FORECASTING TERM STRUCTURE OF HIBOR SWAP RATES

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ABSTRACT

To investigate yield curve dynamics, researchers have employed a wide variety of models, including the famous Nelson-Siegel level, slope, and curvature factors, and principal components analysis, among others. In this paper, we decompose the term structure of HIBOR (Hong Kong Interbank Offered Rate) swap rates by means of the Nelson-Siegel factors and principal components analysis, and employ autoregressive and vector autoregressive for ex ante forecasting the yield curve by predicting the dynamic factors and components. We compare the results of a broadly empirical prediction with benchmark models such as random walk and yield levels. Further, we survey the predictability in the shape of the swap yield curve for these models. Our results appear to show that the Nelson-Siegel model with autoregressive process on factor changes is the most efficient model for forecasting HIBOR swap yields.

JEL: E43; E47; C53

KEYWORDS: term structure, Nelson-Siegel model, principal component analysis, HIBOR swap rate

INTRODUCTION

Accurate estimations of the future term structure of interest rates have become a critical issue for financial market participants. Improved methodology for fitting term structure modeling has been gaining increased attention, but advances in term structure forecasting are relatively rare. Forecasting the term structure plays an important role in derivatives pricing and for risk management hedging purposes. As we know that the arbitrage-free models and equilibrium models are the two popular term structure modeling approaches. The arbitrage-free models (see Ho and Lee, 1986; Hull and White, 1990; Heath, Jarrow, and Morton, 1992) focus on perfectly fitting the term structure at a point in time, notably few literature about forecasting has been developed. The equilibrium models, i.e. affine models (see Vasicek, 1977; Cox, Ingersoll, and Ross, 1985; Duffie and Kan, 1996) focus on modeling the dynamics of the instantaneous rate. The literature on affine models term structure forecasting, such as de Jong (2000) and Dai and Singleton (2000), only consider instantaneous short rates and focus on in-sample fit. Moreover, Duffee (2002) demonstrates disappointing out-of-sample forecasting power for affine models.

In contrast to the arbitrage-free and affine models, Nelson and Siegel (1987, hereafter NS) introduce a parametrically parsimonious model that has the ability to show the shapes of the yield curves: monotonic, humped, and S-shaped. The curve provides a good fit to the cross section of yields in many countries and has become a widely used model among central banks and market participants. In practice, we are not able to detect the spot rates for all periods because of the limited amount of bonds issued in a particular country, especially in emerging markets. However, the NS model overcomes this limitation and captures the entire behavior of the spot rate curve using only three major parameters. In addition to the NS model, principal component analysis (PCA) is another popular technique applied to describe yield curve behavior in a parsimonious manner, notably large empirical tests for PCA have been developed in the literature.

In emerging markets, government bond markets lack liquidity, so the swap rates become the important proxies for the spot rates for these financial participants. The goal of this paper is to investigate the most efficient model for forecasting term structure of the HIBOR swap rates yield curve and survey the predictability in the shape of the swap yield curve for different models. In this paper, we first decompose the term structure of HIBOR swap rates by means of NS and PCA, and employ autoregressive of order 1 AR(1) and multivariate vector autoregressive VAR(1) models for *ex ante* forecasting. To examine out-of-sample forecasting performance, we further take first differences for three factors and compare them with benchmark models, such as random walk and yield levels with AR(1) and VAR(1). The model has its best forecasting performance in terms of minimum root mean squared forecast errors (RMSEs). Moreover, we investigate the out-of-sample predictive power of the shape of the swap yield curve, including level, slope, and curvature.

LITERATURE REVIEW

This section summarizes the previous studies using various models to forecast the term structure of interest rates. NS introduce a parsimonious three-component model that has the ability to describe the shapes of the yield curves. The three time-varying parameters may be interpreted as factors. It is well known that the three-component model has competitive in-sample predictions. Despite of NS provide a good cross-sectional interpolation of yields across maturities, the NS model is not well suited to out-of-sample time series forecasts, because these factors do not stay constant over time. To enhance yield curve forecasts, Diebold and Li (2006, hereafter DL) reformulate the parsimonious three-factor yield curve of the NS model. DL proposes and estimates autoregressive models for the factors, and then forecasts the yield curve by forecasting the factors. Taking a dynamic factor approach, DL constructs the term structure of government bond yields by means of level, slope, and curvature factors, as well as macroeconomic variables, such as inflation and the federal funds rate. The DL model with AR(1) factor dynamics at 1-year-ahead horizons turns out to outperform various standard benchmark models like random walk, and the Fama-Bliss (1987) forward rate regression model.

Since DL provides only short-term out-of-sample forecasts (e.g., 1-, 6-, and 12-month-ahead forecasts), Yu and Zivot (2011) extend the DL dynamic forecast model to a wider empirical perspective. These authors evaluate the long-term (e.g., 36- and 60-month-ahead) forecasts of Treasury bonds and corporate yields, and compare the forecasting performances of the one-step (i.e., state space approach) with the two-step (i.e., NS) models. Yu and Zivot (2011) suggest that the simple two-step dynamic NS model retains the long list of robustness checks on out-of-sample forecast accuracy, especially for investment grade bonds at short-term horizons and for high yield bonds at long-term horizons. Meanwhile, the forecasting performance is improved by incorporating macroeconomic variables. Their findings also suggest that the state space model is not necessarily better than the simple NS with AR (1) model.

