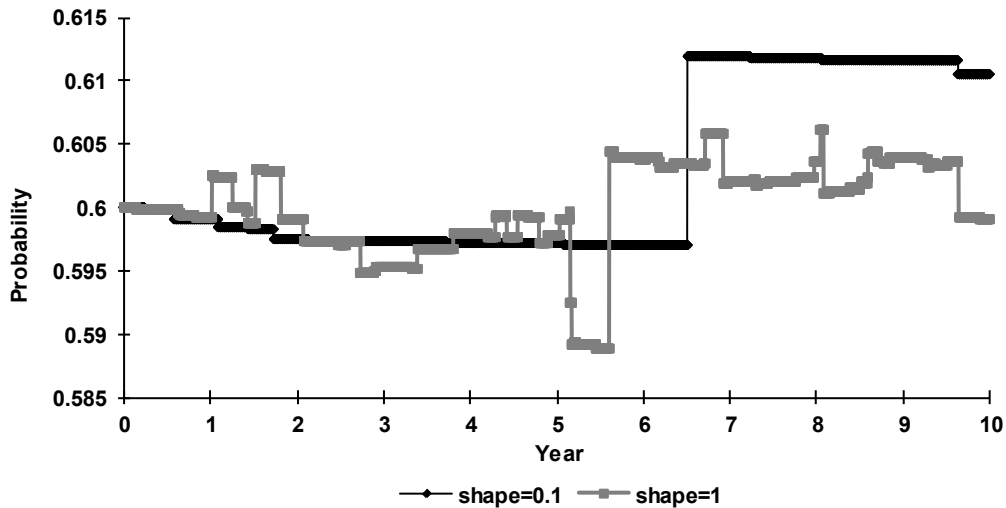
Figure 1: Debt Capacity Ratio in Pessimistic Information



This figure shows the jump behavior of debt capacity ratio when there are unusual events ( $\text{shape}=0.1$ ) versus no unusual events ( $\text{shape}=1$ ) under pessimistic information structure ( $\theta_g=-0.05$ ).

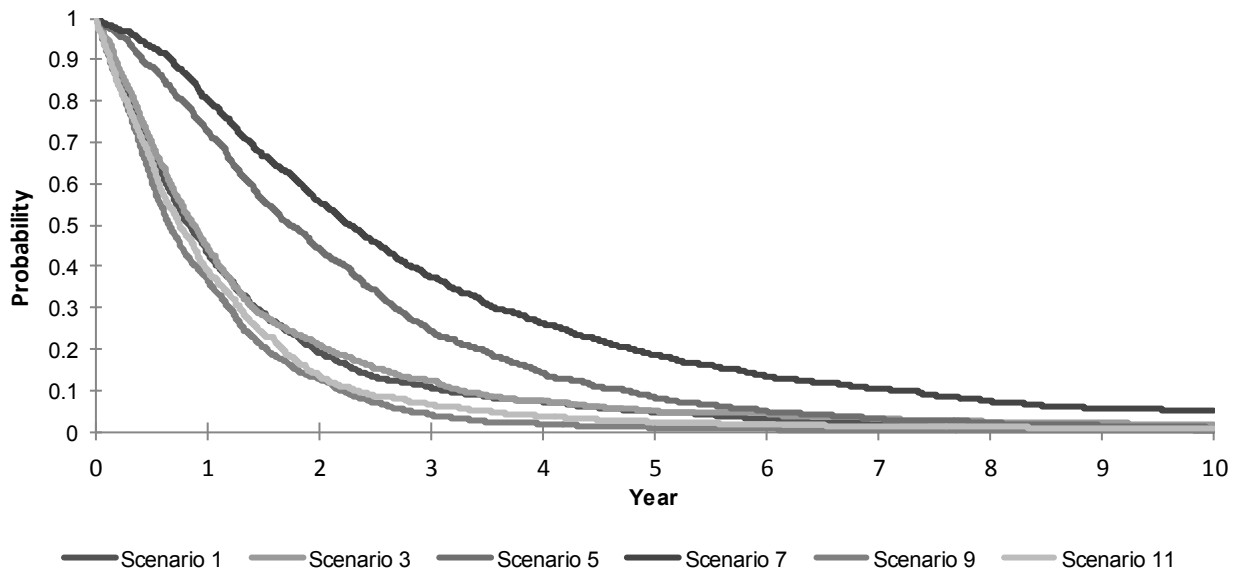
Figures 3-4 (Figures 5-6) plot the bank's survival probability curves  $S(t)$  for scenarios 1 to 12 (scenarios 13 to 24) corresponding to low interest rate (high interest rate) environment, respectively. In Figures 3 and 5, the survival curves for scenarios with unusual economic events, i.e.,  $\nu=0.1$ , are given; while survival curves for scenarios without unusual economic events, i.e.,  $\nu=1$ , are given in Figures 4 and 6.

