VOLATILITY AND COMPOUNDING EFFECTS ON BETA AND RETURNS

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ABSTRACT

Previous research indicates that long-term investors are not compensated for beta or volatility risk. This study shows these two results are at least partly due to the mathematics of compounding exacerbated in high volatility markets. Theoretical beta portfolios defined to perform exactly as the Capital Asset Pricing Model (CAPM) would predict on a monthly basis, show that high beta portfolios dramatically outperform in low volatility environments and underperform in high volatility environments. Empirically sorted beta portfolios confirm the results and show in a low volatility environment, high beta portfolios outperform low beta portfolios by 0.42% a month and underperform by 0.51% in high volatility environments. When combining the two market environments, the inevitable result shows no relationship between beta and return.

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KEYWORDS: Beta, Compounding, Volatility

INTRODUCTION

The single factor market model, and by extension, the capital asset pricing model (CAPM) is one of the most tested models in finance. Originally set forth by Sharpe (1964), Litner (1965) and Mossin (1966), the model concludes with the simplest proposition in finance: greater risk should be compensated with a greater return over time. However, over the last 30+ years, empirical validation of this premise has been stymied. One crucial overlooked factor often referred to as the compounding problem help explains why this has been the case.

The problem can be elucidated by considering the following: Assume a 0% risk free rate and an index return that falls 10% in period 1 and increases 12% in period 2. Over both periods, the index has a cumulative return of 0.8%. Now consider a 2.0 beta portfolio. It falls 20% in period 1 and increases 24% in period 2. Over both periods, its cumulative return is -0.8%. A 0.8 beta portfolio actually returns 0.83% over the two periods. Thus, over the two periods, the high beta portfolio underperforms the market and a 0.8 beta portfolio actually outperforms the market despite the overall positive market return. This compounding problem is exacerbated through time the higher the beta and the greater the level of volatility.

Now consider the CAPM. Traditional testing of the CAPM first employed by Fama & Macbeth (1973) tests whether the coefficient on a regression running the excess return of the stock against beta is significant and positive. The regression takes the general form of

$$R_{it} - R_{ft} = \alpha_t + \gamma_t \beta_i + \mu_i \quad (1)$$

where R_{it} - R_{ft} is the return on the ith stock minus the risk-free rate and α_t and γ_t are regression coefficients. Betas (β) are calculated based on time period t-1 returns and the above regression is run to see if beta is significantly related to excess returns in time t, i.e. is γ_t positive and significantly different from zero.

The inherent problem with this is that monthly returns are usually used to calculate betas. Thus, empirical betas to some extent are just a measure of relative monthly volatility. The greater the relative volatility for a given stock, the greater its beta and theoretically, it should be associated with higher returns. However, the higher the beta, the greater the compounding problem will be when calculating a return over an extended period. In periods of very little volatility, beta should positively relate to excess returns. In periods of high volatility, the compounding problem will be the deciding factor and higher beta stocks will not demonstrate higher returns even if the index return is positive.

The findings of this study reconfirm that low beta portfolios perform just as well and even outperform high beta portfolios over long time horizons. However, in low volatility markets, high beta portfolios significantly outperform low beta portfolios. In contrast, the returns of high beta portfolios are significantly negative in high volatility markets. The compounding problem easily explains this phenomenon.

Thus, beta appears to be a relevant risk factor only for short-term and more active investors. Over longer horizons, high beta portfolio's significant losses during volatile periods offset their outperformance in low volatility environments. These losses cause the long-run beta-return relationship to break down due to the simple mathematics of compounding.

The remainder of this paper organizes as follows: A short literature review precedes the derivation of a mathematical model showing how the compounding problem affects higher beta portfolios. A description of the data and methodology follows. Thereafter, the results section demonstrates the magnitude of the compounding problem using "perfect" monthly betas and empirically shows how beta and volatility sorted portfolios perform in both high and low volatility environments. A simple ex ante trading model is also tested. The paper finishes with some concluding comments.