In addition to extensions of the NS model, principal component analysis (PCA) is another popular technique applied to describe yield curve behavior in a parsimonious manner. A strand of the literature focuses on applying PCA to yield curve decomposition; for example, Litterman and Scheinkman (1991) use PCA to describe US bond returns as mainly determined by three factors: level, slope and curvature. Knez, Litterman, and Scheinkman (1994) provide a four-factor model by observing money market returns, and argue that the additional factor is related to private issuer credit spread. Further, Duffie and Singleton (1997) suggest that the swap contracts comprise default risk, hence arbitrage-free or equilibrium term structure models developed for default-free government bond markets are not applicable to the swap market. They propose a multifactor model for interest rate swap and conclude that both credit and liquidity risks impact the US swap spread. Moreover, Blaskowitz and Herwartz (2009) decompose the EURIBOR (European Interbank Offered Rate) swap term structure by means of PCA and apply AR models for adaptive forecasting. These authors find that the PCA/AR model shows additional gains in directional accuracy and forecast value when compared to the benchmark models.

On the other hand, there is little literature focusing on the predictability of the shape of the yield curve; some exceptions are Dolan (1999), Fabozzi, Martellini, and Priaulet (2005), and Diebold and Li (2006). Dolan (1999) supports the notion that the parameters of the yield curve, estimated from the NS model, can be used to predict changes in curvature of the forward curve centered at the three-year maturity; further, Dolan's empirical evidence advocates that the curvature parameter is correlated closely with the curvature at the four-year spot rate. Fabozzi *et al*. (2005) use government bond yields and swap rates to estimate autoregressive models for predicting NS level, slope, and curvature dynamics, and conclude that while the level and slope factors are not stationary, the curvature factor seems to be so. Further, Fabozzi *et al*. (2005) provide out-of-sample predictability for the shape of the yield curve and suggest that changes in level of interest rates are not predictable at the monthly level, but that the slope of yield curve has the best predictive power. The nonstationary processes of factor dynamics compel us to consider the first difference of factor dynamics for the AR and VAR models in this paper.

It is clear that much of the literature focuses on developed countries. However, in emerging markets with underdeveloped government bond markets, the swap curve is more complete than the sovereign yield curve. Swap spreads represent the difference between swap rates and sovereign yields. They reflect the difference in default risk for the financial sector, and have become quite important for emerging markets. Thus, swap spreads represent a data set for examining how both default and liquidity risks influence securities returns. Hong Kong has one of the most open local financial markets in Asia. In addition to its multi-currency and multi-functional financial platform, Hong Kong is the only Chinese Yuan (CNY) bond market outside mainland China. The China factor allows Hong Kong to have a unique competitive advantage that is difficult for other financial centers to replicate.

Huang, Neftci, and Guo (2008) find that the Hong Kong swap curve is richer in information than the sovereign yield curve and the Libor-type interest rates that are the standard measures of market liquidity. The increasing and special importance of the Hong Kong interest rate swap market and the lack of attention paid to forecasting the term structure encourage us to bridge this gap in the literature by forecasting the term structure of swap rates and the predictability in the shape of the swap yield curve using HIBOR (Hong Kong Interbank Offered Rate) swap rates.

The remainder of this paper is organized in the following sections. In Methodology, we introduce a detailed description of the data and the methodology of forecasting the term structure. In Results and Discussion, we proceed to an empirical analysis, including evaluation of out-of-sample forecasting performance. In Conclusions, we set forth our conclusions.

METHODOLOGY

In this paper, we propose an explicitly out-of-sample forecasting perspective to compare a total of 11 models, including the NS, PCA, random walk, and yield level models.

Data

We use the weekly interest rate swap of HIBOR, taken form DATASTREAM over the period April 29 2002 to July 25 2011. The maturities of the swap yields include 1, 2, 3, 4, 5, 7, and 10 years, and the total number of observations in each maturity sample is 483. In Table 1, we present descriptive statistics for the HIBOR swap yields. It is clear that the swap yield curve is upward sloping.

Table 1: Descriptive Statistics (swap rates in percent)

Maturity (yr)	Mean	StD	Median	Max	Min	Max	Observations
1	2.142	1.524	1.680	0.400	0.310	5.040	483
2	2.533	1.346	2.290	2.170	0.440	5.060	483
3	2.908	1.189	2.860	2.780	0.670	5.110	483
4	3.211	1.071	3.280	1.900	0.980	5.160	483
5	3.452	0.992	3.610	3.780	1.310	5.200	483
7	3.765	0.895	4.000	4.520	1.940	5.590	483
10	4.059	0.865	4.270	3.240	1.870	6.040	483

This table shows descriptive statistics of location and dispersion for weekly data for the period April 29 2002 through July 25 2011.

Model

The models of swap yields we consider are: i. NS with AR(1) factor dynamics (NSAR); ii. NS with VAR(1) factor dynamics (NSVAR); iii. NS with AR(1) factor change dynamics (NSARD); iv. NS with VAR(1) factor change dynamics (NSVARD); v. PCA with AR(1) component dynamics (PCAR); vi. PCA with VAR(1) component dynamics (PCAVAR); vii. PCA with AR(1) component change dynamics (PCARD); viii. PCA with VAR(1) component change dynamics (PCVARD); ix. Random walk (RW); x. AR(1) on yield levels (YDARL); xi. VAR(1) on yield levels (YDVARL). The VAR(1) on yield levels assumes that there are neither integrated nor co-integrated series in the data. We describe these models as follows:

i. NSAR

The term-structure model introduced by NS and reformulated by DL as a modern three-factor model of level, slope and curvature:

$$y_t(m) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t m}}{\lambda_t m}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t m}}{\lambda_t m} - e^{-\lambda_t m}\right), \quad (1)$$

where y_t denotes the yield at time t, and m is the maturity of the bond. λ_t denotes the exponential decay rate. The three parameters β_{1t}, β_{2t}, and β_{3t} are interpreted as three latent dynamic factors. The loading on β_{1t} is 1, which is a constant in all maturities; hence it is interpreted as a level factor or long-term factor. The loading on β_{2t} is $(1 - e^{-\lambda_t m})/\lambda_t m$, a function that starts at 1, but that decays quickly and monotonically to zero when m increases; hence it is interpreted as a slope factor or short-term factor. The loading on β_{3t} is $\left[(1 - e^{-\lambda_t m})/\lambda_t m - e^{-\lambda_t m}\right]$, which starts at 0 (not short-term) with a humped shape in the middle; hence, it is interpreted as a curvature factor or medium-term factor.

