PRICING OF PAYMENT DEFERRED VULNERABLE OPTIONS AND ITS APPLICATION TO VULNERABLE RANGE ACCRUAL NOTES

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ABSTRACT

This paper derives a pricing model for payment deferred vulnerable options and applies the results to the pricing of vulnerable range accrual notes. The valuation model for vulnerable options takes into account the possibility of the option writer defaulting. However, when the payment date is set later than the option maturity date, the valuation model will be incomplete if the default risk between the option maturity and payment dates is not explicitly incorporated. We extend the current available models and our results show that the default risk of the option writer will further reduce the option value if the payment date is after the maturity date. The analysis of vulnerable range accrual note, which contains multiple payment deferred vulnerable options, is also performed. Due to the product design, the pricing model for vulnerable range accrual notes shows that the relationship between volatility and note value is not monotonic but depends on whether the underlying price is within, outside, or on the range boundary.

JEL: G12; G13

KEYWORDS: Reduced form model, vulnerable options, vulnerable range accrual notes

INTRODUCTION

This paper derives a pricing model for payment deferred vulnerable options and applies the results to the pricing of vulnerable range accrual notes. The valuation model for vulnerable options takes into account the fact that the option writer may default. However, when the payment date is set later than the option maturity date, a common arrangement in the OTC structured products market, the valuation model will be incomplete if the default risk between the option maturity and payment dates is not explicitly incorporated. We extend the current available models, which usually assume that the option maturity and payment dates are identical. Our results show that the default risk of the option writer will further reduce the option value if the payment date is after the maturity date.

One practical application of the payment deferred vulnerable option valuation model is in the valuation of vulnerable range accrual notes. Range accrual notes are structured products. Its payoff is defined as the interest payment computed as the proportion of the number of days that the reference underlying asset price lies within a specified range times the interest rate specified at the initiation of the note. The specified interest rate is usually set much higher than the interest rate currently available on the market. Therefore, it gives the note holders a chance to get higher earnings. For this reason, the range accrual note is attractive to investors, especially in a low interest rate environment. The analysis of vulnerable range accrual note, which contains multiple payment deferred vulnerable options, is also performed.

The paper is organized as follows. Section 2 provides a pricing model for payment deferred options. Since range accrual notes can be regarded as combinations of range options, which are combinations of digital options, Section 3 discusses the valuation of digital options and range options. Section 4 then applies the results in Sections 2 and 3 to the pricing of vulnerable range accrual notes. Finally, Section 5 presents our conclusions.

LITERATURE REVIEW

Black and Sholes (1973) value options by constructing a no-arbitrage portfolio and employ the partial differential equation (PDE) technique to derive the closed-form solution for European options. The martingale pricing method (Harrison and Kreps, 1979; Harrison and Pliska, 1981) is efficient in reducing the complexity of pricing processes, and it is now widely used in option valuation. Cox, Ross, and Rubinstein (1979) propose the binomial option pricing model that can handle various types of options, especially American options. For more complex options such as path-dependent options, it is more suitable to apply the Monte Carlo simulation method (Boyle, 1977).

To address the credit risk that the option writer may default, valuation models based on the structural approach (Merton, 1974) have been proposed by various researchers. Notably, Johnson and Stulz (1987), Klein (1996), and Klein and Inglis (1999, 2001) derive pricing models for plain vanilla vulnerable options, assuming that the possible default time may occur on the option maturity date. Liao and Huang (2005) extend the model to assume that the possible default time may be anytime before the option maturity date.

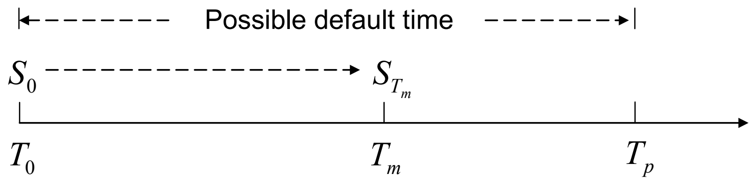
The structural approach for pricing European vulnerable options usually assumes that the evolution of the total asset value of the option writer follows a stochastic process in addition to the process followed by the underlying asset price of the option. As well-documented as it is in literature, it is difficult to estimate the volatility parameter for the process followed by the total asset value of the option writer. Empirically, when the two processes are not required simultaneously, the volatility parameter estimated for the stock price is used as a proxy for the total asset value (Gray *et al*., 2007). However, the legitimacy of this practice has always been questioned. The reduced form approach may circumvent this problem in the pricing of vulnerable options, as proposed by Jarrow and Turnbull (1995) and Hull and White (1995), among others. Therefore, in this paper, we employ the reduced form model to describe the default process of the option issuer and extend the research of vulnerable option pricing to payment deferred vulnerable option.