Figure 2: Debt Capacity Ratio in Optimistic Information



This figure shows the jump behavior of debt capacity ratio when there are unusual events ( $shape=0.1$ ) versus no unusual events ( $shape=1$ ) under pessimistic information structure ( $\theta_g=-0.00$ ).

Figure 3: Survival Curves for Equilibrium Interest Rate 0.05

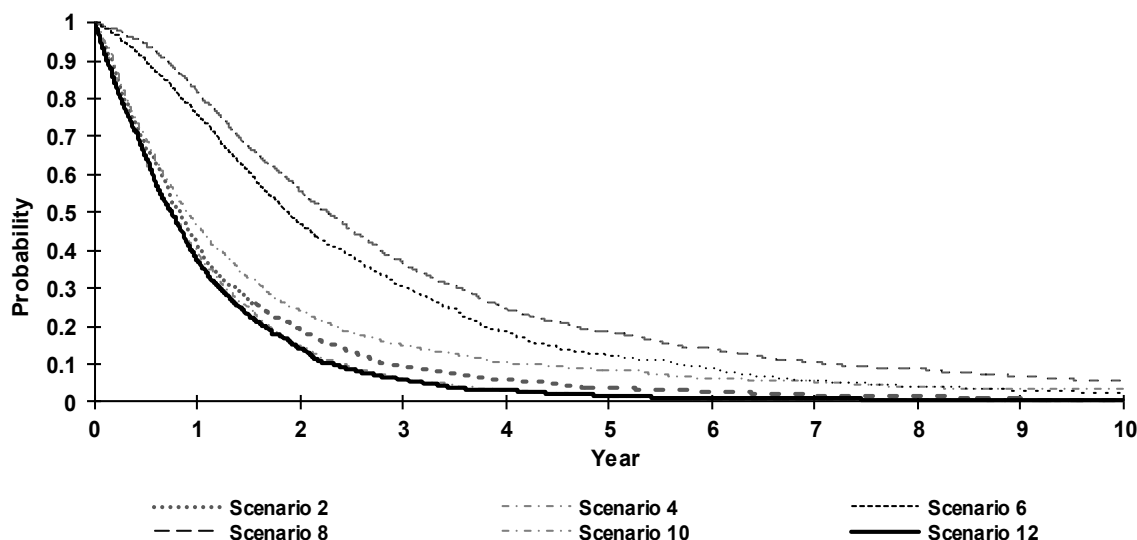


This figure shows the survival curves for scenarios 1,3,5,7,9,11 with unusual events ( $shape=0.1$ ) in low interest rate environment ( $\pi=0.05$ ). Among them, scenarios 1,5,9 are in pessimistic information structure ( $\theta_g=-0.05$ ); whereas scenarios 3,7,11 are in optimistic information structure ( $\theta_g=0.00$ ). The volatility of collateral asset's fundamental of scenarios 1,3 is low ( $\sigma_2=0.1$ ), of scenarios 5,7 is medium ( $\sigma_2=0.2$ ), whereas of scenarios 9,11 is high ( $\sigma_2=0.5$ ).

In either of the Figures 3-6, the medium collateral asset's volatility ( $\sigma_2=0.2$ ) exhibits the upper most survival curves (in cyan curves), while the high volatility ( $\sigma_2=0.5$ ) results in the lowest survival curves (in green curves). Similarly, in Table 1, the medium ( $\sigma_2=0.2$ ) and high volatility ( $\sigma_2=0.5$ ) give the highest and lowest mean survival times, respectively. This implies that banks relying on short-term debts have longer survival times if medium-risk collateral assets are invested. Nevertheless, high risk collateral assets significantly reduce bank's survival times, in both low and high interest rate environment.