LITERATURE REVIEW

Although the compounding problem highlighted above seems relatively obvious, the havoc it causes did not garner serious scrutiny until the significance of its effect manifested itself in the recently created leveraged ETF market. With funds creating daily betas up to 3.0, the effects of compounding condensed into a relatively short period. Despite near perfect daily betas, leveraged funds had dismal annual performance regardless of underlying index returns. Several studies expounded on this issue and specifically identified compounding as the singular most important explanation, (Trainor & Baryla, 2008; Carver, 2009; Avellaneda & Zhang, 2009; Lu, Wang, & Zhang, 2009; Cheng & Madhavan, 2009).

Although betas discussed in the leveraged ETF market are absolute betas since they multiply the actual index return and not the excess return relative to the risk-free rate, the compounding problem can be just as serious when employing the CAPM. Studies running the gamut from Fama & MacBeth (1973) to Fama & French (1992) conclude that the CAPM does not empirically hold. This even led Hulbert (1992) to exclaim, "Beta is dead." Fama & French (2004) continue to reiterate this conclusion although the debate continues, (Grauer & Janmaat, 2009). These studies do not identify the compounding problem.

Although not expressly recognized, Pettengill, Sundaram, & Mathur (1995, 2002) mitigate the compounding issue when they tested beta by separating the market into up and down monthly return periods. Not surprisingly, they show that beta actually explains 70% or more of the deviation across portfolios. Their explanation centered on the difference between ex-ante expectations and ex-post realizations, but unwittingly, they also simultaneously eliminated most of the compounding problem by not combining the up months with the down.

More recently, it has also been shown that low volatility and low beta stocks outperform high volatility and high beta stocks by more than 1% a month, (Ang, Hodrick, Xing, & Zhang, 2006, 2009; Baker, Bradley & Wurgler, 2011). Once again, this phenomenon appears to have its roots within the compounding problem and Baker, Bradley & Wurgler (2011) even make reference to this issue but do not develop the idea to any extent.

Thus, stocks or portfolios may be doing exactly what their betas imply they do given market volatility issues. If one measures beta on a monthly basis, it should not be surprising that its usefulness and relationship to return is also going to be on a short-term basis. Although the market has averaged a positive return over time, both the length of time studied and the underlying volatility has broken the link between beta and long-run returns. The next section demonstrates the theoretical underpinnings of why this is the case.

THE COMPOUNDING PROBLEM

To demonstrate the compounding problem, assume the market portfolio (M) follows a geometric Brownian motion. From Ito's lemma, it follows that:

$$dM/M = \mu dt + \sigma dW \tag{2}$$

where μ is the mean, σ is volatility, and W is standardized Brownian motion.

The continuously compounded return for the market from i =0 to t is:

$$M_{R_t} = \mu_t - \sigma_t^2/2 \tag{3}$$

Now consider any portfolio with a return perfectly related to the market by beta (B). The return model for this portfolio (P) is:

$$dP/P = \beta dM/M = \beta\mu dt + \beta\sigma dW \tag{4}$$

The cumulative compounded return for this portfolio from i = 0 to t is then:

$$P_{R_t} = \beta\mu_t - \beta^2\sigma_t^2/2 \tag{5}$$

This simply shows that the return and standard deviation of the return process is some beta multiple of the market portfolio. If the portfolio continuously follows the market relative to its beta, then the relationship between PR_t and βMR_t is

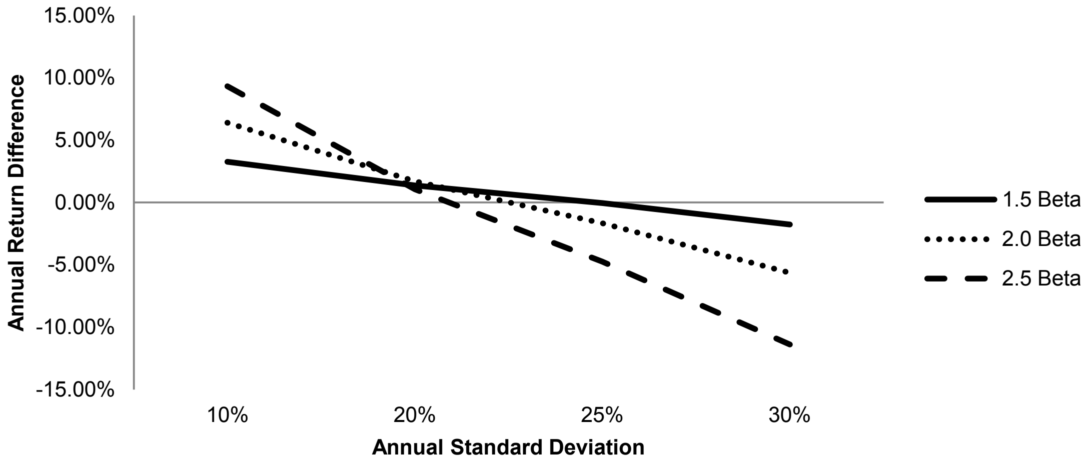
$$P_{R_t} - \beta M_{R_t} = -[(\beta^2 - \beta)/2]\sigma_t^2 \tag{6}$$