Turnbull (1995) assumes the term structure is exogenous and derives the closed-form solution of an interest rate range note. Navatte and Quittard-Pinon (1999) price range notes through a derivation of the embedded European range digital option value. They assume that the interest rate dynamics follows a one-factor linear Gaussian model and employ the change of numeraire approach to derive analytical solutions. Nunes (2004) extends the Navatte and Quittard-Pinon model to a multifactor HJM term structure. Eberlein and Kluge (2006) generalize the afore-mentioned results to the multivariate levy term-structure. Our paper differs from the above in that the option is assumed to be vulnerable.

PRICING OF PAYMENT DEFERRED OPTIONS

A payment deferred option can be described with Figure 1. In Figure 1, T_m is the option maturity date, on which the payoff is decided; T_p is the payment date; S_0 is the underlying price at the initiation of the option; and S_{T_m} is the underlying price on the option maturity date. The difference between a payment deferred option and a plain vanilla option is that the payment date of a plain vanilla is set identical to the option maturity date, and the payment date of a payment deferred option is set later than the option maturity date. This situation is usually seen in some structured notes, such as range accrual notes.

Figure 1: The Vulnerable Payment Deferred Option



This figure illustrates that the possible default time of the option writer is in the time period between the initiation T_0 *and the payment date* T_p *of the option. Once the option writer default, the option holder will not receive the exercise value when the option is in the money at the maturity date.*

Assuming that the underlying asset price follows a geometric Brownian motion, its dynamic under risk neutral probability measure Q is governed by the following process:

$$\frac{dS_t}{S_t} = (r-q)dt + \sigma\, dW_t^Q \tag{1}$$

where S_t is the underlying asset price at time t; r is the risk free rate; q is the dividend rate of the underlying asset; σ is the volatility of the underlying asset price return; W_t^Q is the Wiener process under probability measure Q; and $dW_t^Q \sim N(0, dt)$. Using Itô Lemma, we obtain the following equation:

$$d \ln S_t = (r - q - \frac{\sigma^2}{2})dt + \sigma\, dW_t^Q \tag{2}$$

By integrating both sides of Equation (2), the dynamic process of the asset price can be described as:

$$\ln S_{T_m} = \ln S_t + (r - q - \frac{\sigma^2}{2})(T_m - t) + \sigma\,(W_{T_m}^Q - W_t^Q) \tag{3}$$

where $(W_{T_m}^Q - W_t^Q) \sim N(0, T_m - t)$.

Assuming that the default time of the option writer τ_D follows an exponential distribution:

$$\Pr^Q(\tau_D \le T_p | F_t) = 1 - e^{-\lambda(T_p - t)} \tag{4}$$

where $\Pr^Q(\bullet)$ is the probability function in risk neutral probability measure, F_t is the information set at time t, λ is the default intensity of the option writer, and λ is a constant.

Following Jarrow and Turnbull (1995) and Hull and White (1995), we assume that the process for the asset underlying the option is independent of the credit risk of the option writer. This assumption amounts to considering that the option writer is a large, well-diversified financial institution, a realistic assumption as observed in emerging markets. Suppose that the loss rate of the issuer default is β, $0 \le \beta \le 1$, and that the issuer default process is independent of the dynamic process of the underlying price. The value of a payment deferred vulnerable call option $C(t)$ can then be derived as follows:

