

# OPTIMAL BUY-BACK CONTRACTS WITH ASYMMETRIC INFORMATION

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## ABSTRACT

*When demand is uncertain and it is costly for the retailer to forecast demand information more accurately, the supplier faces a moral hazard problem. The supplier wishes to induce the retailer to forecast more accurate information which will improve the total profit of the supply chain. This paper provides a theoretical analysis of the optimal buy-back contract, in which the supplier chooses the wholesale and buy-back price to maximize his profits given that the retailer's inventory order level and private information acquisition decision are both chosen to maximize the retailer's profits. In contrast to the standard buy-back contract model in which the first best of the system can always be implemented, our model suggests that the supplier pays not only the cost of acquiring information, but also the information rent to induce the retailer to invest in acquiring information. Consequently, the first best of the system cannot be always implemented. Our model explains that Vendor Managed Inventory systems are prevalent while the retailer is better informed than the supplier. Nevertheless, the standard buy-back contract theory contradicts with the empirical facts.*

KEYWORDS: forecast demand, asymmetric information, vendor inventory systems, optimal buy-back

JEL: C60, D8, M11

## INTRODUCTION

We consider a vertical system consisting of an upstream firm that sells through a downstream firm facing uncertain demand. Suppose the upstream firm is the supplier, the downstream firm the retailer. As is standard in the literature (see especially Pasternack, 1985), the supplier is limited to a uniform wholesale and a uniform buy-back price, the retailer's quantity level (inventory level) is not contractible, and the retailer faces an exogenous retail price. To avoid the double marginalization of the decentralized system, return for full or partial credit is offered to the downstream firm. The first best can be implemented if the buy-back contract is well designed.

In many situations, the retailer is better informed about demand information than the supplier. Kulp (2002) considers the precision and reliability of information and the retailer's willingness and ability to share information, and how that will affect the way manufacturers and retailers structure their relationship. Since she uses a price-only contract, double marginal cost is generated when the retailer makes a decision, despite the fact that the retailer has more precise information. This situation differs from that when the supplier makes a decision to overcome double marginal cost based on less precise information. But if we allow a return policy, the supplier can give adequate wholesale prices and buy-back rates, and the first best can be implemented in case the retailer makes a quantity decision when he is better informed. This means the traditional buy-back system dominates the VMI (Vendor Managed Inventory) system, which is not optimal in this situation. As a result, these findings contradict the empirical results of Kulp (2002). In contrast, our theoretical findings in this paper are consistent with these empirical results.

We go beyond this basic framework by considering moral hazard problem for the retailer to forecast more accurate demand information. Whether the retailer obtains information more accurately is unobservable. The supplier chooses the wholesale and buy-back price to maximize his profits, given that the retailer's

inventory order level and information acquisition decision are both chosen to maximize the retailer's profit. This optimal buy-back contract remains a challenging problem. The approach in Pasternack (1985) to solve for the optimal buy-back contract does not work here. As shown in our paper, the first best outcome of the system can not always be implemented; therefore a necessary condition for this approach fails. Lariviere (1999) shows that the standard first order condition method which is used in solving standard optimal contract does not work in buy-back contract problem because the Hessian of the supplier's profit is positive. In this paper, we provide method to study this problem.

## LITERATURE REVIEW

The role of information on contracting in a vertical system has been an important topic in economics and marketing. Rey and Tirole (1986) model the tradeoff between the manufacturer's desire to provide insurance to retailers and his desire to avoid agency costs. Desiraju and Moorthy (1997) show how performance requirements may improve the working of a distribution channel when the retailer is better informed about demand conditions than the manufacturer. Ha (2001) considers the problem of designing a contract to maximize the supplier's profit in a one-supplier, one-buyer relationship for a short-life-cycle product under asymmetric cost information. Blair and Lewis (1994) consider the optimal retail contracts with asymmetric information and moral hazard. Lariviere (2002) examines both the price-based returns mechanism and the quantity-based returns mechanism when there is a positive probability that the retailer is capable of gaining improved demand information through costly forecasting. He shows that buy-backs generally result in greater supplier profit than quantity flexibility contracts unless forecasting is very expensive. A return policy can be implemented via prices (Pasternack, 1985; Donohue, 2000) or quantity limits (Pasternack, 1985; Tsay, 1999). Under either, the first best outcome of the system can be implemented.

Kulp (2002) considers the precision and reliability of information and the retailer's willingness and ability to share information and how that will affect the way manufacturers and retailers structure their relationship. She uses a price-only contract and double marginal cost can explain her empirical results. However, firm can offer buy-back contracts to overcome the double marginality. Our results are consistent with these empirical findings with buy-back contract.

## THE BASIC MODEL

The following are some notations we will use in this paper:

$p$ : the market price of the retailer.

$c$ : the production cost per unit.

$q$ : the quantity that the retailer orders from the supplier.

$w$ : the wholesale price of the supplier to the retailer.

$b$ : the buy-back price of the supplier.

$e$ : the cost to invest in acquiring information.

$D$ : the number of consumers willing to pay the exogenous price.

$P_h$ : the prior probability that the market demand is high.

$P_l$ : the prior probability that the market demand is low.

$\theta$ : the posterior probability that the market demand is high if the good news is received.

$\Pi$ : the joint profit of the system.

$\pi_r$ : the profit of the retailer.

$\pi_s$ : the profit of the supplier.

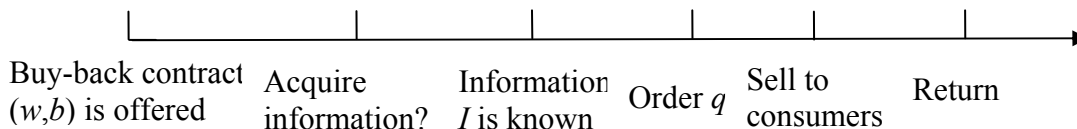
$I$ : the information that is available to retailer.  $I \in \{0,1,2\}$ . 0 means the retailer receives no information; 1 means the retailer receives good information, which indicates the market demand is high; 2 means the retailer receives bad information, which means the market demand is low.

In this model, there is an upstream firm and a downstream firm in the market. We will call the upstream firm a supplier and the downstream firm a retailer. The supplier produces products and sells through the retailer. The retailer sells the product to the consumer at a fixed price  $p$ .

The timing is as shown in Figure 1:

1. The supplier offers a buy-back contract  $(w,b)$  where  $w$  is the wholesale price and  $b$  is the buy-back price. We suppose both  $w$  and  $b$  are constant. (We will discuss possible generalizations later.)
2. The retailer decides to accept or reject the contract.
3. If the retailer accepts the contract, he will decide whether to invest in acquiring information. It costs him  $e$  to retrieve the information. If he does not invest in acquiring information, the cost is 0. If the retailer rejects the contract, the game is over and the profits of both the supplier and the retailer are 0.
4. If the retailer invests in acquiring information, he obtains the information  $I \in \{1,2\}$ , where  $I=1$  means he received good news and  $I=2$  means he received bad news. If he did not invest in acquiring information, the information he has is  $I=0$ .
5. Based on the information  $I$ , the retailer decides the quantity  $q$  to order to maximize his expected profit.
6. Uncertainty is realized. The retailer returns the remaining inventory to the supplier at price  $b$ .