This shows for any positive variance, that the portfolio will not maintain a return that is beta times the underlying index return. The greater the variance and the higher the beta, the faster this relationship degenerates. In absolute terms, the return difference between the portfolio and the market is:

$$P_{R_t} - M_{R_t} = (\beta - 1)\mu_t + (1 - \beta^2)\sigma_t^2/2 \tag{7}$$

This states that for any given positive market return, if the volatility is high enough, the cumulative return for a portfolio with $\beta > 1$ will be less than the market portfolio.

Figure 1: Annualized Return Difference



This figure shows the difference in returns between beta portfolios and the index for standard deviations ranging from 10 to 30%.

Using Equation (7), Figure 1 shows the difference in returns between portfolios with betas of 1.5, 2.0 and 2.5 relative to the index over a year assuming an 8% cumulative annualized return. As Figure 1 shows, the relationship between beta and return turns negative as volatility levels increase. A 2.5 beta portfolio will underperform the index when volatility exceeds 21% and a 1.5 beta portfolio underperforms when volatility reaches 25%.

As a numerical example, if $\beta = 2.0$, then equation (7) is equal to $\mu_t - 1.5\sigma_t^2$. Thus, when $\mu_t = 1.5\sigma_t^2$ there will be no relationship between beta and return and when $\mu_t < 1.5\sigma_t^2$, the relationship will actually be negative. To quantify this further, if the annualized market standard deviation is 30%, then 1.5 times the daily volatility is equal to 0.00036 assuming 250 trading days in a year. A daily return of 0.036% gives a cumulative annualized return of 9.4% and under these conditions, a portfolio with a $\beta = 2.0$ will have zero excess return relative to the market. Any return less than this will result in a 2.0 beta portfolio underperforming the index.

Although the CAPM is by definition a single period model, in reality both beta and a portfolio's return are measured over time. The results above clearly show how the compounding problem causes the expected positive relationship between beta and return to break down as volatility increases or market returns decrease.

DATA AND METHODOLOGY

Although the long-term empirical relationship between beta and returns appears to be non-existent, part of the reason could be due to beta drift after creating portfolios. To eliminate this issue among others that plague beta sorted portfolios, theoretical beta portfolios ranging from 0.5 to 3.0 are created so that their returns are always equal to exactly what the CAPM would imply on a monthly basis. The actual return each month of the Center for Research in Security Price's (CRSP) value weighted index from January 1926 to December 2009 is used as the market proxy and the 30-day t-bill rate is used for the risk-free rate.

Excess returns for each beta portfolio are regressed on the actual beta values using equation (1) described earlier. In addition, market periods are separated into high and low volatility environments based on whether they are above or below the average volatility. For the CRSP value weighted index, the average standard deviation from 1926-2009 is 15.8%. Welch's (1947) t-test for the difference in means between volatility environments is conducted for each beta ranked portfolio. This test is appropriate since the comparison of means occurs between different volatility environments. The actual test statistic is:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \tag{8}$$

where $\bar{x}_1$ and $\bar{x}_2$ are the sample means and s_1^2 and s_2^2 are the sample variances.

Additionally, the same procedure above is applied to CRSP's NYSE/AMEX Scholes-William sorted beta portfolios and volatility sorted portfolios from 1970 to 2009. The smallest and largest beta and volatility portfolios are not used due to extreme values.