$$
\begin{aligned}
C(t) &= e^{-r(T_p-t)}\left\{E^Q\left[(1-\beta)(S_{T_m}-K)^{+}1_{\{\tau_D\le T_p\}}\middle|F_t\right]+E^Q\left[(S_{T_m}-K)^{+}1_{\{\tau_D> T_p\}}\middle|F_t\right]\right\}\\
&= e^{-r(T_p-t)}E^Q\left[(S_{T_m}-K)^{+}-\beta(S_{T_m}-K)^{+}1_{\{\tau_D\le T_p\}}\middle|F_t\right]\\
&= e^{-r(T_p-t)}\left\{E^Q\left[(S_{T_m}-K)^{+}\middle|F_t\right]-\beta E^Q\left[(S_{T_m}-K)^{+}\middle|F_t\right]E^Q\left[1_{\{\tau_D\le T_p\}}\middle|F_t\right]\right\}\\
&= e^{-r(T_p-t)}\left[1-\beta\Pr{}^Q(\tau_D\le T_p|F_t)\right]\left[S_t e^{(r-q)(T_m-t)}N(d_1)-KN(d_2)\right]\\
&= e^{-r(T_p-T_m)}\left[1-\beta\left(1-e^{-\lambda(T_p-t)}\right)\right]\left[S_t e^{-q(T_m-t)}N(d_1)-Ke^{-r(T_m-t)}N(d_2)\right]
\end{aligned}
\tag{5}
$$

where $E^Q[\bullet]$ is the expectation under risk neutral probability measure Q,
$N(\bullet)$ is the cumulative standardized normal distribution function,

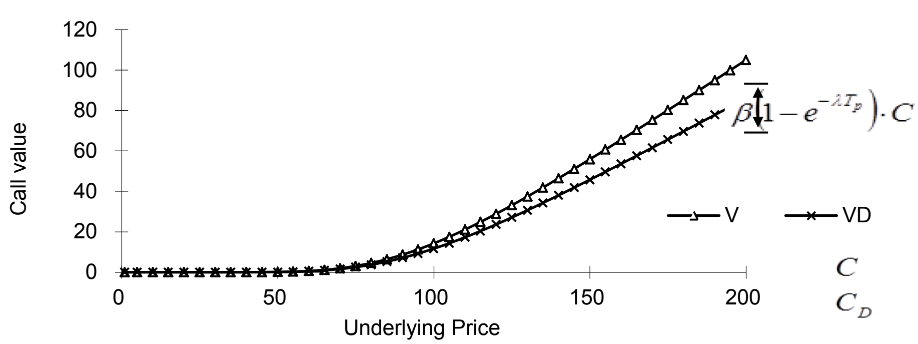
$d_1=\dfrac{\ln\dfrac{S_t}{K}+(r-q+\dfrac{\sigma^2}{2})(T_m-t)}{\sigma\sqrt{(T_m-t)}}$, and $d_2=\dfrac{\ln\dfrac{S_t}{K}+(r-q-\dfrac{\sigma^2}{2})(T_m-t)}{\sigma\sqrt{(T_m-t)}}$.

Similarly, the value of a payment deferred vulnerable put option $P(t)$ is

$$
P(t)=e^{-r(T_p-T_m)}\left[1-\beta\left(1-e^{-\lambda(T_p-t)}\right)\right]\left[Ke^{-r(T_m-t)}N(-d_2)-S_t e^{-q(T_m-t)}N(-d_1)\right]
\tag{6}
$$