Figure 1: Timing



We assume demand is  $D$ , which is a binary discrete distribution. The prior probabilities are that  $Prob(D=h)=P_h$  and  $Prob(D=l)=1-Prob(D=h)=P_l$ .

After the retailer receives information  $I$ , he can update his information and the posterior probabilities are  $Prob(h|I)$  and  $Prob(l|I)$ , where  $Prob(h|I=1) > Prob(D=h)$  and  $Prob(l|I=2) > Prob(D=l)$ . Thus we identify  $I=1$  as good news and  $I=2$  as bad news. Denote  $Prob(h|0) = Prob(h)$  and  $Prob(l|0) = Prob(l)$ . We assume the information is private information so the supplier cannot observe  $I$ .

Without loss of generality, we assume the reserve profit level of the retailer is 0. The retailer's profit is 0 if he rejects the contract. But for any contract, he can always at least get a profit of 0 by ordering a quantity of 0 and do nothing else if he accepts the contract. He can always accept the contract under this assumption. To simplify this model we can assume, without loss of generality, the salvage value is 0 for both parties and the selling cost of the retailer is 0.

Given the timing in our model, there exists a very easy solution to the incentive problem that we study. The upstream firm could sell the whole firm to the downstream firm, demanding a payment just low enough to induce the retailer to accept the contract. Once this transaction takes place, the downstream firm will choose the first best quantity level. Such lump-sum extractions are rarely observed in practice.

The reasons are as follows: First, the upstream firm may face more than one downstream firm. Second, the budget of the downstream firm is constrained. Third, the upstream firm may need in its production some special investment, for example, human capital, which cannot be transferred to the downstream firm directly. Fourth, if there is hidden information for the supplier, the retailer will face agency costs. We assume that the only payments between supplier and retailer are made at a fixed wholesale price and fixed return price (linear price), which are widely used in practice.

### Symmetric Information: Optimal Wholesale Prices without Buy-Back

First, we consider the supplier's strategy without buy-back. We can have better idea why a buy-back contract is interesting and how a buy-back contract works.

We consider the joint profit of the system. That is

$$\Pi = \max_q \Pi(q) = \max_q E\{p \min(D, q) - cq\} \quad (1)$$

When there is no information available and the inventory level is  $q$ , the profit of the system is as follows:

$$\Pi(q) = \begin{cases} (p-c)q & q < l \\ pl - cl + (pP_h - c)(q-l) & l \leq q < h \\ (P_h h + P_l l)p - cq & h \leq q \end{cases} \quad (2)$$

Denote the first best inventory level as  $q^f$ . We have

$$q^f = \arg \max_q \Pi(q) \quad (3)$$

If the best response set is a correspondence that has more than one solution, for simplification, we suppose that the highest possible inventory will be chosen. Therefore the first best inventory level is as follows:

$$q^f = \begin{cases} h & pP_h - c \geq 0 \\ l & pP_h - c < 0 \end{cases} \quad (4)$$

In a decentralized vertical system, the supplier will sell through the retailer. Given the wholesale price  $w$ , the retailer chooses his quantity level  $q$  to order, and the profit is as follows:

$$\pi_r(q) = \begin{cases} (p-w)q & q < l \\ pl - wl + (pP_h - w)(q-l) & l \leq q < h \\ (P_h h + P_l l)p - wq & h \leq q \end{cases} \quad (5)$$

Denote the optimal inventory level for the retailer as  $q^o(w)$ . We have

$$q^o(w) = \arg \max_q \pi_r(q) \quad (6)$$

For  $c \leq w \leq p$ , the optimal inventory level is

$$q^o(w) = \begin{cases} h & pP_h - w \geq 0 \\ l & pP_h - w < 0 \end{cases} \quad (7)$$

Based on this best response function of the retailer, the supplier will choose the wholesale price  $w$  to maximize his profit:

$$\max_w (w - c)q^o(w) \quad (8)$$

Consequently the optimal wholesale price is

$$w = \begin{cases} pP_h & (pP_h - c)h \geq (p - c)l \\ p & (pP_h - c)h < (p - c)l \end{cases} \quad (9)$$

**Proposition 1**-If and only if  $P_h \geq \frac{pl + c(h - c)}{ph}$  or  $P_h < \frac{c}{p}$ , in equilibrium, the retailer will purchase the first best quantity level when the upstream firm uses a price-only contract.

**Proof.** First, we want to show that  $P_h \geq \frac{pl + c(h - c)}{ph}$  or  $P_h < \frac{c}{p}$  are necessary conditions. That is, if

$\frac{pl + c(h - c)}{ph} > P_h \geq \frac{c}{p}$ , the retailer will not purchase the first best inventory in equilibrium.

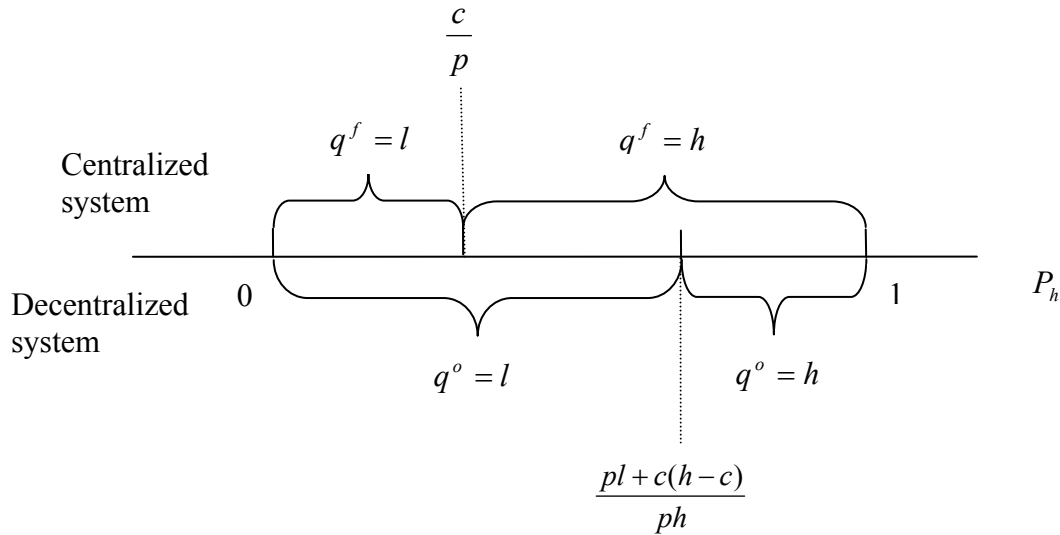
When  $P_h \geq \frac{c}{p}$ , we have  $pP_h - c \geq 0$ . From equation (4), we know that  $q^f = h$ . But  $\frac{pl + c(h - c)}{ph} > P_h$  means  $(pP_h - c)h < (p - c)l$ . From equation (9), we have  $w = p$ . From equation (7), we have  $q^o(w) = l$ . So  $q^o(w) \neq q^f$ .

