GENERALISTS, SPECIALISTS: WHO GET TO THE TOP

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ABSTRACT

This paper tries to analyze the three aspects of hierarchy: (1) generalists or specialists: which should get to the top? (2) How many agents should get to the top? (3) Can the agents who should be at the top in the optimal hierarchy really get to the top? Using a T-period model with promotion, the paper finds that the optimal hierarchy form depends on the size of the externality of coordinating multiple assets by generalists. How many agents should be at the top depends on the elasticity of the externality of coordinating multiple assets. Finally, promotion opportunity gives agents who should at the top more incentive to exert effort, and thus are more likely to get promoted.

KEYWORDS: hierarchy, incentive, promotion, promotion opportunities, generalists, specialists

JEL: D23, D01, L23, M51

INTRODUCTION

This paper aims to identify which hierarchy form should be employed by a certain firm. In this paper, I will study three aspects of hierarchy forms: (1) generalists or specialists, which type should get to the top? (2) How many agents should get to the top? (3) Can the agents who should be at the top really get to the top? The first two aspects deal with the design of optimal hierarchy form, and the third aspect verifies the feasibility of the optimal hierarchy form. The model is suitable for large corporations rather than small owner-managed firms.

The paper defines hierarchy form in terms of authority as in Aghion and Tirole (1997) and Hart and Moore (2005); i.e., the upper-level agent has authority over his subordinates, the lower-level agents. The special feature of this paper is involving incentives in the hierarchy design. Thus, the paper considers not only the ex ante incentive, but also the ex post efficiency. With a T-period model, using backward induction, the paper tries to analyze all the three aspects of hierarchy mentioned above.

The paper is motivated by the broadly observed phenomenon that fresh graduates first work at entry levels, and within several years, some of them are promoted while others are not. Generalists and specialists differ in potential productivity, which is unobservable and non-contractible at the time of hiring. Specialists can learn the specialties from experience and have a higher productivity in working with single asset than a generalist after the initial period. Generalist cannot accumulate specialty experience, but he will have a higher productivity in coordinating multiple assets than a specialist after familiar with the working environment in the initial period. Only two hierarchy forms are considered, pyramid form (generalists at the top) and inverted pyramid form (specialists at the top).

Who should be at the top depends on the size of the externality of coordinating multiple assets. If the size of the externality is large, generalist at the top is desirable; if it is small, specialist at the top may be desirable. The optimal number of the agents at the top depends on the optimal span of control that depends on the elasticity of the externality of coordinating multiple assets. In the pyramid form, if the size of the externality is very sensitive to the number of the assets, i.e., if the elasticity of the externality is large, the span of control should be large, and optimally there should be fewer generalists at the top; otherwise, more agents should be at the top. Finally, the T-period model with promotion can give agents who should at the top more incentive to work harder and thus are more likely to be promoted to the top.

The paper is organized as follows. In Section 2, I will review the recent literature on hierarchy. Section 3 introduces the model and assumptions. Section 4 analyzes who should get to the top. Section 5 analyzes how many agents should get to the top. Section 6 analyzes the initial period and discusses the incentive of getting to the top. Section 7 is conclusion.

LITERATURE REVIEW

Hierarchy has become a hot issue since the internal organization of the firm has attracted more attention of not only the scholars in management science but also economists. Many scholars argue that hierarchy is indispensable in large organizations. The authority system provided by hierarchical structure makes it possible that unambiguous accountability is preserved in organizations with large numbers of people (Jacques, 1990). In addition, hierarchical structure plays an important role in processing information by decomposing large organizations into small information processing units (Williamson, 1985).