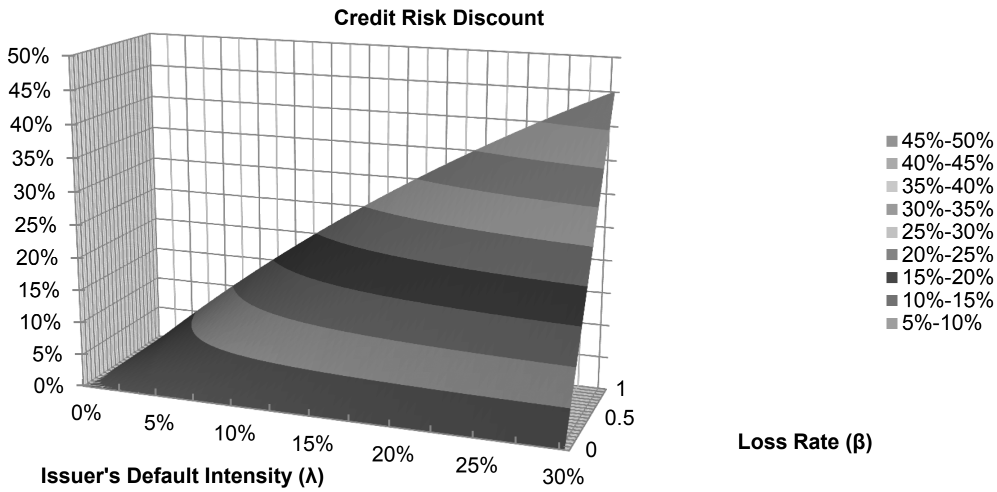
Equations (5) and (6) show that the default risk of the option writer will further reduce the option value if the payment date is later than that of the maturity. When $T_p=T_m$, the payment deferred vulnerable options is reduced to a plain vanilla vulnerable option. In Equation (5), we can see that the value of a payment deferred vulnerable option is the option value without the issuer default risk multiplied by one minus a credit risk discount $\beta\left(1-e^{-\lambda T_p}\right)$, which equals the loss rate β multiplied by the issuer default probability $1-e^{-\lambda T_p}$ before the payment date. The difference in call option values between without issuer default risk C and with issuer default risk C_D is illustrated in Figure 2. Furthermore, as shown in Figure 3, the credit risk discount for the issuer default risk is monotonically positively correlated to the loss rate β and the issuer default intensity λ.

Figure 2: The Call Option Values Without Issuer Default Risk C and With Issuer Default Risk C_D ($K=100$, $T=1$, $\sigma=30\%$, $r=5\%$, $\beta=1$, $\lambda=20\%$)



This figure shows that the option value without issuer default risk is higher than those with issuer default risk and the difference is $\beta\left(1-e^{-\lambda T_p}\right)$*, where* β *is the loss rate,* λ *is the issuer default intensity, and* T_p *is payment date.*

Figure 3: Credit Risk Discount for Issuer Default Risk



This figure shows that the credit risk discount due to the possibility of issuer default is positively correlated to the default intensity of the issuer and the loss rate.

PRICING OF VULNERABLE DIGITAL OPTIONS AND RANGE OPTIONS

Since range accrual notes can be regarded as combinations of range options, which, in turn, are combinations of digital options, we discuss the valuation of digital options and range options in this section. The final payoff of a digital option is decided by the condition that the underlying asset price satisfies certain specifications. The payoff of a digital call (DC) option can be written as follows:

$$DC_T = \begin{cases} H, & if\ S_{T_m} \geq K \\ 0, & otherwise \end{cases} \tag{7}$$

where H is the fixed amount received by the option holder if the digital call option is in the money on the maturity date. Following the settings of the underlying asset price dynamic, the issuer default process, the option maturity date, and the option payment date in the previous section, the value of a vulnerable digital call option will be:

$$\begin{aligned} DC(t) &= e^{-r(T_p-t)}\left\{E^Q\left[(1-\beta)H\cdot 1_{\{K\leq S_{T_m}\}}\cdot 1_{\{\tau_D\leq T_p\}}\middle|F_t\right]+E^Q\left[H\cdot 1_{\{K\leq S_{T_m}\}}1_{\{\tau_D>T_p\}}\middle|F_t\right]\right\} \\ &= e^{-r(T_p-t)}H\cdot E^Q\left[1_{\{K\leq S_{T_m}\}}-\beta\cdot 1_{\{K\leq S_{T_m}\}}\cdot 1_{\{\tau_D\leq T_p\}}\middle|F_t\right] \\ &= e^{-r(T_p-t)}H\cdot\left\{E^Q\left[1_{\{K\leq S_{T_m}\}}\middle|F_t\right]-\beta E^Q\left[1_{\{K\leq S_{T_m}\}}\middle|F_t\right]E^Q\left[1_{\{\tau_D\leq T_p\}}\middle|F_t\right]\right\} \\ &= e^{-r(T_p-t)}H\cdot\left[1-\beta\Pr^Q(\tau_D\leq T_p|F_t)\right]\left[\Pr^Q(K\leq S_{T_m}|F_t)\right] \\ &= e^{-r(T_p-t)}H\cdot\left[1-\beta\left(1-e^{-\lambda(T_p-t)}\right)\right]N(d_2) \end{aligned} \tag{8}$$