Then, we want to show that  $P_h \geq \frac{pl + c(h - c)}{ph}$  or  $P_h < \frac{c}{p}$  are sufficient conditions. That is, in equilibrium the retailer purchases the first best inventory.

When  $P_h \geq \frac{pl + c(h - c)}{ph}$ , we have  $w = pP_h$  and  $q^o(w) = h = q^f$ . When  $P_h < \frac{c}{p}$ , we have  $w = p$  and  $q^o(w) = l = q^f$ . So the first best outcome can be obtained in the decentralized system.

In addition, it is easy to check that  $\frac{pl + c(h - c)}{ph} > \frac{c}{p}$ . In a decentralized vertical system, because of the double marginalities, in equilibrium the retailer will not purchase the first best inventory level if  $\frac{pl + c(h - c)}{ph} > P_h > \frac{c}{p}$  given that the upstream firm uses a price-only contract in equilibrium. The distortion exists.

Figure 2: Centralized and Decentralized Systems



Optimal Buy-Back Contracts under Symmetric Information

In the buy-back contract context, the retailer can return the remaining inventory at the buy-back price to the supplier. Given the wholesale price  $w$  and the buy-back price  $b$ , the objective function of the retailer is the following:

$$\max_q \pi_r(w, b, q) = \max_q E\{p \min(D, q) - wq + b \max(0, q - D)\} \tag{10}$$

The supplier will choose the wholesale price  $w$  and buy-back price  $b$  to maximize his profit. Thus the objective function of the supplier is

$$\max_{w,b} \pi_s(w, b) = \max_{w,b} E\{wq(w, b) - b \max(0, q(w, b) - D)\} \tag{11}$$

Subject to

$$q(w, b) = \arg \max_q \pi_r(w, b, q) \tag{12}$$

We can rewrite equation (10) as

$$\max_q \begin{cases} (p - w)q & q < l \\ pl - wl + (pP_h - w + bP_l)(q - l) & l \leq q < h \\ (hP_h + lP_l)(p - b) - (w - b)q & h \leq q \end{cases} \tag{13}$$

Denote the optimal quantity as  $q^o(w, b)$ . For  $b \leq w \leq p$ , we have

$$q^o(w, b) = \begin{cases} h & pP_h + bP_l - w \geq 0 \\ l & pP_h + bP_l - w < 0 \end{cases} \quad (14)$$

So the maximized profit of the retailer is

$$\pi_r(w, b) = \begin{cases} (p-w)l + (pP_h - w + bP_l)(h-l) & pP_h + bP_l - w \geq 0 \\ (p-w)l & pP_h + bP_l - w < 0 \end{cases} \quad (15)$$

In this situation, the profit of the supplier is

$$\pi_s(w, b) = E\{wq^o(w, b) - b \max(0, q^o(w, b) - D)\} \quad (16)$$

We are interested in the first best outcome of the vertical system. From Proposition 1, we know that if some conditions of demand distribution are satisfied, the retailer will purchase the first best quantity when the supplier uses a wholesale price-only contract. But that depends on the distribution of demand function. Here, we want to find the contract that does not depend on the distribution function of demand so the retailer will purchase the first best quantity independent of the distribution function.

**Theorem 1** *When the information is complete, if the supplier offers a contract  $\{w_v, b_v\}$  where*

$$w_v = p(1-v) + cv \text{ and } b_v = p(1-v) \quad (17)$$

*and  $0 < v < 1$ , then the retailer orders the first best quantity, i.e.  $q^o(w_v, b_v) = q^f$ . The retailer's profit is  $\pi_r(w_v, b_v) = v\Pi$ . The supplier's profit is  $\pi_s(w_v, b_v) = (1-v)\Pi$ .*

**Proof.** From equation (17), we have that

$$pP_h + bP_l - w = (pP_h - c)v \quad (18)$$

Since  $0 < v < 1$ , we have  $\text{sign}(pP_h + bP_l - w) = \text{sign}(pP_h - c)$ . Comparing equation (4) with equation (14), we have  $q^o(w) = q^f$ . For each sold product, the marginal revenues for the supplier and retailer are  $(p-c)(1-v)$  and  $(p-c)v$  respectively. While for each unsold product, the marginal revenues for the supplier and retailer are  $-c(1-v)$  and  $-cv$ . So retailer profit is  $\pi_r(w_v, b_v) = v\Pi$ , and supplier profit is  $\pi_s(w_v, b_v) = (1-v)\Pi$ .

A similar result can be found in Pasternack (1985) when demand distribution is continuous. This theorem is a restatement of his result for the discrete situation.

When the partition of the profit between retailer and supplier is given, the optimal buy-back contract is unique when demand distribution is continuous. But by Proposition 1, this contract cannot always be unique for the discrete distribution. The outcome under the optimal contract in Theorem 1 is the first best, and the optimal contract does not depend on the distribution function of demand.

Intuitively, the retailer will purchase the first best quantity level if the wholesale price and the buy-back price are well designed. The supplier and the retailer share the first best profit of the system by the proportion  $(1-\nu)$  and  $\nu$  respectively. Since the wholesale and buy-back prices are set by the supplier,  $\nu$  is then determined by the supplier. When the supplier wants to maximize his profit, he can set the wholesale price and buy-back price as  $w_\nu = p(1-\nu) + c\nu$  and  $b_\nu = p(1-\nu)$  with  $\nu$  close to 0. In this case, we notice that both the wholesale price and buy-back price are close to  $p$ . The profit of the retailer is close to 0.

**Corollary 1** *When  $I$  is common knowledge, the optimal buy-back contract for the supplier is  $w_\nu = b_\nu = p$ . In this case the contract is  $w_\nu = p(1-\nu) + c\nu$ ,  $b_\nu = p(1-\nu)$  and let  $\nu \rightarrow 0$  so  $w=b=p$  is the limit situation. Then the supplier gets the first best profit of the whole system and the retailer's profit is 0.*

When the supplier is the principal, there are two approaches for him to implement the first best of the system: Vendor Managed Inventory System (VMI) allows the supplier to make quantity decision and the buy-back contract allows the retailer to make quantity decision. From the equation (17), the optimal buy-back contract does not depend on the distribution function of the supplier. Hence the retailer will make decision depending on his demand information only.

When the demand information is the common knowledge, both VMI system and buy-back contract can implement the first best and they are equivalent for the supplier. When the supplier has more accurate prior demand information, VMI system can obtain the first best based on supplier's information so that VMI system is superior to buy-back contract. But when the retailer has more accurate prior demand information, the buy-back contract can obtain the first best based on retailer's information, therefore the optimal buy-back contract is superior to VMI system. In practice, we find the VMI system is prevalent even when the retailer can have an easier access to more accurate information. Since it is costly for the retailer to invest in acquiring information, we need consider not only the cost of acquiring information but also the information rent due to asymmetric information.

