RISK ANALYSIS USING REGRESSION QUANTILES: EVIDENCE FROM INTERNATIONAL EQUITY MARKETS

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ABSTRACT

In this paper we study risk management based on the quantile regression. Unlike the traditional VaR estimation methods, the quantile regression approach allows for a general treatment on the error distribution and is robust to distributions with heavy tails. We estimate the VaRs of five international equity indexes based on AR-ARCH model via quantile regressions. The empirical application show that the quantile regression based method is well suited to handle negative skewness and heavy tails in stock return time series.

JEL: G110; G150; C18

KEYWORDS: Value at risk, international equities, quantile regression, risk analysis

INTRODUCTION

The Value-at-Risk (VaR) is the loss in market value over a given time horizon that is exceeded with probability *p,* where *p* is often set at 0.01 or 0.05. In recent years, VaR has become a popular tool in the measurement and management of financial risk. This popularity is spurred both by the need of various institutions for managing risk and by government regulations [see Dowd (1998), Saunders (1999), Blankley, Lamb and Schroeder (2000) for more detailed description of the SEC disclosure requirements].

Traditional methods of VaR estimation are either based on distributional assumptions such as normality, or nonparametric smoothing that suffers from curse of dimensionality. In this paper, we estimate VaR via quantile regression ARCH models. This model has the advantage of computational convenience, as well as the robustness properties of the quantile regression method. The estimation procedure can be easily implemented on a regular personal computer. The estimation programs are available in standard statistical packages such as S-Plus, and can also be easily written in other programming languages. In addition, since GARCH models can be asymptotically represented by ARCH processes, an ARCH model with an appropriate chosen number of lags can practically provide a good approximation

.We estimate VaR in international equity markets using weekly return series for four major world equity market indexes: the U.S. S&P 500 Composite Index, the Japanese Nikkei 225 Index, the U.K. FTSE 100 Index, and the Hong Kong Hang Seng Index. We consider a combination of AR (in mean) and ARCH (in volatility) for the return series. The empirical results indicate that the quantile regression based method provides good coverage rates, and is better than the traditional estimation based on normality.The remainder of the article is organized as follows. Section 2 reviews relevant literature. Section 3 describes the quantile regression approach to VaR estimation, and provides data descriptions. Empirical results regarding estimated VaR and model performance are reported and discussed in section 4. Finally, Section 5 contains the concluding remarks.

LITERATURE REVIEW

Although VaR is a relatively simple concept, its measurement is in fact a challenging task. Currently there are two broad classes of methods in estimating VaR [see Beder (1995) and Duffie and Pan (1997) for surveys on this topic]. The first approach is based on the assumption that financial returns have normal (or conditional normal) distributions. Under this assumption, the estimation of VaR is equivalent to estimating conditional volatility of returns. Since there is a large and growing literature on volatility modeling itself, this class is indeed a large and expanding world (see, e.g., Jorian (1997)). However, there has been accumulated evidence that portfolio returns are usually not normally distributed. In particular, it is frequently found that market returns display negative skewness and excess kurtosis in the distribution of the time series. These findings suggest that VaR estimation by a more robust method is needed.

The second class of VaR estimators is based on computing the empirical quantile nonparametrically (see, e.g. Jeong 2009); for example, using rolling historical quantiles. Although local, nearest neighbor and kernel methods are somewhat limited in their ability to cope with more than one or two covariates. Other approaches in estimating VaR include the hybrid method by Boudoukh, Richardson and Vhitelaw (1998), and the method based on the extreme value theory [see, for example, Boos (1984), McNeil (1998), and Neftci (2000)].We believe that the quantile regression method is well suited for estimating VaRs. Quantile regression was introduced by Koenker and Bassett (1978) and has now become a popular robust approach for statistical analysis. The quantile regression method is an extension of the empirical quantile methods. While classical linear regression methods based on minimizing sums of squared residuals enable one to estimate models for conditional mean functions, quantile regression methods offer a mechanism for estimating models for the conditional quantile functions, thus quantile regression is capable of providing a complete statistical analysis of the stochastic relationships among random variables (see, e.g. Powell (1986), Gutenbrunner and Jureckova (1992), Buchinsky (1994), and Koenker and Portnoy (1996) among others for subsequent development in quantile regression theory.