RESULTS

Theoretical Beta Portfolios

Table 1 shows the cumulative value of a $1 investment beginning in 1926 for the various beta portfolios. By December 2009, a $1 investment in the index is worth $2,286. For a 2.0 beta portfolio, this value reaches $12,007. Thus, in theory, seeking out higher beta portfolios could lead to vastly superior returns. However, note that a 2.5 beta portfolio only accumulates to $6,982 and a 3.0 beta portfolio only accumulates to $1,093. Thus, for long-term investors, the compounding issue clearly shows that investing in portfolios with significantly higher betas is not associated with higher returns. This result is robust as it holds for the last 40 years as well.

Table 1: Theoretical Beta Returns

	Beta 0.5	Beta 1.0	Beta 1.5	Beta 2.0	Beta 2.5	Beta 3.0
1926 -2009 Cum. Value	$311	$2,286	$7,900	$12,007	$6,982	$1,093
1926 -2009 Annual Ret.	7.07%	9.64%	11.28%	11.83%	11.11%	8.69%
1970 -2009 Annual Ret.	8.19%	10.07%	11.28%	11.75%	11.41%	10.15%

This table shows the average cumulative value along with the annualized return for the CRSP Value Weighted index.

On a shorter-term basis, Table 2 shows the rolling annual average returns across these beta portfolios. The results show a monotonically increasing relationship between beta and return. Thus, for yearly time horizons, there is a theoretical positive relationship between beta and returns. However, portfolios with betas of 2.0 and greater are associated with extreme risk as all suffer annual losses on at least one occasion of more than 90%. Despite the fact the overall rolling average annual return increases for higher beta portfolios, the compounding issue over longer time horizons not only mitigates, but also overwhelms the shorter-term higher returns.

Because higher levels of volatility exacerbate the compounding problem, it is of note to see what occurs to beta portfolios during periods of above and below average volatility. Table 2 shows the advantage of owning higher beta portfolios during periods of below average volatility. While the index returns 15.83%, a 3.0 beta portfolio returns 46.70%. The results are qualitatively the same for the last 40 years as well.

Table 2: Theoretical Beta Annual Returns

	Beta 0.5	Beta 1.0	Beta 1.5	Beta 2.0	Beta 2.5	Beta 3.0
1926 - 2009						
Average Annual Ret.	7.57%	11.71%	16.14%	20.88%	25.90%	31.18%
Min. Annual Ret.	-39.21%	-65.84%	-82.42%	-92.01%	-97.03%	-99.28%
Max. Annual Ret.	68.40%	156.35%	260.22%	372.36%	481.57%	573.90%
Low Vol. Annual Ret.	9.54%	15.83%	22.66%	30.05%	38.05%	46.70%
High Vol. Annual Ret.	2.94%	2.02%	0.86%	-0.64%	-2.62%	-5.22%
t-test for difference between means	3.83***	13.81***	8.32***	8.79***	9.36***	10.06***
1970 - 2009						
Average Annual Ret.	8.31%	10.92%	13.62%	16.40%	19.24%	22.12%
Low Vol. Annual Ret.	9.36%	13.35%	17.60%	22.11%	26.87%	31.90%
High Vol. Annual Ret.	5.93%	5.40%	4.61%	3.48%	1.96%	-0.03%
t-test for difference between means	3.33***	4.17***	4.68***	5.12***	5.55***	5.98***

*This table shows annual rolling returns using the CRSP Value Weighted index for theoretical beta portfolios ranging from 0.5 to 3.0. High volatility periods are defined as periods greater than the average for the entire period while low volatility periods are those periods less than the average. Volatility is defined as the annualized standard deviation of 12 monthly returns. ***Significant at the 1% level.*

Conversely, during periods of above average volatility, returns decline monotonically from 2.94% for a 0.5 beta portfolio to -5.22% for a 3.0 beta portfolio. These results also hold over the last 40 years with returns falling from 5.93% to -0.03%. In addition, for every beta portfolio, mean returns are significantly greater during low volatility periods and significantly lower during high volatility periods. Welch's t-test confirms this with t-values ranging from 3.33 to 13.81, all of which are significant at the 1% level.