Generally, hierarchy has been modeled in two ways. In one way, the firm is defined as the owner of a set of assets, and it authorizes agents the right to use these assets. Each asset represents a decision on the use of the asset. Thus, in this framework, hierarchy can be interpreted as a sequence of commands over assets. For a subset of the assets k, the most senior agent exercises authority, unless he delegates the authority to the next agent(s) in the sequence. Aghion and Tirole (1997) study delegation in a setting where two agents, a boss and his subordinate, have incongruent objectives. They argue that delegation involves a tradeoff between increase in subordinate's incentive and cost of loss of control. Hart and Moore (2005) study the optimal hierarchical structure given that coordinators and specialists have different tasks. Based on certain assumptions, they conclude that coordinators should be senior to specialists, "crisscross" hierarchies are never optimal, and the optimal hierarchy is a pyramid form under certain condition.

Another way of modeling hierarchy treats the firm as an information processor, and it solves tasks by collecting, communicating and confirming information. In this framework, hierarchy can be interpreted as the locus of the communication of the information. New information is acquired and processed at the lower level and then transferred upstream to the boss, while the command of the boss is passed downstream to the lower-level agents. However, communication is imperfect and costly, not only because communicating and absorbing new information cost time, but because information may be contaminated or lost in the communication process. The cost of communication depends on the nature of the information. "Specific knowledge" is more costly to transfer than "general knowledge" (Jensen and Meckling, 1992). Thus, there is a tradeoff between specialization and communication. Bolton and Dewatripont (1994) argue that if the returns to specialization outweigh costs of communication, it is efficient for several agents to collaborate within a firm. Jensen and Meckling (1992) also argue that it is desirable for groups of individuals to exercise decision rights jointly because of bounded rationality (an individual has limited mental capability) and the inalienability of rights within an organization.

THE MODEL

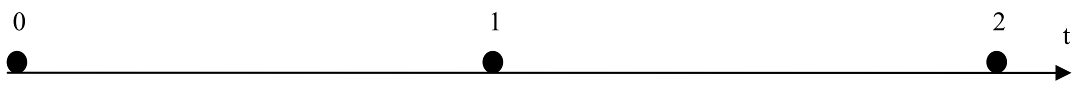
The model is a T-period internal labor market model. The organization form is a "hierarchy over assets", which is contractible ex ante at the beginning of period 1. The hierarchy modeling is in spirit of Hart and Moore (2005): there is a chain of commands over each asset, and the most senior person with 'an idea' exercises authority over the asset. In more detail, in the hierarchy, if the most senior agent who is senior to others on all of his working assets has an idea, then he can exercise his idea and generates value, while any agents who work with any of these assets and junior to him cannot exercise their idea even though they have one. On the other hand, if the most senior agent does not have an idea on the assets, he will pass the authority to his subordinates, the lower-level agents.

Assume there are n assets in a firm. The size of n depends on the size of the firm that is assumed exogenous. Assets are identical; each single asset can produce the same value $V(1)$, and any combination of $k > 1$ assets can produce the same value $V(k)$. In period 1, the firm hires N agents, both generalists and specialists, in the competitive external labor market. By signing contract, the firm commits ex ante to promoting Q_0 percentage of agents in period 2.

Generalists and specialists differ in potential productivity, but among the same type, agents are identical. Assume asymmetric information at hiring, so that the firm does not know agents' types in period 1, while each agent knows his own type. Because of the lack of information about agents' types, the firm has to treat all the agents in the same way in period 1. Assume in period 1, all newly hired agents are treated as lower-level agents. Each agent will work on a set of assets consisting of $k = n/N$ assets, and will be paid the same wage, W_0. With effort level e_1, an agent can generate value $V(k)$ with probability $P(e_1, k)$. This is referred as "an agent has an idea" (Hart, Moore, 2005). Effort is unobservable, but the value generated is observable.

At the beginning of period 2, Q_0 percentage of agents with the highest value generated will be promoted to the upper level in the hierarchy and they will have authority over a different set of assets consisting of j assets. The others who do not get promoted will stay at the lower level and still have authority over k working assets. In period 2, the upper-level agents are paid W^u, and lower-level agents are paid W^l; both W^u and W^l depend on the performance of the agents. An agent can choose to quit or stay at the beginning of period 2. The time line is shown in figure 1. By repeating this hire-promotion process for H periods, the optimal hierarchy form will be achieved and stable.