In recent years, quantile regression estimation for time-series models has gradually attracted more attention. In particular, Koul and Saleh (1992) studied quantile regression methods for the traditional autoregressive processes and Koul and Mukherjee (1994) studied quantile regression in long-memory models. Portnoy (1991) studied asymptotic properties for regression quantiles with m-dependent errors, his analysis also allows for nonstationarity with a nonvanishing bias term. Koenker and Zhao (1996) extended quantile regression to ARCH models. Engle and Manganelli (1999) propose a CaVaR model based on the regression quantiles. Recently, Koenker and Xiao (2006) studied the quantile autoregression (QAR) models that can capture systematic influences of conditioning variables on the location, scale and shape of the conditional distribution of the response. Bouyé and Salmon (2008); Chen, Koenker and Xiao (2009) employ parametric copula models to generate nonlinear-in-parameters quantile autoregression models.The ARCH/GARCH models have been proved to be extremely successful in modeling financial returns. For this reason, much of the literature in VaR estimation considers ARCH type models. However, estimation of these models in the literature is usually based on the assumption that financial returns have normal (or conditional normal) distributions. There is accumulating evidence that financial time series display negative skewness and excess kurtosis. Extreme realizations of returns can adversely affect the performance of estimation and inference designed for Gaussian conditions; this is particularly true of ARCH and GARCH models whose estimation of variances are very sensitive to large innovations. For this reason, we propose using quantile regression methods to estimate VaR in ARCH models.

DATA AND METHODOLOGY

Data

The data used in the following empirical analysis are the weekly return series, from September 1976 to August 1999, of five major world equity market indexes: the U.S. S&P 500 Composite Index, the Japanese Nikkei 225 Index, the U.K. FTSE 100 Index, the Hong Kong Hang Seng Index, and the Singapore Strait Times Index. The FTSE 100 Index data are from January 1983 to September 1999. Table 1 reports some summary statistics of the data.

Table 1: Summary Statistics of the Data

	S&P 500	Nikkei 225	FTSE 100	Hang Seng	SingaporeST
Mean	0.0016	0.0011	0.0023	0.0029	0.0015
Std. Dev	0.0211	0.0242	0.0218	0.0393	0.0326
Max	0.0818	0.1214	0.0982	0.1542	0.1096
Min	-0.1666	-0.1089	-0.1782	-0.3497	-0.4747
Skewness	-0.5343	-0.2873	-1.0405	-1.1830	-2.6771
Excess Kurtosis	3.3987	3.1968	8.4345	7.0583	37.9887
AC(1)	0.0045	-0.0366	0.0534	0.1162	0.0685
AC(2)	0.0005	0.0971	0.0523	0.0922	0.0072
AC(3)	0.0084	0.0247	-0.0181	-0.0016	0.0324
AC(4)	-0.0082	-0.0417	-0.0190	-0.0677	0.0101
AC(5)	-0.0215	-0.0057	-0.0084	-0.0470	0.0448
AC (10)	-0.0256	-0.0167	0.0125	-0.0241	-0.0213

This table shows the summary statistics for the weekly returns of five major equity indexes of the world. AC(k) denotes autocorrelation of order k. The sample period is from September 1976 to August 1999, except for FTSE 100 which starts in January 1983. The source of the data is the online data service Datastream.

The mean weekly returns of the five indexes are all over 0.1% per week, with the Hang Seng Index producing an average return of 0.29% per week, an astonishing 32-fold increase in the index level over the 24-year sample period. In comparison, the Nikkei 225 index only increased by 3-fold. The Hang Seng's phenomenal rise does not come without risk. The weekly sample standard deviation of the index is 3.93%, the highest of the five indexes. In addition, over the sample period the Hang Seng suffered four larger than 15% drop in weekly index level, with maximum loss reaching 35%, and there were 23 weekly returns below -10%! As has been documented extensively in the literature, all five indexes display negative skewness and excess kurtosis. The excess kurtosis of Singapore Strait Times Index reached 37.99, to a large extent driven by the huge one week loss of 47.47% during the 1987 market crash. The autocorrelation coefficients for all five indexes are fairly small. The Hang Seng Index seems to display the strongest autocorrelation with the AR(1) coefficient equal to 0.116 and AR (2) coefficient equal to 0.092.

Estimating VaR by Regression Quantiles

For ease of exposition, we define Value-at-Risk as the *percentage* loss in market value over a given time horizon that is exceeded with probability *p*. That is, for a time series of returns on an asset $\{r\}_{t=1}^{n}$, find VaR_t such that

$$Pr(r_t < -VaR_t | I_{t-1}\ ) = p, \quad (1)$$

Where I_{t-1} denotes the information set at time *t - 1*. From this definition, it is clear that finding a VaR essentially is the same as finding a 100*p%* conditional quantile. Koenker and Bassett (1978) show how a simple minimization problem yielding the ordinary sample quantiles in the location model can be generalized naturally to the linear model, generating a new class of statistics called regression quantiles.

To motivate regression quantile, let's first consider estimating a simple *pth* sample quail- tile. It is clear that the estimator is the solution to the following minimization problem

$$\min_{b \in \Re} \left[\sum_{t \in \{t: r_t \geq b\}} p|r_t - b| + \sum_{t \in \{t: r_t < b\}} (1-p)\,|r_t - b| \right]. \tag{2}$$

When the quantile is the median, $p= 0.5$, we have an important special case: the estimator that minimizes the sum of absolute residuals - the median estimator.