Table 3: Theoretical Beta Regression Results

	Coefficient Estimate		R-square
1926-2009	Constant	Beta (t-stat)	
Avg. Excess Return	-0.0129	0.0945 (44.94)***	99.8
Low Vol. Excess Return	-0.0250	0.1485 (34.53)***	99.6
High Vol. Excess Return	0.0107	-0.0311 (-8.49)***	96.4
1970-2009	Constant	Beta (t-stat)	
Avg. Excess Return	-0.0026	0.0553 (110.34)***	99.9
Low Vol. Excess Return	-0.0127	0.0902 (47.74)***	99.9
High Vol. Excess Return	0.0200	-0.0236 (-8.81)***	95.1

*This table shows the regression results based on equation (1) which regresses the excess annual returns derived from theoretical beta portfolios on beta. ***Significant at the 1% level.*

Table 3 shows the regression results based on equation (1) relating beta to excess returns. These results statistically confirm beta is positively related to excess returns in low volatility periods and significantly negative during high volatility periods at the 1% level. It is also apparent the overall positive annual relationship is driven solely by the magnitude of the positive relationship during low volatility periods. For the overall period, a 0.1 increase in portfolio beta is associated with a 1.485% increase in excess return during low volatility periods. However, this is offset by a 0.311% decrease in high volatility environments. For long-term investors, these swings in returns and the mathematics of compounding cause the annual positive relationship to disintegrate over time.

Beta Sorted Portfolios

The results above show how even theoretically derived portfolios that produce perfect monthly returns in accordance with the CAPM can still show a negative relationship between beta and returns when volatility is high. Empirically derived beta portfolios suffer from the same compounding issue and are subject to even further error due to beta instability and the unique returns to the component stocks in each portfolio.

Table 4 shows the average monthly returns for the CRSP NYSE/AMEX Scholes-William sorted beta portfolios from 1970 to 2009. Portfolios rank from smallest to largest and beta values range from 0.67 to 1.42. Confirming previous research, there is no discernible relationship between beta and returns. The regression results in Table 5 show the coefficient for beta equal to 0.0003 with an insignificant t-stat of 0.57 and an r-square of only 5.1%. It should be noted that due to survivorship bias and the equal weighting used for the components stocks, the monthly returns for all the portfolios are quite high.

Table 4: Empirical Beta Sorted Portfolio Monthly Returns

	Beta 1	Beta 2	Beta 3	Beta 4	Beta 5	Beta 6	Beta 7	Beta 8
Beta	0.67	0.83	0.93	1.03	1.11	1.20	1.28	1.42
Return	1.52%	1.59%	1.63%	1.60%	1.53%	1.57%	1.60%	1.58%
Low Vol.	1.95%	2.14%	2.26%	2.21%	2.24%	2.27%	2.35%	2.37%
High Vol.	0.85%	0.72%	0.64%	0.64%	0.43%	0.47%	0.43%	0.34%
t-test for difference between means	2.56**	2.88***	3.05***	2.77***	3.03***	2.78***	2.79***	2.63***

*This table shows monthly average returns from January 1970 to December 2009 for Scholes-William sorted beta portfolios. High volatility periods are defined as periods greater than the average for the entire period while low volatility periods are those less than the average. Volatility is defined as the annualized standard deviation of daily returns within each month. ***Significant at the 1% level;**5% level.*

Table 5: Beta Sorted Regression Results

	Constant	Beta (t-stat)	R-square
Avg. Excess Return	0.0108	0.0003 (0.57)	5.1%
Low Vol. Excess Return	0.0124	0.0050 (5.65)***	93.2%
High Vol. Excess Return	0.0083	-0.0068 (-9.11)***	91.2%

*This table shows the regression results based on equation (1) which regress the average excess monthly returns derived from beta sorted portfolios on beta. ***Significant at the 1% level.*

During periods of low volatility, the advantage of owning higher beta portfolios is clear as the lowest beta portfolio returns 1.95% while the highest beta portfolio returns an average of 2.37%. This may not appear economically significant, but on an annual basis, it is 5% greater and the regression results shown in Table 5 confirm beta is significant at the 1% level with a t-stat of 5.65. During high volatility environments, portfolio returns monotonically decrease from 0.85% to 0.34% per month. In this case, beta is negatively statistically significant at the 1% level with a t-stat of -9.1. Both of these regressions show beta explains more than 90% of the difference in excess returns across beta portfolios. This reaffirms the theoretical results in the previous section. In addition, at every beta level, the low volatility return exceeds the high volatility return in both an economic and statistically significant sense at the 1% or 5% level with t-stats ranging from 2.56 to 3.05. Thus, Tables 4 and 5 also show that the relationship between beta and returns is not dead.