Similarly, the payoff of a digital put (DP) option can be written as follows:

$$DP_T = \begin{cases} H, & if\ S_{T_m} \leq K \\ 0, & otherwise \end{cases} \tag{9}$$

Therefore, the value of a vulnerable digital put (DP) option is:

$$DP(t) = e^{-r(T_p - t)} H \cdot \left[1 - \beta\left(1 - e^{-\lambda(T_p - t)}\right)\right] N(-d_2) \tag{10}$$

One type of digital option is the range option, which has a payoff if the underlying asset price lies in the specified range on the maturity date. The payoff of a range option can be written as follows:

$$RO_T = \begin{cases} H, & \text{if } D \leq S_{T_m} \leq U \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

where D and U are the lower and upper boundaries of the target range, respectively.

If an investor longs a digital call option with the exercise price D and shorts a digital call option with the exercise price U, the payoff is the same as buying a range option with the lower boundary D and the upper boundary U. Thus the value of a vulnerable range option will be:

$$\begin{aligned} RO(t) &= DC_{K=D}(t) - DC_{K=U}(t) \\ &= e^{-r(T_p - t)} H \cdot \left[1 - \beta\left(1 - e^{-\lambda(T_p - t)}\right)\right]\left[N(d^D) - N(d^U)\right] \end{aligned} \tag{12}$$

where $N(\cdot)$ is the cumulative standardized normal distribution function,

$$d^D = \frac{\ln\frac{S_t}{D} + (r - q - \frac{\sigma^2}{2})(T_m - t)}{\sigma\sqrt{T_m - t}}, \text{ and } d^U = \frac{\ln\frac{S_t}{U} + (r - q - \frac{\sigma^2}{2})(T_m - t)}{\sigma\sqrt{T_m - t}}.$$

PRICING OF VULNERABLE RANGE ACCRUAL NOTES

The final payoff of a range accrual note can be divided into the principal and interest payments. The principal payment is similar to the cash flow of a zero coupon bond and is decided by the principal guarantee ratio, such as 100%, 95%, etc. The interest payment of a range accrual note is defined as the proportion of the number of days that the reference underlying asset price lies within a specified range times an interest rate specified at the start of the contract.

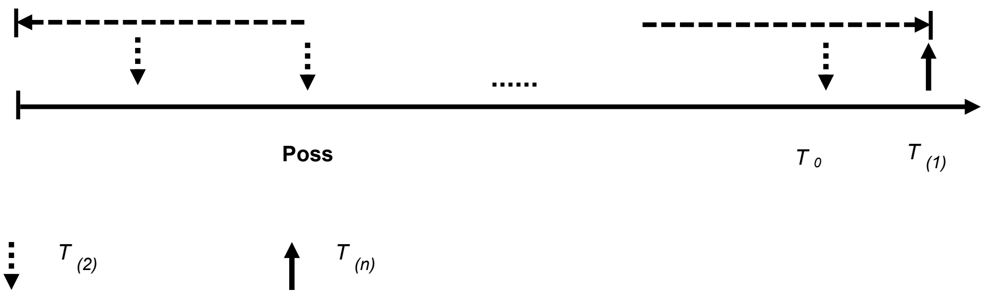
Assume that the nominal of the note is F; the principal guarantee ratio is α; the target range is $[D, U]$, D and U are the lower and upper boundaries of the target range respectively; the note maturity and interest payment date is T_p; the specified interest rate is R; the observation frequency is n; and the observation dates are $T(1)$, $T(2)$, …, $T(n)$. The payment ratio is ϕ, usually sets at around 1 over 260. Some security firms may set this ratio at 1 over 255, and others according to the proportion of trading days in one year. As shown in Figure 4, the interest payment on date T_p can be written as:

$$Coupon = \sum_{i=1}^{n} \phi \cdot R \cdot F \cdot I_{T(i)} \tag{13}$$

where $I_{T(i)} = \begin{cases} 1, & \text{if } D \leq S_{T(i)} \leq U \\ 0, & \text{otherwise} \end{cases}$