Such a device can be generalized to regressions. If we define a k by 1 vector of regressors, x_t, and consider the regression model

$$r_t = b'x_t + u_t \tag{3}$$

with i.i.d. residual $\{u_t\}$, then, conditional on the regressor x_t, the *p-th* quantile of r_t is given by

$$F_{r_t}^{-1}(p|x_t) = \inf\{y|F_{rt}(y|x_t) \geq p\} = b'x_t + F_u^{-1}(p).$$

where F_u (.) is the cumulative distributional function of the residual. Conventionally the first component of the regressors x_t is an intercept term and we have

$$F_{r_t}^{-1}(p|x_t) = b_1 + F_u^{-1}(p) + b_2x_{2t} + \ldots + b_kx_{kt} = b(p)'x_t,$$

Where

$$b_{(p)} = (b_1 + F_u^{-1}(p), b_2, \ldots, b_k).$$

The regression quantile process corresponding to model (3) is determined by the following optimization problem

$$\hat{b} = arg \min_{b \in \Re^k} \left[\sum_{t \in \{t: r_t \geq x_t b\}} p|r_t - x_t' b| + \sum_{t \in \{t: r_t < x_t b\}} (1-p)|r_t - x_t' b| \right]. \tag{4}$$

The estimator $\hat{b}(p)$ generalizes the concept of *pth* sample quantile to the *pth* regression quantile. In this ease, the least absolute error estimator is the regression median. i.e., the regression quantile for $p = 0.5$. Koenker arid Bassett (1978) show that $\hat{b}(p)$ is a root-n consistent estimator of $\hat{b}(p)$. and $\sqrt{n}\left(\hat{b}(p) - b(p)\right)$converges weakly to a normal distribution.

The quantile regression theory can be extended to time series models with conditional heteroskedasticity. Consider a return process $\{r_t\}$ generated by the following regression model with conditional heteroscedasticity

$$r_t = \alpha' x_t + u_t \tag{5}$$

where the error term satisfies

$$u_t = (\gamma_0 + \gamma_1 |u_{t-1}| + \ \dots + \gamma_q \left|u_{t-q}\right| \mathcal{E}_t, \qquad (6)$$

with $\gamma_0 > 0, (\gamma_1, \dots, \gamma_{q)}^{'} \in \Re_+^q$, then this is a time series with ARCH effect. Here we assume that the innovations $\{\mathcal{E}_t\}$ have a general distribution *F()*, including the normal distribution and other commonly used distributions in financial applications with heavy tails. In model (5), x_t is the vector of regressors which may include lag values of the dependent variable. When $x_t = (1, r_{t-1}, \dots, r_{t-p})',$, model (5) reduces to the case of Koenker and Zhao (1996).

By definition, VaR_t at p-percent level is just the conditional quantile of r_t in the model of (5) and (6) given information to time t - 1, i.e.I_{t-1}. Thus, the conditional value at risk *(VaR_t)* at p-percent level is

$$-VaR_t(p) = \alpha^{'} x_t + \left(\gamma_0 + \gamma_1 |u_{t-1}| + \ \dots + \gamma_q \left|u_{t-q}\right| \right) F^{-1}(p) = \alpha\alpha^{'} x_t + \gamma(p)^{'} Z_t$$

Where

$$Z_t = (1, |u_{t-1}|, \dots, \left|u_{t-q}\right|)^{'}$$

and

$$\gamma(p)^{'} = (\gamma_{0,} \gamma_1, \dots, \gamma_q) F^{-1}(p).$$

Quantile regression provides a direct approach of estimating the *γ(p)* and other parameters, thus delivering an estimator of *$VaR_t(p)$*. In particular, the ARCH parameters 7, *γ(p)* can be estimated by the following problem

$$\hat{\gamma}(p) = \underset{\gamma \in \Re^k}{\arg min} [\sum_{t \in \{t: u_t \geq z_t^{'} \gamma\}} p \left|u_{t-} \ z_t^{'} \gamma\right| + \sum_{t \in \{t: u_t \ < z_t^{'} \gamma\}} (1 - p) \left|u_t - \ z_t^{'} \gamma\right|] \qquad (7)$$

Koenker and Zhao (1996) show that $\hat{\gamma}$(p) is a root-n consistent estimator of y (p). In practice, we can replace u_t and Z_t by their (say, OLS) estimators and, under mild regularity conditions, the resulting $\hat{y}$(p) is still a root-n consistent estimator of γ (p).

Quantile regression method has the important property that it is robust to distributional assumptions. This property is inherited from the robustness property of the ordinary sample quantiles. Quantile estimation is only influenced by the local behavior of the conditional distribution of the response near the specified quantile.Computation of the regression quantiles by standard linear programming techniques is very efficient. It is also straightforward to impose the nonnegativity constraints on all elements of *Y*. Barrodale and Roberts (1974) proposed the first efficient algorithm for L_1- estimation problems based on modified simplex method. For very large quantile regression problems there are some important new ideas that speed up the performance of computation relative to the simplex approach underlying the original code. Portnoy and Koenker (1997) describe an approach that combines some statistical preprocessing with interior point methods and achieves faster speed over the simplex method for very large problems.