We first use the univariate AR(1) model to estimate the three factors $\hat{\beta}_{1t}$, $\hat{\beta}_{2t}$, and $\hat{\beta}_{3t}$ by least squares, period by period, then predict yields through the NS model for forecasting factors as follows:

$$\hat{y}_{t+h|t}(m) = \hat{\beta}_{1,t+h|t} + \hat{\beta}_{2,t+h|t}\left(\frac{1-e^{-\lambda_t m}}{\lambda_t m}\right) + \hat{\beta}_{3,t+h|t}\left(\frac{1-e^{-\lambda_t m}}{\lambda_t m} - e^{-\lambda_t m}\right), \quad (2)$$

where h = 1, 6, 12, 24, 36, and 48 weeks; $\hat{\beta}_{j,t+h|t} = \hat{c}_j + \hat{\gamma}_j\hat{\beta}_{jt}$, j = 1, 2, 3, and $\hat{c}_j$ and $\hat{\gamma}_j$ are obtained by regressing $\hat{\beta}_{jt}$ on an intercept and $\hat{\beta}_{j,t-h}$.

ii. NSVAR

Based on a VAR(1) model for $\hat{\beta}_t = (\hat{\beta}_{1t}, \hat{\beta}_{2t}, \hat{\beta}_{3t})'$, the predicted term structure of yield is

$$\hat{y}_{t+h|t}(m) = \hat{\beta}_{1,t+h|t} + \hat{\beta}_{2,t+h|t}\left(\frac{1-e^{-\lambda_t m}}{\lambda_t m}\right) + \hat{\beta}_{3,t+h|t}\left(\frac{1-e^{-\lambda_t m}}{\lambda_t m} - e^{-\lambda_t m}\right), \quad (3)$$

where $\hat{\beta}_{t+h|t} = \hat{c} + \hat{\Gamma}\hat{\beta}_t$, and $\hat{c}$ and $\hat{\Gamma}$ are obtained by regressing $\hat{\beta}_t$ on an intercept and $\hat{\beta}_{t-h}$.

iii. NSARD

Following Equation (1), we use the univariate AR(1) model to estimate the three factors $\hat{\beta}_{1t}$, $\hat{\beta}_{2t}$, and $\hat{\beta}_{3t}$ by least squares, period by period, and then take $\hat{\beta}_{1t}$, $\hat{\beta}_{2t}$, and $\hat{\beta}_{3t}$ first differences as

$\Delta\hat{\beta}_{j,t} = \hat{\beta}_{j,t} - \hat{\beta}_{j,t-1}$, j = 1, 2, 3.

The forecasting coefficients $\Delta\hat{\beta}_{j,t+h|t}$ are then

$$\Delta\hat{\beta}_{j,t+h|t} = \hat{c}_j + \hat{\gamma}_j \cdot \Delta\hat{\beta}_{j,t}, \quad (4)$$

where h = 1, 6, 12, 24, 36, and 48 weeks.

iv. NSVARD

Following Equation (1), we use the multivariate VAR(1) model to estimate the coefficients vector $\hat{\beta}_t = (\hat{\beta}_{1t}, \hat{\beta}_{2t}, \hat{\beta}_{3t})'$, and then take the first difference as

$\Delta\hat{\beta}_t = \hat{\beta}_t - \hat{\beta}_{t-1}$.

The forecasting coefficients vector $\Delta\hat{\beta}_{j,t+h|t}$ is then

$$\Delta\hat{\beta}_{t+h|t} = \hat{c} + \hat{\Gamma}\Delta\hat{\beta}_t. \quad (5)$$

v. PCAR

We first decompose the yield covariance matrix of y_t as $Z\Lambda Z'$ by means of PCA in a time window of size m, $Z\Lambda Z' = \frac{1}{m}\sum_{t=T^*-m+1}^{T^*} y_t\, y_t'$, where the diagonal elements of Λ are the eigenvalues, and the columns of Z are the eigenvectors. In order to obtain the first three principal components, $\hat{p}_{1t}, \hat{p}_{2t}$, and $\hat{p}_{3t}$, we take the largest three eigenvalues and their associated eigenvectors. This can be written as $\hat{p}_{j,t} = z_j' y_t$, j = 1, 2, 3. In the next step, we take the AR(1) model to produce h-step-ahead forecasts for the principal components

$$\hat{p}_{j,t+h|t} = \hat{c}_j + \hat{\gamma}_j\hat{p}_{j,t}, j=1, 2, 3. \quad (6)$$

Finally, we use the PCA model to produce h-step-ahead forecasts of the yields

$$\hat{y}_{t+h|t}(m) = z_1(m)\hat{p}_{1,t+h|t} + z_2(m)\hat{p}_{2,t+h|t} + z_3(m)\hat{p}_{3,t+h|t}, \quad (7)$$

where $z(m)$ is the element for the eigenvector z in maturity m.