Figure 1: Time Line



0	1
The firm hires N agents	Promotion
Sign contract	Lower-level agents work with k assets, and are paid W^l
All agents work at lower level	Upper-level agents work with j assets, and are paid W^u
with k assets, and are paid W_0	Agents choose to stay or quit

The advantages of the proposed contract are as follows. In period 1, since the firm does not know agents' types, there is a hidden information problem. By providing a promotion opportunity, the principal can give agents incentive to reveal their types. In period $t \geq 2$, since effort level is unobservable and non-contractible, there is a hidden action problem (moral hazard). However, since the hierarchy is formed, and each agent has been allocated to the proper position, payment based on performance could be an effective and fair way to give agents incentive.

Types and Productivities

Specialists and generalists differ in term of their potential productivities. In period 1, all agents have the same productivity. With effort level e_1, an agent can generate value $V(k)$ from k assets with probability $P(e_1, k)$. Since period 2, a specialist has accumulated the specialties from period 1 and has a higher productivity in working with a single asset than a generalist. Therefore, in period $t \geq 2$, with effort level e_t, $P^s(e_t, 1) > P^g(e_t, 1)$ (the superscripts s and g stand for specialist and generalist respectively, and the subscripts t stands for the time period). A generalist cannot accumulate specialty experience, but he will have a higher productivity in coordinating multiple assets than a specialist after familiar with the working environment in period 1. Therefore, in period $t \geq 2$, with effort level e_t, $P^g(e_t, k) > P^s(e_t, k)$, for any $k > 1$. Assume that there is no more productivity improvement after period 2.

Preferences

Assume that all agents are risk neutral and live for T periods, $T \geq H$. Each agent has reservation utility $\bar{u}$, and he maximizes his expected utility, which is a linear function of the total expected income net of the cost of effort. For simplicity, assume no discounting (relaxing this simplification will not affect the results).

Assume that the probability of being promoted for an agent is Q. Then, the total expected wage income of an agent i is:

$$E(W^i) = W_0 + \sum_t [\, QW^u + (1 - Q)\, W^l], \quad i = g, s \tag{1}$$

And the expected utility of an agent i is:

$$E(U^i) = E(W^i) - C(e^i{}_1) - \sum_t [\, Q\, C(e^{ij}{}_t) + (1 - Q)\, C(e^{ik}{}_t)], \quad i = g, s \tag{2}$$

where k and j are the number of assets a lower-level agent and an upper-level agent have authority over, respectively.

The firm is risk neutral. The firm's object is twofold. First, firm wants to choose the optimal hierarchy form that can maximize the expected profit. Second, firm wants to hire and promote the proper agents to realize such a hierarchy.

<u>Other Assumptions</u>

Assumption (1)*:* Probability of generating value is a function of type i, effort level e, and number of working assets k; $P^i(e, k) \in [0, 1]$ is increasing and concave in e, and decreasing and convex in k.

$P^{i\prime}{}_e(e, k) > 0$, $P^{i\prime\prime}{}_{ee}(e, k) < 0$, $P^{i\prime}{}_k(e, k) < 0$, $P^{i\prime\prime}{}_{kk}(e, k) < 0$, $P^{i\prime\prime}{}_{ek}(e, k) > 0$

The intuition is that each agent has bounded rationality. Given that each agent has limited time and energy, working with more assets requires the agents process more information, and thus lower the probability of generating idea at each effort level. Effort increases the probability of generating idea, but has diminishing returns.

Assumption (2): Cost of effort $C(e)$ is increasing and convex in e.

$C'(e) > 0$, $C''(e) > 0$

Assumption (3)*:* The value of k assets, $V(k)$, is an increasing and convex function of k.

$V'(k) > 0$, $V''(k) > 0$

It can be interpreted as a positive externality of working with multiple assets; that is, there is increasing returns to scale of assets worked together by one agent.

Assumption (4)*:* The expected value function $P(e_t\,,\, k)V(k)$ ($t \geq 2$) is concave in k for generalists and decreasing in k for specialists.