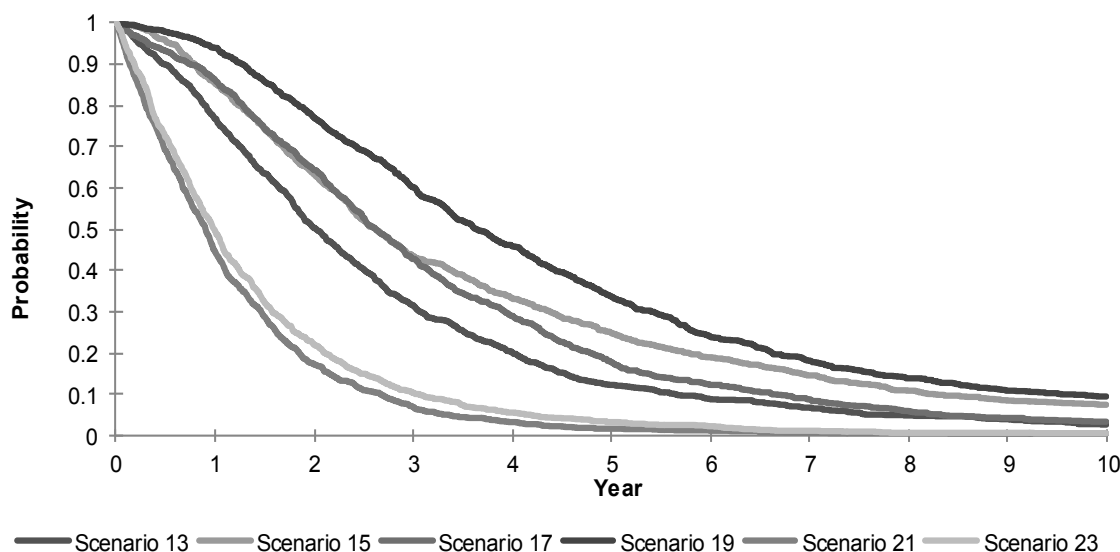


Figure 4: Survival Curves for Equilibrium Interest Rate 0.05



This figure shows the survival curves for scenarios 2,4,6,8,10,12 without unusual events ( $shape=1$ ) in low interest rate environment ( $\pi=0.05$ ). Among them, scenarios 2,6,10 are in pessimistic information structure ( $\theta_g=-0.05$ ); whereas scenarios 4,8,12 are in optimistic information structure ( $\theta_g=0.00$ ). The volatility of collateral asset's fundamental of scenarios 2,4 is low ( $\sigma_2=0.1$ ), of scenarios 6,8 is medium ( $\sigma_2=0.2$ ), whereas of scenarios 10,12 is high ( $\sigma_2=0.5$ ).

Figure 5: Survival Curves for Equilibrium Interest Rate 0.07

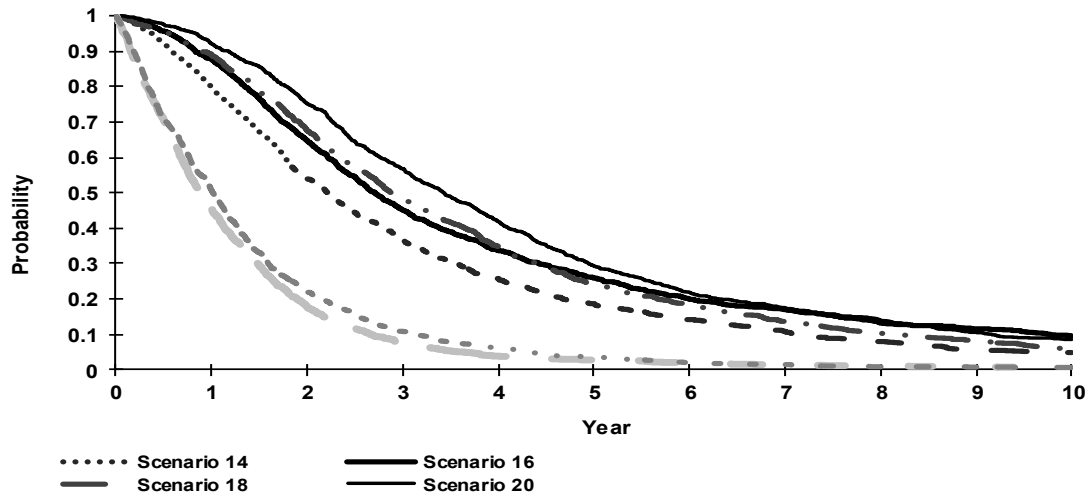


This figure shows the survival curves for scenarios 13,15,17,19,21,23 with unusual events ( $shape=0.1$ ) in high interest rate environment ( $\pi=0.07$ ). Among them, scenarios 13,17,21 are in pessimistic information structure ( $\theta_g=-0.05$ ); whereas scenarios 15,19,23 are in optimistic information structure ( $\theta_g=0.00$ ). The volatility of collateral asset's fundamental of scenarios 13,15 is low ( $\sigma_2=0.1$ ), of scenarios 17,19 is medium ( $\sigma_2=0.2$ ), whereas of scenarios 21,23 is high ( $\sigma_2=0.5$ ).