vi. PCAVAR

Following PCAR, the first three principal components are vector $\hat{p}_t = (\hat{p}_{1t}, \hat{p}_{2t}, \hat{p}_{3t})'$. This can be shown as $\hat{p}_t = z'y_t$. In the next step, we take the VAR(1) model to produce h-step-ahead forecasts for the principal components

$$\hat{p}_{t+h|t} = \hat{c} + \hat{\Gamma}\hat{p}_t. \quad (8)$$

Finally, we use the PCA model to produce h-step-ahead forecasts of the yields

$$\hat{y}_{t+h|t}(m) = z_1(m)\hat{p}_{1,t+h|t} + z_2(m)\hat{p}_{2,t+h|t} + z_3(m)\hat{p}_{3,t+h|t}, \quad (9)$$

where $z(m)$ is the element for the eigenvector z in maturity m.

vii. PCARD

Following PCAR, we obtain $\hat{p}_{1t}, \hat{p}_{2t}$, and $\hat{p}_{3t}$, from $\hat{p}_{j,t} = z_j'y_t$, j = 1, 2, 3. In the next step, we take the first difference of the three principal components

$$\Delta\hat{p}_{j,t} = \hat{p}_{j,t} - \hat{p}_{j,t-1}, j = 1, 2, 3.$$

We then take the AR(1) model to produce h-step-ahead forecasts for the principal components

$$\Delta\hat{p}_{j,t+h|t} = \hat{c}_j + \hat{\gamma}_j\Delta\hat{p}_{j,t}, j = 1, 2, 3. \quad (10)$$

Finally, we use the PCA model to produce h-step-ahead forecasts of the yields.

viii. PCVARD

Following PCVAR, we obtain $\hat{p}_t = (\hat{p}_{1t}, \hat{p}_{2t}, \hat{p}_{3t})'$ from $\hat{p}_t = z'y_t$. In the next step, we take the first difference of the three principal components

$$\Delta\hat{p}_t = \hat{p}_t - \hat{p}_{t-1}.$$

We then take the AR(1) model to produce h-step-ahead forecasts for the principal components

$$\Delta\hat{p}_{t+h|t} = \hat{c} + \hat{\Gamma}\Delta\hat{p}_t. \quad (11)$$

Finally, we use the PCA model to produce h-step-ahead forecasts of the yields.

ix. RW

The h-step-ahead forecast of the term structure of the swap rates is

$$\hat{y}_{t+h|t}(m) = y_t(m). \quad (12)$$

x. YDARL

We use a univariate AR(1) model to produce h-step-ahead forecasts of the yield level

$$\hat{y}_{t+h/t}(m) = \hat{c}(m) + \hat{\gamma} y_t(m). \quad (13)$$

xi. YDVARL

We use a multivariate VAR(1) model to produce h-step-ahead forecasts of the yield level

$$\hat{y}_{t+h/t} = \hat{c} + \hat{\Gamma} y_t, \quad (14)$$

where $y_t = [y_t(1), y_t(2), y_t(3), y_t(4), y_t(5), y_t(7), y_t(10)]'$.

RESULTS AND DISCUSSION

In this section, we assess the out-of-sample predictive accuracies of various models. We estimate and forecast recursively. A 360-week rolling window of data is used to calibrate the model; for example, the first estimation uses 360-week data from April 29, 2002 to March 16, 2009 to estimate the model parameters, then proceeds with the out-of-sample 1-, 6-, 12-, 24-, 36-, and 48-week-ahead predictions. Further, the second estimation uses a 360-week rolling window of data from May 6, 2002 to March 23, 2009 for the next out-of-sample prediction. Therefore, there are total of 77 estimations. We estimate all competitor models in the same way.

To evaluate out-of-sample forecasting performance, we use root-mean-squared error (RMSE),

$$\left[\Sigma\left(y_{t+h} - \hat{y}_{t+h|t}^{\text{Model}}\right)^2 / n\right]^{1/2},$$

where y_{t+h} is the realized yield, $\hat{y}_{t+h|t}^{\text{Model}}$ is the prediction estimated by the models, and n is the total number of estimations, i.e. 77.

The forecast performance for the shape of the swap yield curve is decomposed as level, slope, and curvature. We also use RMSE to evaluate out-of-sample forecasting performances

$$\left[\Sigma\left(\beta_{j,t+h} - \hat{\beta}_{j,t+h|t}^{\text{Model}}\right)^2 / n\right]^{1/2},$$

where $\beta_{j,t+h}$ is the realized parameters of the j factor, extracted from the realized yield rates, $\hat{\beta}_{t+h|t}^{\text{Model}}$ is the prediction estimated by the models, and n is the total number of estimations, i.e. 77.

Forecasting Results

In Tables 2 and 3, we compare h-week-ahead out-of-sample forecasting performance across 11 models for maturities of 1, 2, 3, 4, 5, 7, and 10 years. Table 2 presents short-term RMSEs for the 1- and 6-week-ahead forecasts, and Table 3 presents the long-term RMSEs for the 12-, 24-, 36-, and 48-week-ahead forecasts, respectively. The last column in each model shows that model's average RMSE across the 1-, 2-, 3-, 4-, 5-, 7-, and 10-year maturities. The bold number in the last column represents the lowest RMSE, and is the best forecasting model among the relative models.

From Panel A of Table 2, PCVARD has the lowest average RMSE, 0.110 for 1-week-ahead forecast; NSARD, PCARD, and RW have the second lowest average RMSEs, 0.111. On the other hand, NSARD has the lowest average RMSE, 0.223, and PCVARD has the second lowest average RMSE, 0.226 for 6-week-ahead forecasts in Panel B. Although the average RMSE for NSARD in a 1-week-ahead forecast is not the lowest RMSE, it is not significantly different from the RMSE of PCVARD. Our results show that both PCVARD and NSARD provide the superior forecasting performance for short-term predictive horizon.