From the assumption (4) and the assumption on the types and productivities, one can conclude that in the optimal hierarchy, generalists who have higher probabilities of having ideas on multiple assets and can generate higher expected values from multiple assets should actually work on multiple assets. In contrast, specialists who have higher probabilities in having ideas on individual assets and can generate higher expected values from individual assets should work on individual assets.

Assumption (5)*:* Assume there are only two hierarchy levels, an upper level and a lower level.

Optimal Wages and Incentive of Delegation

Before proceeding to the hierarchy form, let us look at the optimal wage payments for the agents in hierarchy in period $t \geq 2$. Effort level is unobservable, but since agents are risk-neutral, the wage payment schedule that makes agents the residual claimants can elicit the first best effort level as if there were no moral hazard problem. If the agent is at the upper level, he is in charge with the hierarchy composed of himself and his subordinates, and then he should be the residual claimant of the value generated by the hierarchy, so he gets

$$W^u = P(e, j)V(j) + \frac{j}{k}[1 - P(e, j)]P(e, k)V(k) - a \qquad \frac{j}{k} > 0 \text{, } a \text{ is a constant} \tag{3}$$

If the agent is at lower level, then he is the residual claimant of the value generated by himself, and he gets

$$W^l = [1 - P(e, j)]P(e, k)V(k) - b \qquad b \text{ is a constant} \tag{4}$$

Furthermore, if a and b are chosen such that it gives the agent an expected utility same as the reservation utility $\bar{u}$, then the principal will get the same expected profit as if there were no moral hazard problem. It is worth to note that this wage payment schedule implies that the upper-level agents have incentive to monitor his subordinates, though I do not model monitoring explicitly.

In addition, this wage payment schedule makes sense of the incentive of delegation. If the upper-level agent does not "have an idea", then delegating the authority right to his subordinates can increase the potential value generated by the hierarchy, and this potentially increases his own wage payment. Thus, in this model, unlike Hart and Moore (2005), the delegation decision is endogenized in the model. The upper-level agent will always delegate authority to his subordinates if he does not have an idea.

WHO SHOULD GET TO THE TOP

I will focus on two kinds of hierarchy forms, pyramid form (generalists at the top) and inverted pyramid form (specialists at the top). Crisscross form such as matrix form is not considered here. As Hart and Moore (2005) argued, "crisscross form is never optimal" under the assumption that generalist is not a multifaceted specialist who have ideas about small subsets of the assigned working assets. This is also true here, because of the assumption of positive externality of coordinating assets.

Using backward induction, solve the model starting from period $t \geq 2$. The optimal hierarchy form is the one that maximizes the firm's expected profit. Since agents are identical among the same type, the optimal hierarchy is symmetric. For simplicity, we look at the following two hierarchy forms.

Definition: *Hierarchy form gss* is a pyramid form of hierarchy where a generalist is at the upper level and two specialists are at the lower level (the left one in figure 2).

Definition*: Hierarchy form ssg* is an inverted pyramid form of hierarchy where two specialists are at the upper level and a generalist is at the lower level (the right one in figure 2).

Figure 2: Hierarchy Forms

In the optimal contract, the firm sets two pairs of effort-wage in period $t \geq 2$, to maximize his expected profit subject to the agents' participation constraints and incentive compatibility constraints. Under the hierarchy form gss, the firm's problem is:

$$\max_{a,\, b} E\pi_{gss} = P^{g2}(e^{g2}, 2)V(2) - W^{u} + 2[1 - P^{g2}(e^{g2}, 2)]P^{s1}(e^{s1}, 1)V(1) - 2W^{l} \tag{5}$$

subject to:

$$E(U^{g}) = W^{u} - C(e^{g2}) = P^{g2}(e^{g2}, 2)V(2) + [1 - P^{g2}(e^{g2}, 2)]P^{s1}(e^{s1}, 1)V(1) - a - C(e^{g2}) \geq \bar{u} \tag{6}$$