ARCH VaR Model Selection

Given the model (5) and (6), if the lags are correctly selected we should have Pr$\{ r_t < VaR_t(p)\} = p$ at the true parameter. As a result, $\{e_t: e_t = I[r_t < -VaR_t(p)] - p\}$ should be i.i.d. In contrast, when the lags are incorrectly chosen, $\{e_t\}$ will be serially dependent. Therefore, to test the adequacy of lag choice, it suffices to check whether $\{e_t\}$ is i.i.d.

There have been several statistical procedures for testing the i.i.d. assumption. In the case of Gaussian time series, the standardized spectral density captures all serial dependencies. Consequently, any deviation of the spectral density from uniformity is an evidence of serial dependence and thus we can test serial dependence in $\{e_t\}$ using the standardized spectral density approach (Hong 1996). More generally, for non-Gaussian time series, the higher order spectral method or the generalized spectral method may be used in testing serial dependence (Hong 1999). [see also Cowles and Jones (1937), Ljung and Box (1978), etc., for related topics.]

Another popular method used in selecting lag length is to conduct sequential tests for the significance of the coefficients on lags. Such an approach provides a model selection strategy which chooses between a model with, say, k lags and a model with q=k + l lags. Koenker and Zhao (1996) show that a x^2 test can be constructed for hypothesis of the type H_0: $R_\gamma = 0$. Under Ho, the following Wald statistic converges to a centered chi-square distribution with s degrees of freedom (where s is the number of restrictions)

$$T_n = n\widehat{\omega}^{-2}(R\hat{\gamma}(p))'[R\widehat{D}_1^{-1}\widehat{D}_0\widehat{D}_1^{-1}R']^{-1} R\hat{\gamma}(p), \quad (8)$$

where $\omega^2 = p(1-p)/f(F^{-1}(p))^2$, $D_0 = EZZ'$ and $D_1 = EZZ'/\sigma$. This procedure can be applied to testing the significance of lag coefficients. If we are choosing between k lags and $q = k + 1$ lags, let R be a diagonal matrix with the k+1 to q-th diagonal elements equal to ones and others equal to zeros,

$$R = diag[0, \ldots, 0, 1, \ldots, 1], \quad (9)$$

then the corresponding statistic T_n in (8) is used in testing the significance of coefficients $\gamma_{k+1}, \ldots, \gamma_q$. In practice, we select a priori a big enough number q_{max} , then we choose the lag length from possible values $\{1, \ldots, q_{max}\}$. The procedure starts with the most general model which has q_{max} lags and tests whether the last lag coefficient is significant. If it is, then q_{max} is chosen. Otherwise, we estimate the model with q_{max}- 1 lags. This is a sequential procedure which is repeated until a rejection occurs.

RESULTS

For each time series of the five international equity index, we first conduct model specification analysis and choose the appropriate lags for the mean equation and the ARCH component. Based on the selected model, we use Equation (1) to obtain a time series of residuals. The residuals are then used in the ARCH VaR estimation described in (7).

Model Specification Analysis

We conduct sequential tests for the significance of the coefficients on lags. The inference procedures we use here are asymptotic inferences. For estimation of the covariance matrix, we use the robust HAC (Heteroskedastic and Autocorrelation Consistent) covariance matrix estimator of Andrews (1991) with the data-dependent automatic bandwidth parameter estimator recommended in that paper. First of all, we choose the lag length in the autoregression

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \ldots + \alpha_s r_{t-s} + u_t, \tag{10}$$

using a sequential test of significance on lag coefficients. The maximum lag length that we start with is s = 9, and the procedure is repeated until a rejection occurs. Table 2 reports the sequential testing results for the S&P 500 index.

Table 2: VaR Model Mean Specification Test for the S&P 500 Index

Round	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
α_0	3.6193***	3.5529***	3.7008***	3.5246**	3.6824**	3.7304***	3.6843***	3.6453***	3.8125***
α_1	-1.8916	-1.9735*	-1.9903*	-2.0020**	-1.9996*	-1.9735*	-2.0147**	-2.004**	-2.104**
α_2	0.2432	0.2143	0.1474	0.0786	0.0872	0.0833	0.0935	0.0942	
α_3	-0.8676	-0.8220	-0.7795	-0.7899	-0.8123	-0.8162	-0.8162		
α_4	-0.1470	-0.1780	-0.1717	-0.1412	-0.1610	-0.1612			
α_5	-0.5730	-0.5771	-0.5940	-0.5670	-0.5677				
α_6	-0.6055	-0.5934	-0.6112	-0.6102					
α_7	0.1783	0.1895	0.1895						
α_8	1.3186	1.3191							
α_9	0.2034								

This table reports the test results for the VaR model mean equation specification for the S& P500 Index. The number of lags in the AR component of the ARCH model is selected according to the sequential test. The table reports the t-statistic for the coefficient with the maximum lag in the mean equation. .,**,***, indicate significance at the 10,5 and 1 percent levels respectively.*

The t-statistics of all coefficients are listed for nine rounds of the test. The significance level of the t-ratios are indicated in Table 2. *, **, *** indicates significance at 10, 5 and 1 percent level respectively. The t-statistic of the coefficient with the maximum number of lags does not become significant until s = 1, the 9th round. The preferred model is an AR(1) model. We then report the selected mean equations for all five indexes in Table 3.