Table 2: Swap Yield out-of-sample Short-term Forecast Evaluation

Panel A: 1-week-ahead Forecast								
Models	**1yr**	**2yr**	**3yr**	**4yr**	**5yr**	**7yr**	**10yr**	**Avg**
1.NSAR	0.083	0.140	0.127	0.134	0.137	0.138	0.146	0.129
2.NSVAR	0.068	0.117	0.111	0.121	0.129	0.131	0.141	0.117
3.NSARD	*0.065*	0.108	*0.103*	*0.117*	0.128	*0.128*	*0.131*	0.111
4.NSVARD	0.068	0.110	0.104	*0.117*	0.128	0.129	0.132	0.113
5.PCAR	0.070	0.099	0.107	0.122	0.127	0.130	0.134	0.113
6.PCAVAR	0.067	0.097	0.111	0.127	0.131	0.132	0.137	0.115
7.PCARD	0.070	0.100	0.105	*0.117*	0.125	0.131	0.132	0.111
8.PCVARD	0.067	0.096	0.104	*0.117*	0.125	0.131	0.132	**0.110**
9.RW	0.067	*0.091*	0.108	0.118	*0.124*	0.130	0.137	0.111
10.YDARL	0.068	0.093	0.110	0.120	0.128	0.131	0.136	0.112
11.YDVARL	0.070	0.096	0.111	0.125	0.131	0.133	0.139	0.115
Panel B: 6-week-ahead Forecast								
Models	**1yr**	**2yr**	**3yr**	**4yr**	**5yr**	**7yr**	**10yr**	**Avg**
1.NSAR	0.335	0.426	0.425	0.422	0.407	0.386	0.380	0.397
2.NSVAR	0.152	0.226	0.261	0.288	0.296	0.306	0.325	0.265
3.NSARD	*0.121*	0.168	*0.190*	*0.229*	*0.256*	0.285	0.316	**0.223**
4.NSVARD	0.127	0.174	0.195	0.233	0.259	0.288	0.318	0.228
5.PCAR	0.186	0.192	0.234	0.269	0.274	*0.273*	0.289	0.245
6.PCAVAR	0.154	0.196	0.262	0.304	0.311	0.304	0.321	0.265
7.PCARD	0.135	0.160	0.195	0.235	0.257	0.288	0.319	0.227
8.PCVARD	0.130	*0.157*	0.195	0.235	0.258	0.289	0.320	0.226
9.RW	0.141	0.164	0.196	0.232	0.258	0.284	0.314	0.227
10.YDARL	0.154	0.190	0.230	0.263	0.318	0.308	*0.312*	0.254
11.YDVARL	0.159	0.226	0.284	0.319	0.323	0.316	0.336	0.280

This table shows the results of the out-of-sample short-term forecasting evaluation based on the RMSE. Panels A and B present the results of 1- and 6-week-ahead forecasts, respectively. The last column Avg denotes the average RMSEs for 1, 2, 3, 4, 5, 7, and 10 years. The bold numbers present the lowest average RMSEs among these models. The italicized numbers present the lowest RMSE in a specific maturity. The results show that both PCVARD and NSARD provide the superior forecasting performance for short-term predictive horizon.

In Table 3, we find that NSARD provides the best and most reliable out-of-sample forecasting power in 12-, 24-, 36-, and 48-week-ahead forecasting on average, and the lowest average RMSEs are 0.291, 0.403, 0.446, and 0.416, respectively. PCARD (RMSE: 0.291) has the same RMSE with NSARD in 12-week-ahead forecasts, and the suboptimal model is PCVARD (RMSE: 0.292). Both PCARD (RMSE: 0.404) and PCVARD (RMSE: 0.404) are the suboptimal model in 24-week-ahead forecasts. In addition, for 36- and 48-week-ahead forecasts, PCVARD has the second lowest average RMSEs, 0.448 and 0.418, respectively. In summary, PCVARD always play suboptimal forecasting model across maturities, whatever short-term or long-term forecasts.

Further, our forecasting results for the NSARD or NSVARD are more stable than any other NSAR or NSVAR in each *h*-week-ahead forecast, respectively. We also find that the results of PCA are similar to the NS model, which is especially significant for long-term forecasting. The reason for this phenomenon is derived from the nonstationary processes of the AR or VAR models, which cause the first difference models to outperform the non-difference models. Also, NSAR always has the highest RMSE whatever short-term or long-term forecasting across all maturities. But after taking the first-order difference, it is surprising to observe that NSARD is almost superior for all under five-year maturity forecasting of swap yields in Tables 2 and 3.