$$E(U^{s}) = W^{l} - C(e^{s1}) = [1 - P^{g2}(e^{g2}, 2)]P^{s1}(e^{s1}, 1)V(1) - b - C(e^{s1}) \geq \bar{u} \tag{7}$$

$$\max_{e} E(U^{g}) = P^{g2}(e^{g2}, 2)V(2) + [1 - P^{g2}(e^{g2}, 2)]P^{s1}(e^{s1}, 1)V(1) - a - C(e^{g2}) \tag{8}$$

$$\max_{e} E(U^{s}) = [1 - P^{g2}(e^{g2}, 2)]P^{s1}(e^{s1}, 1)V(1) - b - C(e^{s1}) \tag{9}$$

In this section, the subscript t is ignored because we are considering a single period $t \geq 2$ in this section. Eq. (6) and (7) are the agents' participation constraints, and eq. (8) and (9) are the agents' incentive compatibility constraints. Since the monotone likelihood ratio property holds under the assumptions, one can replace eq. (8) and (9) with their corresponding first-order conditions.

$$P^{g2}{}'(e^{g2}, 2)[V(2) - 2\, P^{s1}(e^{s1}, 1)V(1)] = C'(e^{g2}) \tag{10}$$

$$P^{s1}{}'(e^{s1}, 1)[1 - P^{g2}(e^{g2}, 2)]V(1) = C'(e^{s1}) \tag{11}$$

Eq. (10) and (11) give the optimal effort levels, $e^{g2}*$ and $e^{s1}*$. The comparative statics of eq. (10) suggest that the reaction function of the generalist is downward sloping; that is, the generalist's effort is decreasing in the specialist's effort. The same is true for the specialists; the specialist's effort is decreasing in the generalist's effort.

An agent will accept the contract as long as it gives him an expected utility of at least $\bar{u}$. At the optimal, eq. (8) and (9) are binding. Substitute the optimal efforts in eq. (8) and (9), one will get the optimal $a*$ and $b*$.

$$a* = P^{g2}(e^{g2}*, 2)V(2) + [1 - P^{g2}(e^{g2}*, 2)]P^{s1}(e^{s1}*, 1)V(1) - C(e^{g2}*) - \bar{u} \tag{12}$$

$$b* = [1 - P^{g2}(e^{g2}*, 2)]P^{s1}(e^{s1}*, 1)V(1) - C(e^{s1}*) - \bar{u} \tag{13}$$

Substitute $e^{g2}*$, $e^{s1}*$, $a*$ and $b*$ in eq. (5), the optimal expected profit of the firm becomes

$$E\pi_{gss}* = P^{g2}(e^{g2}*, 2)V(2) + 2[1 - P^{g2}(e^{g2}*, 2)]P^{s1}(e^{s1}*, 1)V(1) - C(e^{g2}*) - 2C(e^{s1}*) - 3\bar{u} \tag{14}$$

Under the hierarchy form *ssg*, firm's problem is:

$$\max_{a,\, b} E(\pi_{ssg}) = 2\, [P^{s1}(e^{s1}, 1)V(1) - W^u] + [1 - P^{s1}(e^{s1}, 1)]^2\, P^{g2}(e^{g2}, 2)V(2) - W^l \tag{15}$$

subject to:

$$E(U^g) = W^l - C(e^{g2}) = [1 - P^{s1}(e^{s1}, 1)]^2 P^{g2}(e^{g2}, 2)V(2) - b - C(e^{g2}) \geq \bar{u} \tag{16}$$

$$E(U^s) = W^u - C(e^{s1}) = P^{s1}(e^{s1}, 1)\, V(1) + (1/2)\, [1 - P^{s1}(e^{s1}, 1)]^2 P^{g2}(e^{g2}, 2)V(2) - a - C(e^{s1}) \geq \bar{u} \tag{17}$$

$$\max_e E(U^g) = [1 - P^{s1}(e^{s1}, 1)]^2 P^{g2}(e^{g2}, 2)V(2) - b - C(e^{g2}) \tag{18}$$

$$\max_e E(U^s) = P^{s1}(e^{s1}, 1)V(1) + (1/2)\, [1 - P^{s1}(e^{s1}, 1)]^2 P^{g2}(e^{g2}, 2)V(2) - a - C(e^{s1}) \tag{19}$$