Volatility Sorted Portfolios

The correlation between volatility sorted portfolios and beta is quite high. Table 6 shows as the standard deviation of the portfolios increases from 12.06% to 26.21%, the corresponding beta of these portfolios increases from 0.71 to 1.37. The average monthly return actually shows that higher volatility portfolios are associated with higher returns. Regressing the returns on standard deviations confirms that this relationship is statistically significant at the 1% level with a 6.6 t-stat, (see Table 7). This is interesting in that these portfolios also have higher betas, which suggests a general positive relationship between beta and return. This is contrary to the lack of relationship found between beta and return for portfolios sorted by beta.

Reaffirming the results from Table 4, high volatility and by association high beta portfolios, outperform their low volatility/low beta counterparts during periods of low volatility. At every ranked standard deviation portfolio level, low volatility portfolios significantly outperform high volatility portfolios with t-stats ranging from 2.28 to 3.39, all of which are significant at the 5% or 1% level.

Comparing returns across ranked standard deviation portfolios in the two volatility environments, Table 7 shows that portfolio returns in high volatility periods are not significantly associated with portfolio standard deviation levels, t-stat of 0.33. During low volatility environments, the relationship is pronounced and significant with a t-stat of 6.39.

Table 6: Volatility Sorted Portfolio Monthly Returns

	Vol. 1	**Vol. 2**	**Vol. 3**	**Vol.4**	**Vol.5**	**Vol. 6**	**Vol. 7**	**Vol. 8**
St. Dev.	12.06%	14.07%	15.79%	17.27%	18.94%	20.87%	23.44%	26.21%
Beta	0.71	0.86	0.97	1.05	1.12	1.21	1.31	1.37
Return	1.10%	1.20%	1.26%	1.27%	1.36%	1.40%	1.35%	1.61%
Low Vol.	1.58%	1.79%	1.90%	2.01%	2.11%	2.15%	2.08%	2.36%
High Vol.	0.34%	0.28%	0.26%	0.12%	0.18%	0.23%	0.19%	0.43%
t-test for difference between means	3.20***	3.35***	3.25***	3.39***	3.18***	2.87***	2.52**	2.28**

*This table shows monthly average returns from January 1970 to December 2009 using CRSP's NYSE/AMEX standard deviation ranked portfolios. High volatility periods are defined as periods greater than the average for the entire period while low volatility periods are those less than the average. Volatility is defined as the annualized standard deviation of daily returns within each month. ***Significant at the 1% level.*

Table 7: Volatility Sorted Regression Results

	Constant	**Volatility (t-stat)**	**R-square**
Avg. Excess Return	0.0030	0.0299 (6.60)***	87.8%
Low Vol. Excess Return	0.0067	0.0468 (6.39)***	87.2%
High Vol. Excess Return	-0.0026	-0.0027 (0.33)	13.2%

*This table shows the regression results based on equation (1) which regress the average excess monthly returns derived from volatility sorted portfolios on the standard deviation. ***Significant at the 1% level.*

Thus, volatility sorted portfolios do show a positive relationship between volatility and return, although it appears most of this is due to the outperformance of high volatility portfolios during below average volatility periods. This provides additional evidence that the compounding problem is an issue but not to the extent, it is in beta-sorted portfolios. It should also be pointed out that these are average monthly returns and do not necessarily imply that a long-term buy-and-hold strategy of high volatility portfolios will outperform. On the contrary, they are more likely to underperform due to the same compounding issue that plagues beta-sorted portfolios.

Ex-Ante Trading

The evidence above shows that high beta portfolios payoff during low volatility periods. However, this is based on ex-post volatility measurements. To the extent volatility is persistence (Mandelbrot, 1963; Engle, 1982; Bollerslev, 1986; Kritzman, 2010) and thus predictable, it may be possible to exploit this relationship and find an ex ante reason to hold high beta portfolios. A simple trading rule based on the volatility phenomena is to hold the particular beta ranked portfolio if the previous month's volatility is less than the average volatility over the preceding 5 years. Otherwise, hold the lowest ranked beta portfolio. The risk-free asset is not used so the risk differential between the trading strategy and the buy and hold position is more comparable.