Table 3: ARCH VaR Models Selected by the Sequential Test

Index	Mean Lag	5% ARCH Lag	1% ARCH Lag
S&P 500	1	7	10
Nikkei 225	2	8	8
FTSE 100	1	6	6
Hang Seng	4	7	9
Singapore ST	1	7	9

This table summarizes the preferred ARCH VaR models for the five global market indexes. The number of lags in the mean equation and the volatility component of the ARCH model are selected according to the test.

Our next task is to select the lag length in the ARCH effect

$$u_t = \left(\Upsilon_0 + \Upsilon_1|u_{t-1}| + \cdots + \Upsilon_q|u_{t-q}|\right). \tag{11}$$

Again, a sequential test is conducted using the results of (8). To calculate the t-statistic, we need to estimate $\omega^2 = p\,(1-p)/f(F^{-1}(p))^2$. There are many studies on estimating $f(F^{-1}(p))$, including Siddiqui (1960) and Sheather and Maritz (1983).

Notice that

$$\frac{dF^{-1}(t)}{dt} = \frac{1}{f(F^{-1}(t))}, \tag{12}$$

following Siddiqui (1960), we may estimate (12) by a simple difference quotient of the empirical quantile function. As a result,

$$f\ (\widehat{F^{-1}}\ (t)) = \frac{2h_n}{\hat{F}^{-1}\ (t+h_n) - \hat{F}^{-1}\ (t-h_n)}, \tag{13}$$

where $\hat{F}^{-1}(t)$ is an estimate of $F^{-1}(t)$ and h_n is a bandwidth which goes to zero as $n \to \infty$. A bandwidth choice has been suggested by Hall and Sheather (1988) based on Edgeworth expansion for studentized quantiles. This bandwidth is of order $n^{-1/3}$ and has the following representation

$$h_{HS} = z_{\alpha}^{2/3}[1.5s(t)/s^{"}(t)]^{1/3}n^{-1/3}, \tag{14}$$

where z_{α_α} satisfies $\Phi(z_{\alpha_\alpha}) = 1 - \alpha/2$ for the construction of 1 - α confidence intervals. In the absence of additional information, *s(t)* is just the normal density. Starting with q_{max} =10, a sequential test was conducted and results for the 5% VaR model of the S&P 500 index are reported in Table 4. We see that in the fourth round, the *t*-statistic on lag 7 becomes significant. The sequential test stops here, and it suggests that ARCH(7) is appropriate.

Table 4: 5% VaR Model ARCH Specification Test for the S&P 500 Index

Round	1st	2nd	3rd	4th
y_0	-20.621*	-25.110*	-27.081*	-19.789*
y_1	2.8911*	3.3601*	3.2420*	3.1658*
y_2	1.9007	2.9561*	2.8366*	2.5561*
y_3	0.9982	1.0886	0.9567	1.2560
y_4	0.7737	1.0099	2.2672	1.5672
y_5	0.6919	0.8564	1.1111	0.8689
y_6	0.2336	0.3366	0.5244	0.2688
y_7	2.3406*	2.5219*	0.2318	2.8891*
y_8	0.4866	0.4688	1.3248	
y_9	1.1644	0.9921		
y_{10}	1.4665			

This table reports the test results for the 5 % VaR model specification for the S& P500 Index. The number of lags in the volatility component of the ARCH model is selected according to the test. The table reports the t-statistic for the coefficient with the maximum lag in the ARCH equation. .,**,***, indicate significance at the 10, 5 and 1 percent levels respectively.*

Based on the model selection tests, we decide to use the AR(1)-ARCH(7) regression quantile model to estimate 5% VaR for the S&P 500 index. We also conduct similar tests on the 5% VaR models for other four indexes and on the 1% VaR models for all five indexes. To conserve space we do not report the entire testing process in the paper. The results are available from the author. The mean equations generally have one or two lags, except the Hang Seng Index, which has a lag of 4 and displays more persistent autoreggressive effect.

For the ARCH equations, at least 6 lags are needed for the indexes. The longest lag, at 10, is for the 10% ARCH VaR model for the S&P 500 index. The 1% VaR models require at least as many lags in the ARCH equation as the 5% VaR model. For the Nikkei 225 and FTSE 100 indexes, the lengths of the ARCH lags are the same for the 1% and 5% VaR models. Since the estimation program for the regression quantile VaR model is very efficient, lags up to 10 in the ARCH equation are very easy to handle.