After comparing Figures 3-4 and 5-6, one finds that the larger equilibrium interest rate ( $\pi=0.07$ ) increases the survival times of a bank. This is due to the fact that higher long-term interest rate implies higher growth rate of the asset's fundamental and therefore larger collateral asset's market value, which implies larger debt capacity and longer survival time. The impact of the long-term equilibrium interest rate is more prominent when the collateral asset's volatility is small ( $\sigma_2=0.1$ ) compared to medium and high

volatilities ( $\sigma_2=0.2$  and  $0.5$ ). Figures 3-6 also show that optimistic information structure ( $\theta_g=0.00$ ) increases bank's survival times compared to the pessimistic information structure ( $\theta_g=-0.05$ ). Table 1 shows the consensus result that the mean survival times are larger in the optimistic information scenarios. After comparing Figures 3 with 4 (Figures 5 with 6), one finds the impact of the information structure on survival times is more prominent when the likelihood of unusual economic events is higher (low shape parameter  $\nu=0.1$ ).

Figure 6: Survival Curves for Equilibrium Interest Rate 0.07



*This figure shows the survival curves for scenarios 14,16,18,20,22,24 without unusual events (shape=1) in high interest rate environment ( $\pi=0.07$ ). Among them, scenarios 14,18,22 are in pessimistic information structure ( $\theta_g=-0.05$ ); whereas scenarios 16,20,24 are in optimistic information structure ( $\theta_g=0.00$ ). The volatility of collateral asset's fundamental of scenarios 14,16 is low ( $\sigma_2=0.1$ ), of scenarios 18,20 is medium ( $\sigma_2=0.2$ ), whereas of scenarios 22,24 is high ( $\sigma_2=0.5$ ).*

The impacts of long-term equilibrium interest rate and information structure almost disappears as the volatility  $\sigma_2$  of the collateral asset's fundamental increases to 0.5. This indicates that when the collateral asset's fundamental deteriorates so the volatility  $\sigma_2$  goes high enough; the collateral asset's fundamental dominates the effects of equilibrium interest rate as well as information structure.

Taken together, the four parameters, i.e., the long-term equilibrium interest rates, the volatility of the collateral asset's fundamental that is independent of the interest rate, the shift parameter that controls for the information structure in the market liquidity, and the likelihood of the occurrence of unusual economic events affect the bank's survival probabilities jointly. Among the four parameters, the impact of the collateral asset's volatility dominates in a way that as the volatility increases to a threshold, the variations for the remaining three parameters make no significant differences for the survival probabilities. In all cases, banks holding collateral assets with medium volatility ( $\sigma_2=0.2$ ) have the longest survival times. When the information structure in the market liquidity is pessimistic, maintaining the long-term interest rate at a lower level can offset the differences between optimistic and pessimistic information structures only for the low and medium volatilities of collateral asset ( $\sigma_2=0.1$  and  $0.2$ ). However, the survival curve for banks holding low volatility collateral assets, i.e.,  $\sigma_2=0.1$ , is approximating to that of high volatility, i.e.,  $\sigma_2=0.5$ , in the low equilibrium interest rate scenarios ( $\pi=0.05$ ). This might provide an explanation of the empirically observed phenomenon that banks default even when the qualities of their fundamentals are still high (He and Xiong, 2010a; Acharya, Gale, and Yorulmazer, 2009).

**CONCLUSION**

In this study, the derivation of the survival/default probabilities for banks holding long-term assets and short-term debts based on a structural model that takes into consideration of collateral asset’s fundamentals and market liquidity is given. The attractive feature of the proposed model includes: (1) An interest rate sensitive fundamental of the collateral assets is assumed; (2) A stochastic interest rate environment is assumed; (2) A purely jump stochastic process is used to model the debt capacity ratio. For simplicity, it is assumed that the interest rate sensitive fundamental and the market liquidity are independent. Nevertheless, as the deterioration of debt market liquidity caused severe financing difficulties for banks, which in turn may exacerbate the fundamental of their assets, therefore the importance of the interaction between the assets’ fundamental and the liquidity should not be ignored (Brunnermeier, 2009; Krishnamurthy, 2010). An extension of the proposed structural model by incorporating the correlation between the interest rate sensitive fundamental and the market liquidity will be studied in the future.