The indicator function I can be viewed as a range option, the payoff of which is one dollar if the option is in the money and zero if it is out of the money. According to the above setting, the interest payment of a range accrual note can be expressed as the sum of the final payoffs of a series of range options. Therefore, the value of the range accrual note at time t can be expressed as the sum of the discounted principal payment and a series of range options. We can derive the generalized pricing formula of the vulnerable range accrual note (denoted as $Note$) as follows:

Figure 4: The Observation Date and Payment Date of the Range Accrual Note



This figure illustrates that the observation dates for determination of the coupon payment and the payment date of the coupon. The possible default time of the issuer is in the time period between the initiation of the note and the payment date. Once the issuer default, the note holder will not receive the coupon payment.

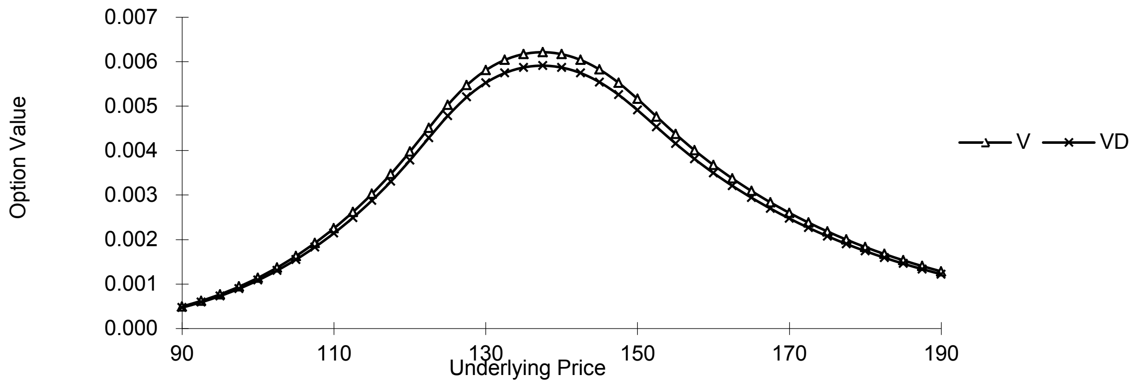
$Note(t)$ = the discounted vulnerable principal payment + the value of a series of vulnerable range options

$$
\begin{aligned}
&= e^{-r(T_p-t)}\left\{E^Q\left[(1-\beta)\alpha F\cdot 1_{\{\tau_D\le T_p\}}\middle|F_t\right]+E^Q\left[\alpha F\cdot 1_{\{\tau_D>T_p\}}\middle|F_t\right]\right\}+\\
&\quad e^{-r(T_p-t)}\left\{E^Q\left[(1-\beta)\cdot\sum_{i=1}^{n}\phi RF I_{T(i)}\cdot 1_{\{\tau_D\le T_p\}}\middle|F_t\right]+E^Q\left[\sum_{i=1}^{n}\phi RF I_{T(i)}\cdot 1_{\{\tau_D>T_p\}}\middle|F_t\right]\right\}\\
&= e^{-r(T_p-t)}\left\{\left[1-\beta(1-e^{-\lambda(T_p-t)})\right]\alpha F+\right.\\
&\quad\left.\sum_{i=1}^{n}\phi RF\left\{E^Q\left[(1-\beta)\cdot I_{T(i)}\cdot 1_{\{\tau_D\le T_p\}}\middle|F_t\right]+E^Q\left[I_{T(i)}\cdot 1_{\{\tau_D>T_p\}}\middle|F_t\right]\right\}\right\}\\
&= e^{-r(T_p-t)}\left\{\left[1-\beta(1-e^{-\lambda(T_p-t)})\right]\alpha F+\left[1-\beta(1-e^{-\lambda(T_p-t)})\right]\phi RF\cdot\sum_{i=1}^{n}E^Q\left[I_{T(i)}\middle|F_t\right]\right\}\\
&= e^{-r(T_p-t)}F\left[1-\beta(1-e^{-\lambda(T_p-t)})\right]\left\{\alpha+\phi R\sum_{i=1}^{n}\left[N(d_i^D)-N(d_i^U)\right]\right\}
\end{aligned}
\tag{14}
$$

where $d_i^D=\dfrac{\ln\dfrac{S_t}{D}+(r-q-\dfrac{\sigma^2}{2})\left[T(i)-t\right]}{\sigma\sqrt{T(i)-t}}$, and $d_i^U=\dfrac{\ln\dfrac{S_t}{U}+(r-q-\dfrac{\sigma^2}{2})\left[T(i)-t\right]}{\sigma\sqrt{T(i)-t}}$.