Estimated VaRs

The estimated parameters for the mean equations for all five indexes are reported in Table 5. The constant term for the five indexes is between 0.1% for the Nikkei and 0.26% for the Hang Seng. As suggested by

Table 1, the Hang Seng seems to display the strongest autocorrelation and this is reflected in the 4 lags chosen by the sequential test.

Table 5: Estimated Mean Equation Parameters

	S&P 500	Nikkei 225	FTSE 100	Hang Seng	Singapore ST
α_0	0.0021***	0.0010***	0.0021***	0.0026***	0.0015***
	(0.0006)	(0.0007)	(0.0007)	(0.0011)	(0.0009)
α_1	-0.0558**	-0.0327**	0.0553**	0.1090**	0.0655**
	(0.0265)	(0.0288)	(0.0270)	(0.0289)	(0.0289)
α_2		0.0953**		0.0880**	
		(0.0288)		(0.0291)	
α_3				-0.0136**	
				(0.0291)	
α_4				-0.0740**	
				(0.0289)	

This table reports the estimated parameters of the mean equation for the five global equity indexes. The standard errors are in parentheses under the estimated parameters.,**,***, indicate significance at the 10, 5 and 1 percent levels respectively.*

Tables 6 and 7 report the estimated ARCH parameters for the 5% VaR and 1% VaR models, respectively. The coefficients on the lagged absolute residuals are mostly positive. The negative coefficients are all statistically insignificant, with the exception of one. The selected ARCH models are relatively long, ranging from 6 lags to 10 lags. This is largely due to the fact that, when the conditional variances have relatively complicated structures, we usually need ARCH models with many lags to deliver good approximations of such general volatility models.

Table 6: Estimated ARCH Equation Parameters for the 5% VaR Model

Parameter	S&P 500	Nikkei 225	FTSE 100	Hang Seng	Singapore ST
y_0	-0.0342***	-0.0395***	-0.0335***	-0.0637***	-0.0456***
	(0.0017)	(0.0022)	(0.0014)	(0.0032)	(0.0025)
y_1	0.2129*	0.0645*	0.0506*	0.1691*	0.1089**
	(0.0672)	(0.0554)	(0.0697)	(0.0795)	(0.0489)
y_2	0.1103**	0.2005**	0.0595*	0.1092**	0.1559*
	(0.0432)	(0.0429)	(0.0686)	(0.0339)	(0.0653)
y_3	-0.0196**	0.1043*	0.0298**	0.2282**	0.0223**
	(0.0156)	(0.0633)	(0.0261)	(0.0376)	(0.0428)
y_4	0.1319*	0.0453**	0.0601*	0.0733**	0.1061*
	(0.0842)	(0.0390)	(0.0883)	(0.0296)	(0.0813)
y_5	0.0167**	0.0996**	-0.0174**	0.0235**	0.1479**
	(0.0192)	(0.0446)	(0.0143)	(0.0371)	(0.0491)
y_6	0.0253*	0.0173**	0.0948**	0.0193*	0.0299**
	(0.0941)	(0.0326)	(0.0478)	(0.0530)	(0.0206)
y_7	0.0002***	0.2553**	0.0948**	0.0917**	0.1036**
	(6.92E-5)	(0.0360)	(0.0478)	(0.0423)	(0.0437)
y_8		0.1374**			
		(0.0447)			

This table reports the estimated parameters of the ARCH equation for the 5% VaR model for the five global indexes. The standard errors are in parentheses under the estimated parameters. .,**,***, indicate significance at the 10, 5 and 1 percent levels respectively.*

Based on our model estimation of the U.S. S&P 500 index, for the 5% VaR, the estimated VaRs generally range between 2.5% and 5%, but during very volatile periods they could jump over 10%, as happened in October 1987. The 1% VaRs lie above the 5% VaRs. The two series overlap each other most of the time, but they are very much separate from each other from 1992 to 1994 when overall market volatility is relatively low. During high volatility periods, there is high variation in estimated VaRs and 5% and 1% VaRs overlap each other more often. Certainly on a particular date, the 1% VaR lies above the 5% VaR.