#### Optimal Contracts under Asymmetric Information: Incentives for Information Acquisition

Then we consider the consequences of asymmetric information. We assume the supplier does not know whether the retailer has invested in acquiring the information or what kind of information he held.

Denote  $\pi_r(w, b, I)$ ,  $\pi_s(I)$ , and  $\Pi(I)$  as the profits of the retailer, the supplier and the system conditional on the information  $I$  respectively. When  $I=0$ , the symmetric information situation occurs as we have discussed.

We now simplify the analysis by assuming that  $P_h=P_l=1/2$  and  $\theta = \text{Prob}(h | I = 1) = \text{Prob}(l | I = 2) > 1/2$ .

The information is generated as follows:

When the real demand is high, the probability of generating good news is  $\gamma$  and the probability for the bad news is  $1-\gamma$ . That is  $\text{Prob}(I=1|h) = \gamma$ . On the other hand, when the real demand is low, the probability of generating bad news is  $\gamma$  and the probability of generating good news is  $1-\gamma$ . That is  $\text{Prob}(I=0|l) = \gamma$ .

In this case, when the retailer invests in acquiring information, the probability of receiving good news is  $\frac{1}{2}$  and the probability of receiving bad news is  $\frac{1}{2}$ .



Given the contract the supplier offered, the retailer will decide whether or not to invest in acquiring information. The retailer would like to retrieve information only if he is better off when he does so. We assume that if the retailer is indifferent between acquiring information or not, he always acquires information. The incentive compatibility constraints should be satisfied. If the given contract  $(w, b)$  satisfies  $\frac{1}{2}\pi_r(w, b, 1) + \frac{1}{2}\pi_r(w, b, 2) - e \geq \pi_r(w, b, 0)$ , the retailer will be better off if he invests in acquiring information. While if the given contract  $(w, b)$  satisfies  $\frac{1}{2}\pi_r(w, b, 1) + \frac{1}{2}\pi_r(w, b, 2) - e < \pi_r(w, b, 0)$ , the retailer will be worse off if he invests in acquiring information. Therefore we can divide the available set into two categories:  $\Omega_Y$  and  $\Omega_N$ . The former is the set of contracts given that the retailer will invest in acquiring information. That is  $\Omega_Y = \{(w, b) : \frac{1}{2}\pi_r(w, b, 1) + \frac{1}{2}\pi_r(w, b, 2) - e \geq \pi_r(w, b, 0)\}$ . The latter is the set of contracts given that the retailer will not invest in acquiring information. That is  $\Omega_N = \{(w, b) : \frac{1}{2}\pi_r(w, b, 1) + \frac{1}{2}\pi_r(w, b, 2) - e < \pi_r(w, b, 0)\}$ .

The supplier will choose the optimal contract to maximize his profit given the retailer's reaction. First, we consider the case that in equilibrium the supplier will not induce the retailer to invest in acquiring information. We have

**Corollary 2** *If the supplier chooses a contract that does not induce the retailer to invest in acquiring information in equilibrium, then that contract is  $w_v = b_v = p$ . In this case, the supplier gets the first best profit of the whole system conditional on not knowing additional information. The retailer's profit is 0.*

That is, if the optimal contract  $(w, b) \in \Omega_N$  in equilibrium, then that contract is  $w_v = b_v = p$ . Denoting the profit of the retailer as  $\pi_s^N$ , we have

$$\pi_s^N = \Pi(0) \tag{19}$$

That is,

$$\pi_s^N = \begin{cases} (p - c)l & \text{if } \frac{p}{2} - c < 0 \\ pl + \frac{1}{2}p(h - l) - ch & \text{if } \frac{p}{2} - c \geq 0 \end{cases} \tag{20}$$

Intuitively, if the retailer does not invest in acquiring information in equilibrium, the symmetric information situation occurs as we have discussed. The supplier will try to set the optimal wholesale price and the optimal buy-back price as in Theorem 1. The retailer will purchase the first best quantity at the same time. The optimal buy-back contract for the system does not depend on distribution of demand. To maximize his profit, the supplier will choose the wholesale price and the buy-back price close to 0. Hence the retailer's profit is close to 0.

Since the retailer's reserve utility is 0, and he cannot retrieve information and order quantity 0, the retailer will always accept the contract in our model.

We consider the case that the retailer would like to invest in acquiring information voluntarily. If the supplier chooses contracts  $(w, b) \in \Omega_Y$  to induce the retailer to retrieve information, the supplier's problem can be stated as follows:

$$\begin{aligned} & \max_{w,b} \frac{1}{2} \pi_s(w, b, 1) + \frac{1}{2} \pi_s(w, b, 2) \\ = & \max_{w,b} \frac{1}{2} E\{wq(w, b, 1) - b \max(0, q(w, b, 1) - D) | 1\} + \\ & \frac{1}{2} E\{wq(w, b, 2) - b \max(0, q(w, b, 2) - D) | 2\} \end{aligned} \quad (P1)$$

Subject to

$$\frac{1}{2} \pi_r(w, b, 1) + \frac{1}{2} \pi_r(w, b, 2) - e \geq \pi_r(w, b, 0) \quad (IC-1)$$

and

$$q(w, b, I) = \arg \max_q E\{p \min(D, q) - wq + b \max(0, q - D) | I\} \quad (IC-2)$$

We can simplify the equation (IC-2) as follows:

$$q(w, b, I) = \begin{cases} h & \text{if } p \text{ Prob}(h | I) + b \text{ Prob}(l | I) - w \geq 0 \\ l & \text{if } p \text{ Prob}(h | I) + b \text{ Prob}(l | I) - w < 0 \end{cases} \quad (IC-2')$$

Further more, we can show that the IC constraint of the retailer (IC-1) is binding.

**Proposition 2** *A necessary condition for the optimal contract  $(w, b) \in \Omega_Y$  in equilibrium is that the IC constraint of the retailer (IC-1) is binding in equilibrium. Hence (IC-1) can be replaced by*

$$\frac{1}{2} \pi_r(w, b, 1) + \frac{1}{2} \pi_r(w, b, 2) - e = \pi_r(w, b, 0) \quad (IC-1')$$

**Proof.** If  $(w, b) \in \Omega_Y$ , the retailer will invest in acquiring information in equilibrium. We suppose that the retailer's constraint (IC-1) is not binding in equilibrium. Then the problem is equivalent to equation (P1) subject to (IC-2'). In this situation, similar to Theorem 1, the supplier can maximize his profit by setting the wholesale price and buy-back price as  $w_v = p(1 - v) + cv$  and  $b_v = p(1 - v)$  with  $v$  close to 0. The supplier gets the profit of the system with complete information. The expected profit of the retailer is 0, but the cost to retrieve the information is  $e > 0$ . This is contradictory to (IC-1) since  $\pi_r(w, b, 0) \geq 0$ .