The vulnerable option pricing model shows that the default risk of the option writer reduces the option value. This feature also shows up in the valuation of the vulnerable range accrual note. Take a range accrual note as an example to illustrate the price behavior of the embedded option. The underlying asset of this range accrual note is an equity security. The other parameters of this range accrual note are as follows: the note maturity and interest payment date T_p is three months after the issue date; the principal guarantee ratio α is 100%; the underlying asset price on the issue date S_0 is 137; the target range is [$90\%\times S_0$, $110\%\times S_0$], which means the lower boundary D is $90\%\times S_0$ and the upper boundary U is $110\%\times S_0$; the payment ratio ϕ is 1/260; the specified interest rate R is 5.3%; the risk free interest rate is set as the three months Commercial Paper (CP) rate on the issue date, which is 0.943%; the volatility of the underlying asset price return σ is 53.38%; and the dividend yield of the underlying asset q is 0. Figure 5 shows the embedded option values of the range accrual note without issuer default risk V and with issuer default risk V_D for different underlying asset prices.

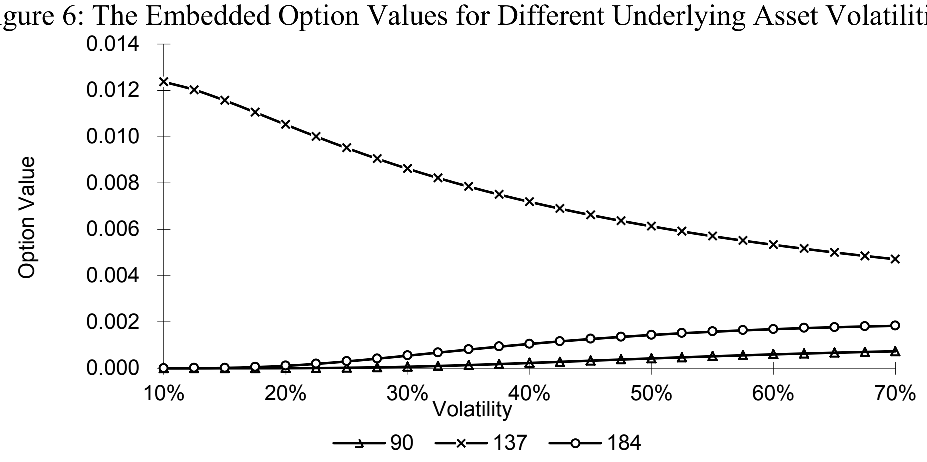
Figure 5: The Embedded Option Values of the Range Accrual Note Without Issuer Default Risk V and With Issuer Default Risk V_D



This figure shows that the option values with issuer default risk V_D *is higher than those without issuer default risk* V *(The issuer loss rate* $\beta = 1$ *; the issuer default intensity* $\lambda = 20\%$ *).*

Further, due to the product design, the pricing model for vulnerable range accrual notes demonstrates that the relationship between the underlying asset volatility and the embedded option value is not monotonic but depends on whether the underlying price is within, outside, or on the range boundary. This is shown in Figure 6. Generally speaking, when the underlying asset price is within the target range, the embedded option value increases with decreasing volatility. On the other hand, when the underlying asset price is outside the target range, the embedded option value increases with increasing volatility.

Figure 6: The Embedded Option Values for Different Underlying Asset Volatilities



This figure shows that when the underlying asset price is within the target range, i.e. S_t *=137, the embedded option value increases with decreasing volatility. On the other hand, when the underlying asset price is outside the target range, i.e.* S_t *= 90 and 184, the embedded option value increases with increasing volatility.*

The reason for the above findings is that when the underlying asset price is within the target range, the lower the volatility is, the lower the probability that the underlying asset price breaches the target range will be, which leads to a higher embedded option value. Conversely, when the underlying asset price is outside the target range, the higher the volatility is, the higher the probability that the underlying asset price moves back into the target range will be, which leads to a higher embedded option value.