Table 3: Swap Yield out-of-sample Long-term Forecast Evaluation

Panel A: 12-week-ahead Forecast								
Models	**1yr**	**2yr**	**3yr**	**4yr**	**5yr**	**7yr**	**10yr**	**Avg**
1.NSAR	0.566	0.687	0.706	0.696	0.664	0.602	0.558	0.640
2.NSVAR	0.203	0.302	0.386	0.434	0.448	0.454	0.470	0.385
3.NSARD	*0.146*	0.186	*0.245*	*0.302*	*0.341*	0.383	0.430	**0.291**
4.NSVARD	0.151	0.189	0.247	0.304	0.343	0.385	0.432	0.293
5.PCAR	0.269	0.272	0.351	0.397	0.399	0.373	0.380	0.349
6.PCAVAR	0.206	0.265	0.387	0.453	0.469	0.449	0.465	0.385
7.PCARD	0.159	*0.163*	0.246	0.309	*0.341*	0.385	0.433	0.291
8.PCVARD	0.156	*0.163*	0.248	0.311	0.343	0.386	0.434	0.292
9.RW	0.179	0.183	0.259	0.306	0.343	*0.381*	*0.422*	0.296
10.YDARL	0.214	0.263	0.357	0.402	0.516	0.469	0.449	0.381
11.YDVARL	0.209	0.322	0.435	0.493	0.503	0.480	0.495	0.420
Panel B: 24-week-ahead Forecast								
Models	**1yr**	**2yr**	**3yr**	**4yr**	**5yr**	**7yr**	**10yr**	**Avg**
1.NSAR	0.878	1.017	1.024	0.982	0.920	0.802	0.715	0.906
2.NSVAR	0.296	0.461	0.583	0.644	0.663	0.662	0.672	0.569
3.NSARD	*0.189*	0.254	*0.355*	*0.429*	*0.474*	0.528	0.591	**0.403**
4.NSVARD	0.192	0.257	0.357	0.432	0.476	0.530	0.593	0.405
5.PCAR	0.407	0.474	0.567	0.607	0.597	0.540	*0.537*	0.533
6.PCAVAR	0.298	0.426	0.581	0.661	0.684	0.652	0.667	0.567
7.PCARD	0.200	0.229	0.356	0.441	0.480	0.524	0.598	0.404
8.PCVARD	0.197	*0.228*	0.357	0.442	0.481	0.525	0.599	0.404
9.RW	0.240	0.270	0.391	0.458	0.491	*0.523*	0.565	0.420
10.YDARL	0.315	0.448	0.620	0.693	0.873	0.736	0.657	0.620
11.YDVARL	0.293	0.473	0.630	0.703	0.720	0.682	0.694	0.599
Panel C: 36-week-ahead Forecast								
Models	**1yr**	**2yr**	**3yr**	**4yr**	**5yr**	**7yr**	**10yr**	**Avg**
1.NSAR	1.094	1.226	1.210	1.140	1.049	0.891	0.764	1.053
2.NSVAR	0.363	0.573	0.703	0.760	0.769	0.743	0.724	0.662
3.NSARD	*0.209*	0.284	*0.408*	*0.488*	*0.536*	0.579	0.622	**0.446**
4.NSVARD	0.214	0.289	0.411	0.491	0.538	0.581	0.623	0.449
5.PCAR	0.511	0.626	0.717	0.741	0.713	0.616	*0.572*	0.643
6.PCAVAR	0.366	0.538	0.700	0.776	0.790	0.730	0.718	0.660
7.PCARD	0.220	0.263	0.411	0.501	0.548	0.573	0.630	0.449
8.PCVARD	0.216	*0.260*	0.410	0.500	0.548	0.573	0.630	0.448
9.RW	0.269	0.331	0.473	0.542	0.570	*0.568*	0.576	0.475
10.YDARL	0.385	0.590	0.806	0.886	1.089	0.881	0.732	0.767
11.YDVARL	0.371	0.569	0.732	0.801	0.809	0.742	0.725	0.678
Panel D: 48-week-ahead Forecast								
Models	**1yr**	**2yr**	**3yr**	**4yr**	**5yr**	**7yr**	**10yr**	**Avg**
1.NSAR	1.248	1.416	1.410	1.329	1.221	1.031	0.865	1.217
2.NSVAR	0.392	0.670	0.823	0.879	0.879	0.838	0.801	0.755
3.NSARD	*0.200*	0.296	*0.426*	*0.478*	*0.496*	0.499	0.520	**0.416**
4.NSVARD	0.205	0.301	0.429	0.481	0.498	0.501	0.521	0.419
5.PCAR	0.588	0.757	0.862	0.875	0.828	0.696	0.617	0.746
6.PCAVAR	0.396	0.633	0.819	0.894	0.900	0.821	0.795	0.751
7.PCARD	0.208	0.270	0.427	0.495	0.516	*0.493*	0.525	0.419
8.PCVARD	0.204	*0.267*	0.426	0.495	0.516	*0.493*	0.526	0.418
9.RW	0.265	0.384	0.540	0.592	0.591	0.525	*0.481*	0.483
10.YDARL	0.427	0.723	0.978	1.056	1.278	1.027	0.834	0.903
11.YDVARL	0.406	0.659	0.841	0.909	0.909	0.822	0.789	0.762

This table shows the results of the out-of-sample long-term forecasting evaluation based on the RMSE. Panels A, B, C, and D present the results for 12-, 24-, 36-, and 48-week-ahead forecasts, respectively. The last column Avg presents the average RMSEs for 1, 2, 3, 4, 5, 7, and 10 years. The bold numbers present the lowest average RMSEs among these models. The italicized numbers present the lowest RMSE in a specific maturity. The results show that NSARD provides the best and most reliable out-of-sample forecasting power for long-term predictive horizon.

Apparently, the NSARD provides more reliable for 1-5 years maturities forecasts and the first-order difference of the estimated parameters deeply improve the predictability of NS model. Although NSARD is not the lowest RMSE for over five-year maturities among swap yields, it is not the worst model. We also find that it is difficult to be sure which is the best model for the over five-year maturities in swap yield forecasting, but it almost certainly RW. These findings suggest that there is poor predictability in long-term rates.

In Table 4, we make comparisons with the overall forecasting horizon performance. The short-term average presents average RMSEs for the 1- and 6-week-ahead forecasts. The long-term average presents average RMSEs for the 12-, 24-, 36-, and 48-week-ahead forecasts. The second-to-last column presents the average RMSEs for short-term and long-term forecasts. We find that NSARD produces superior out-of-sample forecasts of swap yield in the long-term horizon. In the short term, the forecasting performance of NSARD has no significant difference with other models like, PCVARD and RW. Over all, we find the most efficient model is NSARD, and the worst model is NSAR. The results confirm that the first-order difference of the estimated parameters results in more stable forecasting errors.