In addition, we have

**Theorem 2** *Necessary conditions for the optimal contract  $(w, b) \in \Omega_Y$  in equilibrium are*

$$q(w, b, 1) = h \quad (20)$$

$$q(w, b, 2) = l \quad (21)$$

*In other words, if in equilibrium the supplier chooses a contract which induces the retailer to exert effort, the retailer will choose a high level of inventory if he received good news and a low level of inventory if he received bad news.*

**Proof.** If  $(w, b) \in \Omega_N$ , by corollary 2, we have  $\pi_s(0) = \Pi(0)$  in equilibrium. Denote the first best quantity level as  $q^f(0)$ . When  $\frac{p}{2} - c \geq 0$ , from equation (4), we have  $q^f(0) = h$ . If  $(w, b) \in \Omega_Y$ , since  $\theta = \text{Prob}(h | I = 1) = \text{Prob}(l | I = 2) > 1/2$  and the IC constraint (IC-2)', we know  $q(w, b, 1) \geq q(w, b, 2)$ . We want to show that  $q(w, b, 1) = q(w, b, 2)$  is not true in equilibrium. Suppose  $q(w, b, 1) = q(w, b, 2)$  is true in equilibrium. There are two possibilities:  $q(w, b, 1) \geq q(w, b, 2) = h$  or  $q(w, b, 1) \geq q(w, b, 2) = l$ . For both cases, we have  $\frac{1}{2}\pi_s(1) + \frac{1}{2}\pi_s(2) \leq \Pi(0) - e < \pi_s(0)$  since  $e > 0$ . The supplier will not induce the retailer to retrieve information. It is not equilibrium. When  $\frac{p}{2} - c < 0$ , similar reasoning works. We have the contradiction.

Using Theorem 2, the equation (IC-2)' becomes the following:

$$q(w, b, 0) = \begin{cases} h & \text{if } \frac{p+b}{2} - w \geq 0 \\ l & \text{if } \frac{p+b}{2} - w < 0 \end{cases} \quad (\text{IC-2-0})$$

$$q(w, b, 1) = h \quad (\text{IC-2-1})$$

$$q(w, b, 2) = l \quad (\text{IC-2-2})$$

Intuitively, supplier can be better off only if the performance of the whole system is better off. This means the information is “valuable”, i.e. it can help the retailer’s decision. Because it is costly to retrieve information, the added value by acquiring information should be able to cover the cost of inducing the retailer to invest in information acquiring.

When  $(w, b) \in \Omega_Y$ , the equation (IC-2-2) shows that if the given contract  $(w, b)$  satisfies  $\frac{p+b}{2} - w \geq 0$ , it is optimal for an uninformed retailer to choose a high inventory level to maximize his own profit. If the given contract  $(w, b)$  satisfies  $\frac{p+b}{2} - w < 0$ , it is optimal for an uninformed retailer to choose a low inventory level. Therefore we can divide the set of contracts  $\Omega_Y$  into two sub-sets:  $\Omega_h$  and  $\Omega_l$ . The former is the sub-set of contracts under which the optimal inventory for an uninformed retailer is high. That is  $\Omega_h = \{(w, b) : \frac{p+b}{2} - w \geq 0\}$ . The latter is the sub-set of contracts under which the optimal inventory for an uninformed retailer is low. That is  $\Omega_l = \{(w, b) : \frac{p+b}{2} - w < 0\}$ .

Substituting (IC-2-1) and (IC-2-2) into equation (P1), we can rewrite equation (P1) as follows:

$$\max_{w, b} wl + \frac{1}{2}[w - b(1 - \theta)](h - l) - c \frac{h + l}{2} \quad (\text{P2})$$

Substituting (IC-2,0), (IC-2-1) and (IC-2-2) into equation (IC-1') and using notation  $\Omega_h$  and  $\Omega_l$ , we can rewrite equation (IC-1') as follows:

$$\begin{cases} p(1-\theta) + b\theta + \frac{2e}{h-l} = w & \forall (w,b) \in \Omega_h \\ p\theta - w + \frac{b}{2}(1-\theta) = \frac{2e}{h-l} & \forall (w,b) \in \Omega_l \end{cases} \quad (IC)$$

Since the IC constraints have different formulae when  $(w,b)$  belong to different available sets, we need solve this problem in each set and find the solution, which is the maximization of both.

**Proposition 3** *The optimal contract for (P2) subject to the constraint  $(w,b) \in \Omega_h$  is*

$$b_h = p - \frac{2e}{(\theta - \frac{1}{2})(h-l)} \quad (22)$$

and

$$w_h = p - \frac{e}{(\theta - \frac{1}{2})(h-l)} \quad (23)$$

**Proof.** If  $(w,b) \in \Omega_h$  is in equilibrium, the problem (P2) becomes

$$\max_{w,b} wl + \frac{1}{2}[w - b(1-\theta)](h-l) - c\frac{h+l}{2} \quad (P-h)$$

Subject to

$$p(1-\theta) + b\theta + \frac{2e}{h-l} = w \quad (IC-h)$$

and

$$\frac{p+b}{2} - w \geq 0 \quad (h)$$

The solutions to this problem are the equations (22) and (23).

From Proposition 3, we have  $\pi_s(w,b) \leq \pi_s(w_h, b_h)$  for any  $\{w,b\} \in \Omega_h$ .

Since  $\Omega_l$  is not a closed set, we can define  $\bar{\Omega}_l = \{(w,b) : \frac{p+b}{2} - w \leq 0\}$ . We know  $\Omega_l \subset \bar{\Omega}_l$ . We have

**Proposition 4** *The optimal contract for (P2) subject to  $(w,b) \in \bar{\Omega}_l$  is also given by equations (22) and (23), i.e.*

$$b_l = p - \frac{2e}{(\theta - \frac{1}{2})(h-l)} = b_h \quad (24)$$

And

$$w_l = p - \frac{e}{(\theta - \frac{1}{2})(h-l)} = w_h \tag{25}$$

**Proof.** If  $(w, b) \in \bar{\Omega}_l$  is in equilibrium, the problem (P2) becomes

$$\max_{w,b} wl + \frac{1}{2}[w - b(1-\theta)](h-l) - c\frac{h+l}{2} \tag{P-l}$$

subject to

$$p\theta - w + \frac{b}{2}(1-\theta) = \frac{2e}{h-l} \tag{IC-l}$$

and

$$\frac{p+b}{2} - w \leq 0 \tag{l}$$

The solutions for this problem are equations (24) and (25).

Since  $\Omega_l \subset \bar{\Omega}_l$ , equations (24) and (25) are clearly the solutions to the (P2) subject to the constraint  $(w, b) \in \Omega_h \cup \Omega_l$ . If the supplier induces the retailer to invest in acquiring information in equilibrium, the retailer would have chosen  $h$  if he has not acquired information.