Table 8 shows the results for this type of strategy for each beta-sorted portfolio. Unfortunately, this simple trading rule does not suggest moving into high beta portfolios based on ex ante volatility levels will increase returns. There is no statistical difference between using the trading rule and the buy and hold strategy at any beta level. In addition, there is no statistically significant relationship between beta and excess return using the trading rule, t-stat of -1.05 on the beta coefficient using equation (1). It is possible that advanced predictive models of volatility or the use of the Chicago Board Option Exchange's Volatility Index (VIX) will help. However, the results here suggest long-term investors with no priori expectations on future volatility levels are better off investing in low beta portfolios.

Table 8: Volatility Based Trading Returns

	Beta 1	**Beta 2**	**Beta 3**	**Beta 4**	**Beta 5**	**Beta 6**	**Beta 7**	**Beta 8**
Beta	0.67	0.83	0.93	1.03	1.11	1.20	1.28	1.42
Buy and Hold	1.52%	1.59%	1.63%	1.60%	1.53%	1.57%	1.60%	1.58%
Trading Rule	1.52%	1.60%	1.61%	1.56%	1.52%	1.52%	1.53%	1.53%
t-test for difference between means	0.00	0.03	0.06	0.12	0.03	0.14	0.18	0.12

This table shows average monthly returns from 1970 to 2009 that compare a buy-and-hold strategy to a trading rule that involves holding the particular beta sorted portfolio when the previous month's volatility is less than the preceding 60-month average, else hold the lowest beta sorted portfolio.

CONCLUDING COMMENTS

The evidence against a positive relationship between long-run returns and beta is substantial. This study demonstrates that even theoretically derived beta portfolios that explain 100% of the deviation in returns on a monthly basis will not show a positive relationship between beta and return over longer horizons due to the mathematics of compounding. Based on historical returns and volatility, portfolios with betas greater than two appear doomed to underperform over extended holding periods. However, for shorter investment horizons and especially within low volatility environments, there is a rationale for holding higher beta portfolios.

Theoretically, there is a pronounced positive relationship between beta and return in low volatility environments where the effects of compounding are mitigated. From 1926 to 2009, the annualized return in low volatility environments for a 3.0 beta portfolio is 46.7% compared to -5.3% in above average volatility environments. This relationship remains over the last 40 years, and the last 20 as well.

Empirically sorted beta portfolios show the same qualitative type of outperformance in low volatility environments and significant underperformance in high volatility environments. Compounding is clearly an issue in explaining the breakdown between beta and return over longer periods. For portfolios sorted by volatility, the results are not as clear. However, the outperformance of high volatility portfolios appears to be contained within below average volatility environments reinforcing the idea that the compounding problem is a major issue.

Thus, it is the contention of this study that using beta to estimate the risk of a portfolio does have merit. In highly volatile markets when the cost of holding risky portfolios is high, high beta portfolio values fall. However, during low volatility environments, high beta portfolios do realize excess returns.

Unfortunately, a simple trading rule based on ex-ante volatility levels does not appear able to exploit this relationship. More advanced models to predict volatility may be more successful. This study is unable to demonstrate any ex ante rationale for long-term investors to hold high beta portfolios. For those with specific volatility forecasts and short-term horizons, the beta values of portfolios are relevant. If an investor is convinced volatility levels will remain low, investing in high beta portfolios should be associated with higher returns.

Although this study shows the compounding problem is an issue for cumulative returns, it is unable to explain why the monthly average returns of higher beta portfolios are not associated with higher returns. Thus, other factors still plague the empirical results regarding the CAPM. In addition, the return calculations used in this study ignore transaction costs and both CRSP's beta and volatility-sorted portfolios suffer from serious survival bias.

Moving forward, the effects of compounding in other markets remain unexplored. This issue is especially relevant for ETFs that have specialized in narrower markets that are more volatile. In addition, to the extent, the return degradation due to the compounding effect over any horizon can be isolated, a more enlightened answer may be forthcoming as to why size and book-to-market effects are so pronounced. Are these effects consistent regardless of the volatility environment? Finally, future research that controls for the compounding effect along with the use of the CAPM may improve portfolio performance evaluation and give a more accurate measure of manager derived excess return.

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