Table 4: Swap Yield out-of-sample Forecast Evaluation

Models	1-week-ahead	6-week-ahead	12-week-ahead	24-week-ahead	36-week-ahead	48-week-ahead	Short-term	Long-term	Avg	Rank
1.NSAR	0.129	0.397	0.640	0.906	1.053	1.217	0.263	0.954	0.724	11
2.NSVAR	0.117	0.265	0.385	0.569	0.662	0.755	0.191	0.593	0.459	8
3.NSARD	0.111	*0.223*	*0.291*	*0.403*	*0.446*	*0.416*	*0.167*	*0.389*	**0.315**	1
4.NSVARD	0.113	0.228	0.293	0.405	0.449	0.419	0.170	0.392	0.318	4
5.PCAR	0.113	0.245	0.349	0.533	0.643	0.746	0.179	0.568	0.438	6
6.PCAVAR	0.115	0.265	0.385	0.567	0.660	0.751	0.190	0.591	0.457	7
7.PCARD	0.111	0.227	*0.291*	0.404	0.449	0.419	0.169	0.391	0.317	3
8.PCVARD	*0.110*	0.226	0.292	0.404	0.448	0.418	0.168	0.391	0.316	2
9.RW	0.111	0.227	0.296	0.420	0.475	0.483	0.169	0.419	0.335	5
10.YDARL	0.112	0.254	0.381	0.620	0.767	0.903	0.183	0.668	0.506	10
11.YDVARL	0.115	0.280	0.420	0.599	0.678	0.762	0.198	0.615	0.476	9

This table shows the comparisons with the overall forecasting horizon performance. The short-term average presents the average RMSEs of the 1-, and 6-week-ahead forecasts. The long-term average presents the average RMSEs of the 12-, 24-, 36-, and 48-week-ahead forecasts. The second-to-last column Avg presents the average RMSEs for 1-, 6-, 12-, 24-, 36-, and 48-week-ahead forecasts. The bold numbers present the lowest average RMSEs among these models. The italicized numbers present the lowest RMSE in a specific maturity. The results show that NSARD dominates the other models no matter for short-term or long-term forecasts.

According to the macroeconomics literature, unrestricted VAR tend to produce poor predictions. The accuracy and precision of the VAR-based forecast depend on the time heterogeneity properties of the series. Unless the VAR model is stable, it will generate poor forecasting over long horizons. We find that NSVAR is better than NSAR; however, PCAVAR is worse than PCAR. Apparently, AR models do not absolutely outperform VAR in this paper, but after taking the first-order difference, we find that the forecasting performances of NSARD versus NSVARD or PCARD versus PCVARD are almost equal.

Comparison of Forecasting Performance of the Shape of the Swap Yield Curve

In this subsection, we predict the dynamics of changes in level, slope, and curvature coefficients. Table 5 reports the evidence on out-of-sample predictability in the shape of the swap yield curve. We first run in-sample AR(1) regressions of changes in the beta parameters to obtain coefficients for level, slope, and curvature, then compare these with the realized coefficients to calculate RMSEs. If the model is able to reflect the future basic shape of the swap yield curve, then it should be able to reveal a better maturity-specific pattern.

In Table 5, we find that: the NSARD in predicting the level factor has the lowest short-term forecast average RMSE at 0.115, the long-term average RMSE is 0.209, and the total average RMSE is 0.177. For the slope factor, PCAR is best for the short-term (RMSE: 0.222) and long-term (RMSE: 0.467) forecast horizons. Finally, YDARL is superior for short-term (RMSE: 0.162) and long-term (RMSE: 0.297) forecast horizons in predicting the curvature factor. According to Litterman and Scheinkman (1991), the level factor accounts for an average of 89.5 percent of the observed variation in yield changes across maturities. If our data are analogous to those of Litterman and Scheinkman, this may be why the NSARD becomes the best model for forecasting level factor and swap yield curve. Meanwhile, it is interesting to find that the ranks for level factor in Table 5 are similar to that for swap yield in Table 4, especially for ranks from 1 to 5. This also can explain why level factor accounts for a large part of the variation in yield changes.

CONCLUSION

The goal of this study is to investigate the most efficient model for forecasting the HIBOR swap yield curve and survey the predictability in the shape of the swap yield curve for different models. In emerging markets, government bond markets lack liquidity, so the swap rates become the important proxies for the spot rates for these financial participants. Due to the lack of attention paid to forecasting the swap yield and term structures of emerging markets, we employ various models to forecasting HIBOR swap yield and term structures. Accurate estimate of the current term structure of interest rates is an important issue in finance. This requirement can be evaluated using the out-of-sample fit.

In this study, we propose an explicitly out-of-sample forecasting perspective using HIBOR swap yields, and investigate various prediction-horizon forecasts to compare a total of eleven models. At the same time, we try to exploit predictability in the shape of the HIBOR swap term structure. HIBOR swap yield are weekly data, and taken form DATASTREAM over the period April 29 2002 to July 25 2011.

We find that NSARD outperforms other competitors on out-of-sample forecasting accuracy, especially on maturities under five years, for almost all prediction-horizons. Our robust empirical evidence supports the remarkable out-of-sample forecasting performance of the NS dynamic model. Further, we find that a methodology using lagged explanatory variables to predict changes in yield will improve out-of-sample predictability for both the NS and PCA models.