**APPENDIX**

Proof of Lemma 1: Consider the discounted process  $F(t)=R(t)/P(t, T)$ , where  $T>t$ ,  $R(t)$  is the fundamental defined in (2),  $P(t, T)$  is the time- $t$  price of a default-free zero-coupon bond that matures at time  $T$ . Using (2) and Ito’s lemma, one has

$$dF(t)/F(t)=[\sigma_1 - s(t, T)]dZ_1 + \sigma_2 dZ \tag{A1}$$

where  $dZ_1=dZ_r - s(t, T)dt$  and  $s(t, T)=\frac{\sigma_r(1 - e^{-\theta(T-t)})}{\theta}$ ,  $\theta$  and  $\sigma_r$  are the revert rate and volatility for the interest rate  $r$  in (1), respectively. As

$$\exp\left(\int_0^T \frac{\sigma_r^2 e^{-2\theta(T-t)}}{2\theta^2} dt\right) < \infty$$

By Girsanov’s theorem, there exists a forward measure  $Q_1$  equivalent to the risk neutral measure  $Q$ , such that  $Z_1$  defined above is a Brownian motion under  $Q_1$ . Thus,  $F(t)$  is a martingale under  $Q_1$ . See Musiela and Rutkowski (1997) for details. It follows that the market value in (3) can be rewritten as  $V(t)=P(t, T)E_t[f(R(T))]$ , where the expectation is taken under the forward measure  $Q_1$ . Furthermore, from (A1), given the information set  $\mathcal{F}_t$ ,  $F(T)$  is lognormally distributed under  $Q_1$  with mean

$$m = \log(F(t)) - \frac{b^2}{2}$$

and variance in (6). Since  $F(T)=R(T)$ , it can be shown the expectation

$$E_t[f(R(T))] = \int_{\log(R^*)}^{\infty} e^s \phi\left(\frac{s - m}{b}\right) ds = F(t)\Phi(w)$$

where the constant  $w$  is given in (5),  $\phi$  is the standard normal density and the cash flows  $f(R(T))=R(T)$  if  $R(T) \geq R^*$ , else  $f(R(T))=0$ .

Derivation of Eqs. (7)-(8). A variance-gamma process  $X(t)=\theta_g g_t + \sigma_g Z_g(g_t)$  is a Brownian motion with

drift and volatility  $\theta$  and  $\sigma_g$ , respectively, time-changed by a random scale  $g_t=g(t)-g(0)$ , which is the increment of a gamma process  $\{g(s; \nu, \gamma): s>0\}$  with shape parameter  $\nu$  and scale parameter  $\gamma$ , respectively, during time interval  $(0, t]$ . According to Jacod and Shiriyayev (1987), the process  $\exp\{iuX(t)-\psi_t(u)\}$  is a semi-martingale, where  $\psi_t(u)$  is the logarithm of the characteristic function  $E[\exp(iuX(t))]$ , which can be obtained by first conditioning on the gamma distributed random time-change  $g_t$  and applying the characteristic function of a normal distribution, then using the characteristic function of a gamma distribution with shape and scale parameters  $\nu t$  and  $\gamma$ , respectively, to obtain

$$\psi_t(u) = -\nu t \times \ln\left(1 - i \frac{\theta_g}{\nu} u + \frac{1}{2\nu} \sigma_g^2 u^2\right) \tag{A2}$$

As in Madan et al. (1991, 1998), let the scale parameter  $\gamma=1/\nu$ . To ensure positivity, let the debt's capacity ratio follows the semi-martingale process, i.e.,  $\exp\{iuX(t)-\psi_t(u)\}$  with  $u=1/i$ . It follows that the time- $t$  debt's capacity ratio is

$$\alpha_t = \alpha_{t_0} \times \exp\left(X(t) + \nu t \times \ln\left(1 - \frac{\theta_g}{\nu} - \frac{\sigma_g^2}{2\nu}\right)\right) \tag{A3}$$

Let  $t_0=(i-1)\Delta t$  and  $t=i\Delta t$ , the logarithm of the ratio of the debt's capacity ratios in (7) can be obtained from (A3) and noting that  $X(t)=\theta_g g_t + \sigma_g Z_g(g_t)$ , the innovation  $\varepsilon_i = \sigma_g Z_g(g)$  and  $g=g(i\Delta t)-g((i-1)\Delta t)$ . As  $g$  is the increment of a gamma process with shape parameter  $\nu$  and scale parameter  $\gamma=1/\nu$ , respectively, therefore,  $g$  is a gamma-distributed with shape parameter  $\nu\Delta t$  and scale parameter  $\gamma=1/\nu$ , respectively. As the innovation  $\varepsilon_i = \sigma_g Z_g(g)$ , therefore given the increment  $g$ , the innovation  $\varepsilon_i$  is Gaussian distributed. Eqs. (7)-(8) are derived.

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