Eq. (16) and (17) are the agents' participation constraints, and eq. (18) and (19) are the agents' incentive compatibility constraints. The first-order conditions of eq. (18) and (19), eq. (20) and (21), give the optimal effort levels, $e^{s1}*$ and $e^{g2}*$.

$$P^{s1}{}'(e^{s1}, 1)\{V(1) - [1 - P^{s1}(e^{s1}, 1)]P^{g2}(e^{g2}, 2)V(2)\} = C'(e^{s1}) \tag{20}$$

$$P^{g2}{}'(e^{g2}, 2)[1 - P^{s1}(e^{s1}, 1)]^2 V(2) = C'(e^{g2}) \tag{21}$$

Similar to the gss form, the comparative statics of eq. (20) suggest that the reaction function of a specialist is downward sloping; that is, the specialist's effort is decreasing in the generalist's effort. The same is true for the generalist; the generalist's effort is decreasing in the specialist's effort.

At the optimal, eq. (16) and (17) are binding. Substitute the optimal efforts into the binding eq. (16) and (17), one will get the optimal $a*$ and $b*$.

$$a* = P^{s1}(e^{s1}*, 1)\, V(1) + (1/2)[1 - P^{s1}(e^{s1}*, 1)]^2 P^{g2}(e^{g2}*(e^{s1}*), 2)V(2) - C(e^{s1}*) - \bar{u} \tag{22}$$

$$b* = [1 - P^{s1}(e^{s1}*, 1)]^2 P^{g2}(e^{g2}*(e^{s1}*), 2)V(2) - C(e^{g2}*(e^{s1}*)) - \bar{u} \tag{23}$$

Substitute $e^{s1}*$, $e^{g2}*$, $a*$ and $b*$ in eq. (15), the optimal expected profit of the firm becomes

$$E\pi_{ssg}* = 2P^{s1}(e^{s1}*, 1)\, V(1) + [1 - P^{s1}(e^{s1}*, 1)]^2 P^{g2}(e^{g2}*(e^{s1}*), 2)V(2) - 2C(e^{s1}*) - C(e^{g2}*(e^{s1}*)) - 3\bar{u} \tag{24}$$

Lemma 1*:* Under both hierarchy forms, the reaction curves of the upper-level agents and the lower-level agents are downward sloping. That is, under both hierarchy forms, increase in generalist's effort will reduce the specialist's effort, and vice versa.

Proof: See the argument above.

qed.

The finding in Lemma 1 is consistent with Aghion and Tirole (1997): centralization harms the incentive of the agents at lower level; that is, the effort of the upper-level agents will crowd out the effort of lower-level agents. Thus, there is a tradeoff between incentive at lower level and loss of control, since the lower-level agents and the upper-level agents have different decision on the use of the assets. However, as discussed before, in the case of having no idea, upper-level agents will always want to delegate. In addition, the model here is different from the model in Aghion and Tirole (1997) in two senses. First of all, in the model here, both upper-level and lower-level agents are treated as "agent" in an agent-principal problem, and the "principal" is the firm, the one who constructs the hierarchy. Nevertheless, in Aghion and Tirole (1997), the upper-level agent acts as the "principal", and the lower-level agent acts as the "agent". Secondly, in the model here, the expected income of a lower-level agent only depends on the expected value he generated by his own, but in Aghion and Tirole (1997), the expected income of a lower-level agent not only depends on the expected value he generated by himself, but also depends on the principal's expected value generated.

Lemma 2: Given the optimal effort-wage pairs, a generalist will choose ($e^{g2}*$, $a*$) and a specialist will choose ($e^{s1}*$, $b*$) under gss form, and a generalist choose ($e^{g2}*$, $b*$) and a specialist will choose ($e^{s1}*$, $a*$) under ssg form.