Similarly, we also find that when we use lagged explanatory variables to predict changes in the level, slope, and curvature coefficients, rather than past values of these parameters as in Diebold and Li (2006), this enhances the out-of-sample predictability of these models. The evidence for out-of-sample predictability is analogous to Fabozzi *et al*. (2005).

On the other hand, we try to compare eleven models to predict the shape of the swap term structure. The empirical evidence indicates that NSARD provides the best predictability in the level factor of the HIBOR swap yield curve, contrary to the results of Fabozzi *et al*. (2005), by using data from US government bond yield and swap rates. Finally, we conclude that the preferred model should depend on the nature of the data and the forecast horizons. Thus, the main contribution of this paper is to bridge the gap in the literature by forecasting the term structure of swap rates and the predictability in the shape of the swap yield curve and find the most efficient model for predicting the HIBOR swap yield and term structures.

Table 5: Shape of HIBOR SWAP Yield Curve Forecast Evaluation

Models	1. NS-AR	2. NS-VAR	3. NS-ARD	4. NS-VARD	5. PC-AR	6. PCA-VAR	7. PC-ARD	8. PC-VARD	9. RW	10. YD-ARL	11. YD-VARL
Level											
1wk	0.110	0.092	0.084	0.086	0.089	0.088	0.086	0.084	0.085	0.087	0.090
6wk	0.348	0.185	0.146	0.152	0.177	0.178	0.151	0.149	0.153	0.178	0.204
12wk	0.568	0.263	0.177	0.180	0.248	0.255	0.177	0.177	0.187	0.262	0.301
24wk	0.821	0.415	0.231	0.235	0.416	0.406	0.234	0.233	0.253	0.420	0.437
36wk	0.983	0.494	0.231	0.236	0.521	0.484	0.237	0.235	0.271	0.519	0.501
48wk	1.150	0.576	0.195	0.200	0.627	0.565	0.200	0.198	0.281	0.631	0.572
Short- term	0.229	0.139	**0.115**	0.119	0.133	0.133	0.119	0.117	0.119	0.133	0.147
Long- term	0.881	0.437	**0.209**	0.213	0.453	0.428	0.212	0.211	0.248	0.458	0.453
Avg	0.663	0.338	**0.177**	0.182	0.346	0.329	0.181	0.179	0.205	0.350	0.351
Rank	11	7	1	4	8	6	3	2	5	9	10
Slope											
1wk	0.118	0.130	0.122	0.122	0.117	0.121	0.119	0.119	0.125	0.122	0.123
6wk	0.333	0.360	0.356	0.357	0.326	0.352	0.362	0.363	0.362	0.329	0.335
12wk	0.458	0.509	0.502	0.502	0.433	0.498	0.509	0.509	0.501	0.449	0.465
24wk	0.666	0.648	0.705	0.704	0.519	0.638	0.715	0.715	0.679	0.611	0.627
36wk	0.755	0.671	0.771	0.770	0.513	0.659	0.781	0.781	0.722	0.644	0.651
48wk	0.680	0.669	0.679	0.679	0.402	0.656	0.685	0.685	0.605	0.615	0.658
Short- term	0.226	0.245	0.239	0.240	**0.222**	0.237	0.241	0.241	0.244	0.226	0.229
Long- term	0.640	0.624	0.664	0.664	**0.467**	0.613	0.673	0.673	0.627	0.580	0.600
Avg	0.502	0.498	0.523	0.522	**0.385**	0.487	0.529	0.529	0.499	0.462	0.477
Rank	7	5	9	8	1	4	10	11	6	2	3
Curvature											
1wk	0.170	0.160	0.168	0.169	0.115	0.118	0.135	0.135	0.108	0.107	0.111
6wk	0.328	0.274	0.297	0.298	0.219	0.224	0.259	0.259	0.239	0.217	0.234
12wk	0.445	0.312	0.357	0.357	0.264	0.258	0.308	0.307	0.281	0.247	0.280
24wk	0.663	0.390	0.495	0.494	0.389	0.330	0.428	0.427	0.396	0.311	0.367
36wk	0.766	0.391	0.565	0.565	0.452	0.327	0.487	0.487	0.435	0.314	0.358
48wk	0.820	0.388	0.562	0.563	0.485	0.325	0.473	0.473	0.415	0.314	0.366
Short- term	0.249	0.217	0.233	0.234	0.167	0.171	0.197	0.197	0.174	**0.162**	0.173
Long- term	0.674	0.370	0.495	0.495	0.398	0.310	0.424	0.424	0.382	**0.297**	0.343
Avg	0.532	0.319	0.407	0.408	0.321	0.264	0.348	0.348	0.312	**0.252**	0.286
Rank	11	5	9	10	6	2	8	7	4	1	3

This table shows RMSEs of the forecasting the shape of the HIBOR swap yield curve for all h-week-ahead in all modes. The short-term average presents the average RMSEs of the 1- and 6- week-ahead forecasts. The long-term average presents the average RMSEs of the 12-, 24-, 36-, and 48-week-ahead forecasts. The second to last column Avg presents the average RMSEs for 1-, 6-, 12-, 24-, 36-, and 48-week-ahead forecasts. The bold numbers present the lowest average RMSEs among those models. The last raw shows the rank for the average RMSEs. The rank 1 presents the lowest average RMSE, that is, the best forecasting performance.

Limitations

In spite of dynamic NS model has good empirical performance, however, the NS model does not enforce arbitrage-free consistency over time (see Diebold and Li, 2006). It is therefore interesting to extend this research to introduce a closely related generalized Nelson–Siegel model on which the arbitrage-free condition can be imposed. Finally, another possible future research is to extend this study to across different emerging markets, because the preferred model could depend on the nature of the data and the forecast horizons.

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