Table 7: Estimated ARCH Equation Parameters for the 1% VaR Model

Parameter	S&P 500	Nikkei 225	FTSE 100	Hang Seng	Singapore ST
y_0	-0.0523***	-0.0650***	-0.0551***	-0.1126***	-0.0797***
	(0.0039)	(0.0037)	(0.0048)	(0.0080)	(0.0044)
y_1	0.2678**	-0.0536	0.2842	0.3033	0.0893*
	(0.0679)	(0.2367)	(0.1013)	(0.1449)	(0.0564)
y_2	0.1783	0.2336**	0.0435	0.3158	0.3012*
	(0.1291)	(0.0259)	(0.1492)	(0.1858)	(0.0980)
y_3	-0.0878	0.0929*	0.1582	0.3675	0.1141*
	(0.2933)	(0.0946)	(0.1397)	(0.1346)	(0.0584)
y_4	0.1644*	0.0507	-0.0828	0.0463*	0.0562
	(0.0540)	(0.1561)	(0.1424)	(0.0642)	(0.1029)
	0.0971	0.0335*	0.1777	0.1013*	-0.0710
	(0.3457)	(0.0527)	(0.1312)	(0.0658)	(0.1522)
y_6	0.1557	0.0903**	0.1405*	0.0140	0.0183**
	(0.1057)	(0.0418)	(0.0502)	(0.2306)	(0.0305)
y_7	0.1992	0.4277*		0.1121*	0.2293
	(0.1533)	(0.0733)		(0.0659)	(0.1208)
y_8	-0.0938*	0.1707*		0.0891	-0.0222
	(0.0722)	(0.0657)		(0.3770)	(0.1314)
y_9	-0.0394			0.4309	0.1320*
	(0.1267)			(0.1408)	(0.0616)
y_{10}	-0.1892*				
	(0.0696)				

This table reports the estimated parameters of the ARCH equation for the 1% VaR model for the five global indexes. The standard errors are in parentheses under the estimated parameters. .,**,***, indicate significance at the 10, 5 and 1 percent levels respectively.*

The Japan-NIKKEI 225 results show that the estimated weekly 5% and 1% VaRs for the Nikkei 225 Index are quite stable and remain at the 4% and the 7% level from 1976 till 1982. Then the Nikkei 225 Index took off and appreciated about 450% over the next eight years, reaching its highest level at the end of 1989. This quick rise in stock value is accompanied by high risk, manifested here by the more volatile VaR series. In particular, the VaRs fluctuated dramatically, ranging from a low of 3% to a high of 15%. This volatility in VaR may reflect optimistic market outlook at times as well as worry about high valuation and the possibility of a market crash. That crash did come in 1990, and ten years later, the Nikkei 225 Index still hovers around at a level which is about half off the record high in 1989. The 1990s is far from a rewarding decade for investors in the Japanese equity market. The mean annual return from the Nikkei 225 Index is negative and risk is at a high level. Average weekly 5% VaR is about 5%, and about 7% for the 1% VaR. The variation in both series is also very high, bouncing between 13.5% and -1%. The estimated 5% and 1% VaRs for the U.K. Financial Times 100 Index appreciated 7-fold over the 16-year sample period. The 5% VaR is very stable and averages about 3%. They stay very much within the 2% - 4% band, except on a few occasions, such as the 1987 global market crash. The 1% VaR is also more stable than that of the Nikkei 225 Index, mostly ranging between 4% and 8%. Compared with the SP 500 Index and the Nikkei 225 Index, the overlap between the 5% VaR and the 1% VaR is minimal.

The Hong Kong Hang Seng Index produces an average return of 0.29% per week, an astonishing 32-fold increase in the index level over the 24-year sample period. The Hang Seng's phenomenal rise does not come without risk. We mentioned above that the weekly sample standard deviation of the index is 3.93%, the highest of the five indexes. In addition, the Hong Kong stock market has had more than its fair share of the market crashes. If we define a market crash as having the main index drop at least 15% in a week, then Hong Kong experienced four market crashes in 24-year sample period. The average 5% VaR over the sample is about 7%, and the average 1% VaR is about 12%, both the highest among the five indexes. The variation in the estimated VaR is huge, in particular the 1% VaRs. It ranges from 0% to 34%, the largest of the five indexes.Interestingly, for Singapore Strait Times index, the estimated VaRs display a pattern very similar to that of the U.K. FTSE 100 Index, although the former is generally larger than the latter. The higher risk in the Singapore market did not result in higher return over the sample period. Among the five indexes, the Singapore market suffered the largest loss during the 1987 crash, a 47.5% drop in a week. The market has since recovered much of the loss. Among the five indexes, the Singapore market only outperformed the Nikkei 225 Index over the entire 24-year sample period.

Performance of the ARCH Quantile Regression Model

In this section we conduct empirical analysis to help us understand the difference in dynamics between VaRs estimated by regression quantiles and those by volatility models with the conditional normality assumption. There are extensive empirical evidences supporting the use of the GARCH models in conditional volatility estimation. Bollerslev, Chou, and Kroner (1992) provide a nice overview of the issue. Furthermore, Engle and Ng (1993), Glosten, Jagannathan, and D. E. Runkle (1993), Bekaert and Wu (2000), and others have demonstrated that asymmetric GARCH models outperform those that do not allow the asymmetry, i.e., negative return shocks increase conditional volatility more than the positive return shocks. Therefore we estimate asymmetric GARCH(1,1) models and then produce VaR estimates by assuming conditional normality of the return. We also estimated several other ARCH models, with and without the asymmetric volatility specification. The regular GARCH(1,1) and asymmetric GARCH(1,1) produce similar performances in terms of the VaR test described below.