We have

**Theorem 3** *If in equilibrium, the supplier chooses a contract that induces the retailer to invest in acquiring information, then the contract  $(w, b)$  satisfies*

$$w = \frac{p+b}{2} \tag{26}$$

In addition, we have

$$b = p - \frac{2e}{(\theta - \frac{1}{2})(h-l)} \tag{27}$$

and

$$w = p - \frac{e}{(\theta - \frac{1}{2})(h-l)} \tag{28}$$

**Proof.** We can verify that equations (27) and (28) maximize supplier profit subject to the (IC) constraint. As a result they will also maximize supplier profit under constraints (IC-1) and (IC-2). That is, they are optimal contracts if the supplier induces the retailer to invest in acquiring information in equilibrium. We

can check that  $w = \frac{p+b}{2}$  directly from equations (27) and (28).

Theorem 3 shows that if the supplier chooses an optimal contract that induces the retailer to invest in acquiring information in equilibrium, the profit of the supplier is:

$$\pi_s = wl + \frac{1}{2}[w - b(1 - \theta)](h - l) - c \frac{h + l}{2} \tag{29}$$

where  $b$  and  $w$  are defined in equations (27) and (28).

Bringing equations (27) and (28) into equation (29), we can simplify the supplier's profit as

$$\pi_s(e) = pl + \frac{1}{2} p\theta(h - l) - \frac{e}{(\theta - \frac{1}{2})(h - l)} [l + \frac{2\theta - 1}{2}(h - l)] - c \frac{h + l}{2} \tag{30}$$

We can define  $e^*$  by the following equation:

$$\pi_s(e^*) = \pi_s^N \tag{31}$$

We have

**Proposition 5** *In equilibrium, the optimal contract is  $w = p - \frac{e}{(\theta - \frac{1}{2})(h - l)}$  and  $b = p - \frac{2e}{(\theta - \frac{1}{2})(h - l)}$*

*if and only if the cost of effort is  $e \leq e^*$ .*

**Proof.** Since  $l + \frac{2\theta - 1}{2}(h - l) > 0$ , from equation (30), we have  $\frac{\partial \pi_s(e)}{\partial e} < 0$ . That is,  $\pi_s(e)$  is a decreasing function of  $e$ . From equation (31), we know that  $\pi_s(e) \geq \pi_s^N$  if the cost of effort is  $e \leq e^*$ , and  $\pi_s(e) < \pi_s^N$  if the cost of effort is  $e > e^*$ .

We can see that  $e^*$  is the threshold effort level. When the effort is  $e > e^*$ , it is too costly to induce the retailer to retrieve information in equilibrium.

We define  $\frac{e}{(\theta - \frac{1}{2})(h - l)} [l + \frac{2\theta - 1}{2}(h - l)]$  to be the virtual cost. That is, the cost the supplier will pay to

induce the retailer to invest in acquiring information. And we define  $\frac{el}{(\theta - \frac{1}{2})(h - l)}$  to be the information

rent, which is the benefit the retailer extracts from the channel because of his information advantage.

Bringing equations (20) and (30) into equation (31), we have:

$$e^*(m, \sigma, p, \theta, c) = \begin{cases} \max\left(0, \frac{(p\theta - c)(h-l)}{\frac{4l}{(2\theta-1)(h-l)} + 2}\right) & \text{if } \frac{p}{2} - c < 0 \\ \max\left(0, \frac{[c - (1-\theta)p](h-l)}{\frac{4l}{(2\theta-1)(h-l)} + 2}\right) & \text{if } \frac{p}{2} - c \geq 0 \end{cases} \quad (37)$$

We have

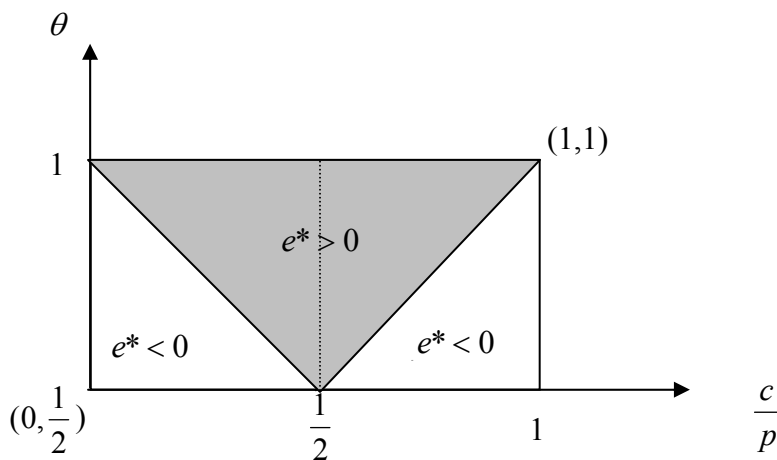
**Proposition 6** *The threshold effort level  $e^*(m, \sigma, p, \theta, c) > 0$  if and only if  $\theta p > c > (1-\theta)p$ .*

**Proof.** We can verify this from equation (32) directly.

If  $e^*(m, \sigma, p, \theta, c) \leq 0$ , it means that the effort is not exerted in equilibrium for any effort level  $e > 0$ .

From Figure 3, we can see that if the parameters fall out of the grey triangle area, in equilibrium the supplier does not induce the retailer to invest in acquiring information, even if the cost of retrieving information is very small.

Figure 3

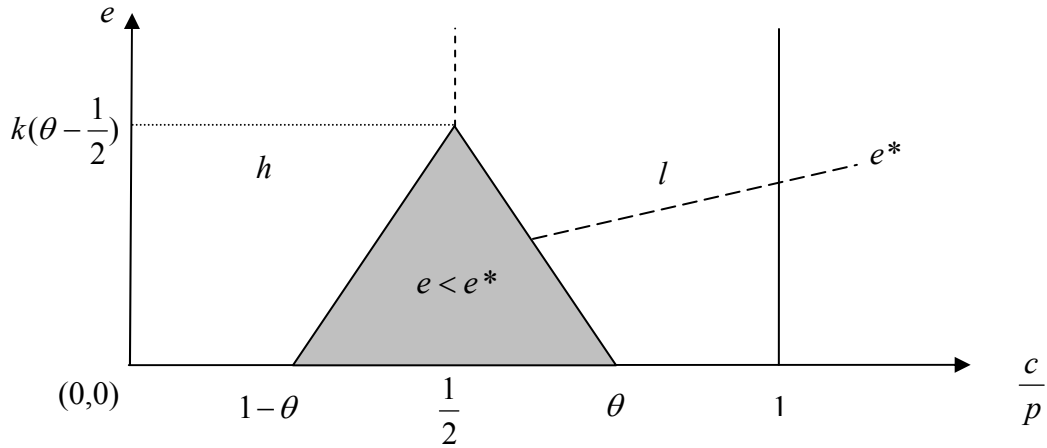


From Figure 4, we know that if the parameters fall out of the grey area, the supplier does not induce the retailer to invest in acquiring information. The boundary of the grey area is the threshold effort level  $e^*$ .