Proof: Because the participation constraints are binding, ($e^{g2}*$, $a*$) for a generalist and ($e^{s1}*$, $b*$) for a specialists satisfy equality of eq. (4) and equality of (5) under gss form. Under the assumption of types and productivities and assumption (4), $P^{s2}(e^{g2}*,k) < P^{g2}(e^{g2}*,k)$, $P^{g1}(e^{s1}*,1) < P^{s1}((e^{s1}*,1)$. Thus ($e^{g2}*$, $a*$) will give a specialist negative expected utility; and ($e^{s1}*$, $b*$) will give a generalist negative expected utility; and therefore a generalist will never choose ($e^{s1}*$, $b*$) and a specialist will never choose ($e^{g2}*$, $a*$) under gss form. Similarly, one can prove that under ssg form, a generalist will always choose ($e^{g2}*$, $b*$) and a specialist will choose ($e^{s1}*$, $a*$).

qed.

Proposition 1: If the gain of coordinating assets by the generalist is large, the hierarchy form gss is optimal; otherwise, the hierarchy form ssg is optimal.

Proof: Subtract eq. (14) by eq. (24), one gets:

$$E\pi_{gss}* - E\pi_{ssg}* = \{P^{g2}(e^{g2}{}_{gss}*,2) - [1-P^{s1}(e^{s1}{}_{ssg}*, 1)]^2P^{g2}(e^{g2}{}_{ssg}*(e^{s1}{}_{ssg}*), 2)\}V(2) - [C(e^{g2}{}_{gss}*) - C(e^{g2}{}_{ssg}*(e^{s1}{}_{ssg}*))] - 2\{P^{s1}(e^{s1}{}_{ssg}*, 1) - [1 - P^{g2}(e^{g2}{}_{gss}*, 2)]P^{s1}(e^{s1}{}_{gss}*(e^{g2}{}_{gss}*), 1)\} V(1) + 2[C(e^{s1}{}_{ssg}*) - C(e^{s1}{}_{gss}*(e^{g2}{}_{gss}*))] \quad (25)$$

It is trivial to show that the first part (first two lines) of the right-hand side of eq. (25) is positive, and the second part (last two lines) is negative. Therefore, if the first part is larger than the second part, i.e., the gain of coordinating multiple assets by the generalist is large, then the generalist should be at the top, and the gss form is optimal; otherwise, the specialists should be at the top, and the ssg form is optimal.

qed.

The intuition of Proposition 1 is that since the effort of an upper-level agent will crowd out the effort of the lower-level agents as shown in Lemma 1, if the potential net output (net of wage payment) produced by a generalist is large, then the principal should not assign him at lower level where his effort will be inhibited. Otherwise, if the coordination is not important, then the specialist should be assigned to the upper level where he will exert a higher effort. Proposition 1 is consistent with the claim in Hart and Moore (2005). Involving incentive in hierarchy makes the necessary and sufficient condition of optimal hierarchy form much more complicated; however, unlike Hart and Moore's model, the agent at the top does not necessarily have lower probability of generating value.

HOW MANY AGENTS AT THE TOP

This section still deals with a single period $t \geq 2$ (the subscript t is ignored in this section). Assume the optimal hierarchy is a pyramid form (like the gss form), and the assumption below applies.

Assumption (6): There are n assets in the firm, and the span of control of an upper-level agent is m; i.e., an upper-level agent is senior to m lower-level agents.

The size of n depends on the size of the firm, which is assumed exogenous. The size of n may be affected by technology, market structure, and industry, but not by the hierarchy structure. Under assumptions (6), there need n/m upper-level agents and n lower-level agents in the optimal hierarchy. Firm's problem becomes

$$\max_{a,\,b} E\pi = \frac{n}{m}[P^{gm}(e^{gm}, m)V(m) - W^u] + n\{[1 - P^{gm}(e^{gm}, m)]P^{s1}(e^{s1}, 1)V(1) - W^l\} \tag{26}$$