CONCLUSIONS

In pricing options, it is common to assume that the payment date is identical to the maturity date. The main concern of this paper is the credit risk of the option writer. Therefore, the credit risk involved between option maturity and payment dates should be incorporated in the pricing of payment deferred vulnerable options. We assume that the underlying asset price follows a geometric Brownian motion, and employ the reduced form model to describe the default process of the option issuer. The research results show that the default risk of the option writer will further reduce the option value if the payment date is after the maturity date. Additionally, an interesting finding about the range accrual note is that the note value will not change monotonically with the volatility but will depend on whether the underlying price is within, outside, or on the range boundary. This is a special feature that the note writer has to focus on when hedging is conducted. In this work, we assume an independent relationship between the underlying asset price and the option writer's default. This assumption is reasonable for a large and well-diversified option writer. In order to handle more general cases, we recommend that future studies should consider the correlation between the underlying asset price and the option writer's default.

REFERNCES

Black, F. and Scholes, M. (1973) "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, vol. 81, p. 637-659.
Boyle, P. (1977) "Options: A Monte Carlo Approach," *Journal of Financial Economics*, vol. 4, p. 323-338.

Cox, J., Ross, S. and Rubinstein, M. (1979) "Option Pricing: A Simplified Approach," *Journal of Financial Economics*, vol. 7, p. 229-264.

Eberlein, E. and Kluge, W. (2006) "Valuation of Floating Range Notes in Levy Term-Structure Models," *Mathematical Finance*, vol. 16, p. 237-254.

Gray, D. F., Merton, R. C. and Bodie, Z. (2007) "Contingent Claims Approach to Measuring and Managing Sovereign Credit Risk," *Journal of Investment Management*, vol. 5(4), p. 1-24.

Harrison, M. and Kreps, D. (1979) "Martingales and Arbitrage in Multiperiod Securities Markets," *Journal of Economic Theory*, vol. 20, p. 381-408.

Harrison, M. and Pliska, S. R. (1981) "Martingales and Stochastic Integrals in the Theory of Continuous Trading," *Stochastic Processes and Applications*, vol. 11, p. 215-260.

Hull, J. C. and White, A. (1995) "The Impact of Default Risk on the Prices of Options and Other Derivative Securities," *Journal of Banking and Finance*, vol. 19, p. 299-322.

Jarrow, R. and Turnbull, S. (1995) "Pricing Derivatives on Financial Securities subject to Credit Risk," *Journal of Finance*, vol. 50, p. 53-85.

Johnson, H. and Stulz, R. (1987) "The Pricing of Options with Default Risk," *Journal of Finance*, vol. 42, p. 267-280.

Klein, P. (1996) "Pricing Black-Scholes Options with Correlated Credit Risk," *Journal of Banking and*

Finance, vol. 20, p. 1211-1229.

Klein, P. and Inglis, M. (1999) "Valuation of European Options Subject to Financial Distress and Interest Rate Risk," *Journal of Derivatives*, vol. 6, p. 44-56.

Klein, P. and Inglis, M. (2001) "Pricing Vulnerable European Options when the Option's Payoff can Increase the Risk of Financial Distress," *Journal of Banking and Finance*, vol. 25, p. 993-1012.

Liao, S. L. and Huang, H. H. (2005) "Pricing Black-Scholes Options with Correlated Interest Rate Risk and Credit Risk: An Extension," *Quantitative Finance*, vol. 5(5), p. 443-457.

Merton, R. C. (1974) "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, vol. 29, p. 449-470.

Navatte, P. and Quittard-Pinon, F. (1999) "The Valuation of Interest Rate Digital Options and Range Notes Revisited," *European Financial Management*, vol. 5, p. 425-440.

Nunes, J. P. V. (2004) "Multifactor Valuation of Floating Range Notes," *Mathematical Finance*, vol. 14, p. 79-97.

Turnbull, S. (1995) "Interest Rate Digital Options and Range Notes," *Journal of Derivatives*, vol. 3(2), p. 92-101.

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