In addition, outside the grey area, if  $\frac{c}{p} < \frac{1}{2}$ , the inventory level in equilibrium is high; if  $\frac{c}{p} > \frac{1}{2}$ , the

inventory level in equilibrium is low. When the parameters are in the grey area, in equilibrium the supplier induces the retailer to invest in acquiring information, and the equilibrium inventory level is high conditional on good news and low conditional on bad news.

Figure 4: If Parameters Fall out the Grey Area, Supplier Doesn't Induce Retailer to Invest in Acquiring Information



Where  $k = \frac{p(h-l)}{\frac{4l}{(2\theta-1)(h-l)} + 2}$  in Figure 4.

In addition, we have

**Corollary 3** *A necessary condition for the supplier to induce the retailer to invest in acquiring information in equilibrium is  $\theta p > c > (1-\theta)p$ .*

### COMPARATIVE STATIC ANALYSIS

In order to give a more illustrative analysis in our model, we use the following notation:

Denote the mean of the demand as  $m$ , and the variance of the demand as  $\sigma^2$ . We have

$$m = E(D) = \frac{h+l}{2} \tag{33}$$

and

$$\sigma = \sqrt{\text{var}(D)} = \frac{h-l}{2} \tag{34}$$

So quantity levels  $h$  and  $l$  can be characterized by parameters  $m$  and  $\sigma$ . We have the following equations:

$$h = m + \sigma \tag{35}$$

$$l = m - \sigma \tag{36}$$



The Optimal Contract

We can rewrite equations (27) and (28) as follows:

$$b(m, \sigma, e, p, \theta, c) = p - \frac{2e}{(2\theta - 1)\sigma} \quad (37)$$

And

$$w(m, \sigma, e, p, \theta, c) = p - \frac{e}{(2\theta - 1)\sigma} \quad (38)$$

We have

**Proposition 7** *If  $e < e^*$ , in equilibrium the optimal contract  $(w(m, \sigma, e, p, \theta, c), b(m, \sigma, e, p, \theta, c))$  satisfies  $b_e < 0$ ,  $w_e < 0$ ,  $b_\sigma > 0$ ,  $w_\sigma > 0$ ,  $b_\theta > 0$ ,  $w_\theta > 0$ ,  $w_p = b_p = 1$  and  $w_m = b_m = w_c = b_c = 0$ .*

**Proof.** From Proposition 5, we know that in equilibrium, the optimal contract satisfies equations (37) and (38). Notice that  $\theta > \frac{1}{2}$ . We can get these results from equations (37) and (38) directly.

When effort is exerted in equilibrium, if the cost of effort increases, the supplier has to compensate for the cost of effort. We know a more generous return policy can help the supplier capture channel profit. When the supplier offers a more generous return policy, she will increase the wholesale price at the same time in order to capture more channel profit. Lowering the wholesale and buy-back prices can offer the retailer more to compensate his cost of effort. The lower buy-back price means less insurance for the retailer, which gives more incentive to invest in acquiring information. So  $b_e < 0$ ,  $w_e < 0$ .

When the variance of demand increases, it is more profitable for the retailer to improve his profit by investing in acquiring information. The supplier would like to increase his own profit by increasing the wholesale price and buy-back price, subject that the retailer's (IC) constraint is still held. We have  $w_\sigma > 0$  and  $b_\theta > 0$ .

It is interesting to note that the wholesale price and buy-back price do not depend on the mean of demand. Intuitively, the mean of demand can be considered the determined part of demand. But the optimal buy-back contract will help solve the uncertainty of the problem. In this case, changing the determined part will not change the optimal buy-back contract.

If the information becomes more informative, i.e. if  $\theta$  increases, it is easier to induce the retailer to acquire the information. The supplier would like to increase his profit by increasing the wholesale price and the buy-back price subject that the retailer's (IC) constraint is still held. So  $b_\theta > 0$ ,  $w_\theta > 0$ .

The optimal buy-back contract does not depend on production cost. Because here, we use the optimal buy-back contract to stimulate the retailer to retrieve information, therefore the retailer does not care about the cost of production. The cost of production can only help the supplier decide whether to induce the retailer to invest in acquiring information. But the optimal buy-back contract would depend on production cost with a downward slope if  $p$  were endogenous.

The Threshold Effort Level

We can rewrite equation (32) as follows:

$$e^*(m, \sigma, p, \theta, c) = \begin{cases} \max\left(0, \frac{(p\theta - c)\sigma}{\frac{m - \sigma}{(2\theta - 1)\sigma} + 1}\right) & \text{if } \frac{p}{2} - c < 0 \\ \max\left(0, \frac{[c - (1 - \theta)p]\sigma}{\frac{m - \sigma}{(2\theta - 1)\sigma} + 1}\right) & \text{if } \frac{p}{2} - c \geq 0 \end{cases} \quad (39)$$

**Proposition 8** *If  $\theta p > c > (1 - \theta)p$ , the threshold effort level  $e^*(m, \sigma, p, \theta, c)$  satisfies  $e_m^* < 0$ ,  $e_\sigma^* > 0$ ,  $e_\theta^* > 0$  and*

$$e_p^* \begin{cases} > 0 & \text{if } \frac{p}{2} - c < 0 \\ < 0 & \text{if } \frac{p}{2} - c \geq 0 \end{cases} \quad (44)$$

$$e_c^* \begin{cases} < 0 & \text{if } \frac{p}{2} - c < 0 \\ > 0 & \text{if } \frac{p}{2} - c \geq 0 \end{cases} \quad (45)$$

**Proof.** From Proposition 6, if  $\theta p > c > (1 - \theta)p$ , we have  $e^*(m, \sigma, p, \theta, c) > 0$ . We can check equation (39) and obtain the results.

When effort is exerted in equilibrium, the retailer will share the channel profit with the supplier. But when effort is not exerted in equilibrium, the supplier will extract all the profit of the system. When the expectation of demand increases, it is more unlikely that the supplier will offer the optimal contract to elicit the retailer to retrieve information. So  $e_m^* < 0$ .

As the variance of demand increases, more accurate information will aid the performance of the system. And it is easier to elicit the retailer to invest in acquiring information. We have  $e_\sigma^* > 0$ .

The impact of the changes in retail price and in production cost is not so obvious. They depend on the relation between price and cost.

When the information becomes more informative, the performance of the system will increase with more accurate information. It is more likely that the effort level will be exerted. That is,  $e_\theta^* > 0$ .

When  $\theta$  is close to  $\frac{1}{2}$ , we can see the threshold level effort goes to infinity. That is,

**Corollary 4** *If  $\theta p > c > (1 - \theta)p$ , we have  $e^* \rightarrow 0$  when  $\theta \rightarrow \frac{1}{2}$ .*

### The Profit of the Retailer

Next, we will see what happens to the profits of the retailer and supplier when some exogenous parameters change.