We estimated the 5% VaRs of the S&P 500 Index estimated by the ARCH regression quantiles. and the asymmetric GARCH(1,1) model with the conditional normality assumptions. We see that these two series actually track each other pretty well, although the VaR series estimated by regression quantile seem to be higher than the GARCH VaR during low volatility periods. However, during very volatile markets, as during the 1987 market crash, the GARCH plus normality approach produces much higher VaR estimates. This could be due to the fact that large return shocks produce large volatility estimates in the GARCH setting. Value at risk after a market crash could be too high based on this approach. The quantile regression approach seems to generate an increased VaR at a more reasonable level.

To measure the relative performance more accurately, we compute the percentage of realized returns that are below the negative estimated VaRs. The results are reported in Table 8. The top panel of the table presents the percentages for the VaRs estimated by the ARCH regression quantile model, and the bottom panel for the VaRs estimated by the asymmetric GARCH model with the conditional normal return distribution assumption. For the 1% VaR, we see that the regression quantile method produces the percentage that is closer to the 1% mark for all five series. The GARCH approach seems to underestimate the VaRs consistently. For the 5% VaR, the regression quantile method produces the percentage that is closer to the 5% mark for all series, except for the FTSE 100. But now the GARCH approach seems to overestimate the VaRs consistently. To look at this more closely we extend the analysis for the S&P 500

Index. We estimate VaRs using the two methods at 2%, 4%, 6%, 10%, 15%. Now we have a total of 7 percentage levels. The regression quantile method produces the closest percentage at all percentage levels, and the percentages scatter around the true value. However, the GARCH method seems to underestimate VaRs for the smaller percentages (1% and 2%), and overestimate VaRs for the larger percentages (larger than or equal to 4%).

Table 8: VaR Model Performance Comparison

% VaR	1%	2%	4%	5%	6%	10%	15%
		VaR by Regression Quantile					
S&P 500	1.339	1.925	4.168	5.2874	6.276	9.791	14.819
Nikkei 225	1.340	2.011	4.312	5.7084	6.581	10.210	14.564
FTSE 100	0.694	1.867	3.658	5.5868	5.232	8.951	12.372
Hang Seng	0.755	2.113	4.222	4.8902	5.512	9.348	13.558
		VaR by GARCH Normality Assumption					
S&P 500	1.1976	1.7964	3.1936	4.0918	4.9900	7.6846	12.4750
Nikkei 225	1.2974	1.9960	3.5928	4.6906	5.2894	8.5828	12.3752
FTSE 100	0.9980	1.6966	2.9940	3.4930	3.8922	6.5868	9.7804
Hang Seng	1.8962	2.8942	3.3932	3.6926	4.1916	7.1856	10.9780
		VaR by RiskMetrics					
S&P 500	0.4990	0.4990	0.6986	0.7984	0.7984	2.0958	3.5928
Nikkei 225	0.5988	0.7984	0.9980	0.9980	1.2974	2.2954	1.4910
FTSE 100	0.1996	0.1996	0.2994	0.7984	0.8982	1.7964	3.6926
Hang Seng	0.7984	0.8982	1.3972	1.3972	1.5968	2.6946	3.7924

This table reports the coverage ratios, i.e., the percentage of realized returns that are below the estimated VaRs. The top panel reports the performance of the VaRs estimated by the ARCH regression quantile model. The middle panel reports the results for VaRs estimated by the asymmetric GARCH model with the conditionally normal return distribution assumption. The bottom panel reports the results for VaRs estimated by the RiskMetrics method.

The five indexes we analyzed are quite different in their risk characteristics as discussed above. The ARCH quantile regression approach seems to be robust and can consistently produce very good estimates of the VaRs at different percentage (probability) levels. The asymmetric GARCH model, being a very good volatility model, is not able to produce good VaR estimates with the normality assumption. The ARCH quantile regression model does not assume normality and is well suited to hand negative skewness and heavy tails.

CONCLUDING COMMENTS

In this paper we estimate value at risk using the quantile regression approach pioneered by Koenker and Bassett (1978). Comparing to the widely use RiskMetric method and other methods based on distributional assumptions, this method does not assume a particular conditional distribution for the returns. This is particularly important in VaR estimation because return data are well-known to be non-Gaussian. We apply the model to weekly return series of five major world equity market indexes: the U.S. S&P 500 Index, the Japanese Nikkei 225 Index, the U.K. FTSE 100 Index, the Hong Kong Hang Seng Index, and the Singapore Strait Times Index. The empirical results found that the quantile regression based method is more robust than RiskMetrics. These results regarding VaR estimation may have important implications for risk management practices.There are several directions for future research. First, our analysis in this paper is based on univariate analysis. Informative covariates may be introduced to improve the accuracy of estimation. Second, nonlinear models such as Copula models and GARCH models may be considered to take into account nonlinearity in financial time series. We hope to explore these in future research.

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