If  $\theta p > c > (1 - \theta)p$  and  $e < e^*$ , in equilibrium the profit of the retailer is as follows:

$$\pi_r = pl + \frac{1}{2}[p\theta + b(1 - \theta)](h - l) - \frac{w}{2}(h + l) \quad (42)$$

We can simplify it thus:

$$\pi_r(m, \sigma, e, p, \theta, c) = \frac{(m - \sigma)e}{(2\theta - 1)\sigma} \quad (43)$$

We can verify that the profit of the retailer is the information rent, by which the supplier will offer the incentive to induce the retailer to invest in acquiring information.

We have

**Proposition 9** *If  $\theta p > c > (1 - \theta)p$  and  $e < e^*$ , in equilibrium the profit of the retailer has the following properties:*

$$\frac{\partial \pi_r(m, \sigma, e, p, \theta, c)}{\partial e} > 0 \quad (44)$$

$$\frac{\partial \pi_r(m, \sigma, e, p, \theta, c)}{\partial m} > 0 \quad (45)$$

$$\frac{\partial \pi_r(m, \sigma, e, p, \theta, c)}{\partial \sigma} < 0 \quad (46)$$

$$\frac{\partial \pi_r(m, \sigma, e, p, \theta, c)}{\partial \theta} < 0 \quad (47)$$

$$\frac{\partial \pi_r(m, \sigma, e, p, \theta, c)}{\partial p} = \frac{\partial \pi_r(m, \sigma, e, p, \theta, c)}{\partial c} = 0 \quad (48)$$

**Proof.** From Proposition 6, if  $\theta p > c > (1 - \theta)p$ , we have  $e^*(m, \sigma, p, \theta, c) > 0$ . We can derive these results from equation (43).

When effort is exerted in equilibrium, information rent is proportional to the effort level. The retailer shares the channel profit according to the improved demand information with the supplier. When the expectation of demand increases, retailer profit also increases. When the variance of demand increases or the information becomes more informative, it is easier for the supplier to elicit the retailer to invest in acquiring information. Information rent decreases, and retailer profit decreases as well.

It is interesting that information rent is unrelated to retail price. This result would not hold if the retail price  $p$  is endogenous.

The optimal buy-back contract is unrelated to the cost of production. Hence the retailer's profit is unrelated to the cost of production.

Profit of the Supplier

We can rewrite equation (30) as follows:

$$\pi_s(m, \sigma, e, p, \theta, c) = (p - c)m - (1 - \theta)p\sigma - e \left[ \frac{m - \sigma}{(2\theta - 1)\sigma} + 1 \right] \tag{49}$$

In this situation, the virtual cost is  $e \left[ \frac{m - \sigma}{(2\theta - 1)\sigma} + 1 \right]$  and the information rent can be written as  $e \left[ \frac{m - \sigma}{(2\theta - 1)\sigma} \right]$ , which is the profit of the retailer.

**Proposition 10** *If  $\theta p > c > (1 - \theta)p$  and  $e < e^*$ , in equilibrium the profit of the supplier has the following properties:*

$$\frac{\partial \pi_s(m, \sigma, e, p, \theta, c)}{\partial e} < 0 \tag{50}$$

$$\frac{\partial \pi_s(m, \sigma, e, p, \theta, c)}{\partial p} > 0 \tag{51}$$

$$\frac{\partial \pi_s(m, \sigma, e, p, \theta, c)}{\partial c} < 0 \tag{52}$$

$$\frac{\partial \pi_s(m, \sigma, e, p, \theta, c)}{\partial \theta} > 0 \tag{53}$$

$$\frac{\partial \pi_s(m, \sigma, e, p, \theta, c)}{\partial m} > 0 \tag{54}$$

$$\frac{\partial \pi_s(m, \sigma, e, p, \theta, c)}{\partial \sigma} \begin{cases} > 0 & \text{if } \frac{(1 - \theta)(2\theta - 1)p\sigma^2}{m\sigma + m - \sigma} < e < e^* \\ < 0 & \text{if } e < \frac{(1 - \theta)(2\theta - 1)p\sigma^2}{m\sigma + m - \sigma} \end{cases} \tag{55}$$

**Proof.** From Proposition 6, if  $\theta p > c > (1 - \theta)p$ , we have  $e^*(m, \sigma, p, \theta, c) > 0$ . We can derive those results from equation (49).

When effort is exerted in equilibrium, the supplier need not only compensate the retailer with the cost of acquiring information, but also offer an incentive for the retailer to invest in acquiring information. The information rent is also an increasing function of the cost of effort, which means when the cost of effort increases, the profit of the supplier decreases.

That is,

$$\frac{\partial \pi_s(m, \sigma, e, p, \theta, c)}{\partial e} < 0.$$

If the retail price increases or production cost decreases, the performance of the system will increase. However the information rent is the same by Proposition 8, and the supplier is better off. By Proposition 8, as the information becomes more informative, the retailer profit or the information rent decreases. However, the performance of the system is better off and supplier profit increases. When the expectation of demand increases, the performance of the system is better off and supplier profit increases at the same time.

When the variance of demand increases, the change in supplier profit is indeterminate: On one hand, the performance of the system is worse off. On the other hand, since the information rent decreases, it is easier for the supplier to elicit the retailer to invest in acquiring information. But when the cost of effort is very small, usually supplier profit decreases.

### SOME EXTENSIONS

It is interesting to extend our model in some different directions:

1. Who will retrieve the information when both the supplier and the retailer are able to retrieve it?
2. The model that the retailer's effort can help improve product sales
3. When both firms' effort can help improve product sales, what should the ownership structure be?
4. Demand can be changed with price. That is, demand is  $D(p)$ , which is a binary discrete distribution dependent on  $p$ . The prior probabilities are  $Prob(D=h(p))=P_h(p)$  and  $Prob(D=l(p))=1-Prob(D=h(p))=P_l(p)$ . In this situation, the standard vertical model is a special case of our model. There is uncertainty of demand in our model, yet it is still technically tractable.
5. The demand function is continuous.
6. We can consider the quantity-based return policy (quantity flexible contracts).

### CONCLUSIONS

The standard buy-back contract model is not consistent with the empirical findings of Kulp (2002) that VMI systems are prevalent when the retailer is better informed than the supplier. We analyze the optimal buy-back contract for a supplier selling to a retailer when demand is uncertain, while the retailer can take a costly unobservable action to forecast demand more accurately. We show when it is costly for the retailer to obtain a better forecast of demand, the first best outcome cannot always be implemented by an optimal buy-back contract. If the retailer invests in acquiring information in equilibrium, the total revenue of the supplier covers not only the cost of investing in acquiring information, but also a positive information rent. Hence, the fact that VMI systems are prevalent can be well explained in our model. In addition, we offer in our model many testable results for which we look forward to having empirical application. Our model is appropriate for analyzing vertical systems when a downstream firm has private information in acquiring more accurate information of uncertain demand. The model is technically tractable while it captures most properties we are interested in, and the explicit solution is available as well.

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