subject to

$$E(U^g) = W^u - C(e^{gm}) = P^{gm}(e^{gm}, m)V(m) + m[1 - P^{gm}(e^{gm}, m)]P^{s1}(e^{s1}, 1)V(1) - a - C(e^{gm}) \geq \bar{u} \tag{27}$$

$$E(U^s) = W^l - C(e^{s1}) = [1 - P^{gm}(e^{gm}, m)]P^{s1}(e^{s1}, 1)V(1) - b - C(e^{s1}) \geq \bar{u} \tag{28}$$

$$\max_e E(U^g) = P^{gm}(e^{gm}, m)V(m) + m[1 - P^{gm}(e^{gm}, m)]P^{s1}(e^{s1}, 1)V(1) - a - C(e^{gm}) \tag{29}$$

$$\max_e E(U^s) = [1 - P^{gm}(e^{gm}, m)]P^{s1}(e^{s1}, 1)V(1) - b - C(e^{s1}) \tag{30}$$

The first-order conditions of eq. (29) and (30), i.e., eq. (31) and (32) give the optimal effort levels, $e^{gm}*(m)$ and $e^{s1}*(m)$.

$$P_e^{gm}{}'(e^{gm}, m)[V(m) - mP^{s1}(e^{s1}, 1)V(1)] = C'(e^{gm}) \tag{31}$$

$$P_e^{s1}{}'(e^{s1}, 1)[1 - P^{gm}(e^{gm}, m)]V(1) = C'(e^{s1}) \tag{32}$$

Substitute $e^{gm}*(m)$ and $e^{s1}*(m)$ into binding participation constraints (equation (27) and (28) with equal signs), one can get the optimal $a*(m)$ and $b*(m)$.

$$a* = P^{gm}(e^{gm}*(m), m)V(m) + [1 - P^{gm}(e^{gm}*(m), m)]P^{s1}(e^{s1}*(m), 1)V(1) - C(e^{gm}*(m)) - \bar{u} \tag{33}$$

$$b* = [1 - P^{gm}(e^{gm}*(m), m)]P^{s1}(e^{s1}*(m), 1)V(1) - C(e^{s1}*(m)) - \bar{u} \tag{34}$$

Thus, the expected profit of the firm with the optimal effort-wage pairs is

$$E\,\pi* = \frac{n}{m}[P^{gm}(e^{gm}*(m), m)V(m) - C(e^{gm}*(m))] + n\{[1 - P^{gm}(e^{gm}*(m), m)]P^{s1}(e^{s1}*(m), 1)V(1) - C(e^{s1}*(m))\} - (m+1)\frac{n}{m}\bar{u} \tag{35}$$

The first-order condition of eq. (35) with respect to m gives the optimal span of control $m*$.

***Lemma 3**:* The optimal span of control is independent of the size of the firm.

Proof: The proof is trivial, as all n's are cancelled out in the first-order condition of eq. (35).

qed.

From assumption (3), there is a positive externality of coordinating multiple assets. Define the *elasticity of externality of cooperating multiple assets* as the sensitivity of potential value increased when an agent working with more assets.

$$\text{Elasticity (externality of cooperating m assets)} = \frac{V'(m)}{\frac{V(m)}{m}} \tag{36}$$

***Proposition 2**:* The more elastic the externality of cooperating is, the larger the span of control should be; otherwise, the span of control should be small.

The proof is tedious and is skipped here, but the intuition is simple. Here, the upper-level agents are generalists, and they work with multiple assets. If their cooperating effect is significant and has large impact on the value, then they should work with more assets, and the hierarchy should have larger span of control; therefore, steeper hierarchy is favorable. Otherwise, there should be small span of control, and therefore flatter hierarchy is favorable.

WHO WANTS TO GET TO THE TOP

Now go back to solve the period 1's problem. Like the previous section, this section also assumes the optimal hierarchy is a pyramid form (like the gss form), and there are n assets and the span of control of upper-level agent is m. In addition, as committed ex ante, Q_0 percentage of agents with the highest value generated will be promoted to the upper level at the beginning of period 2. Assume in period 1, all agents work with one asset, so that Q_0 percentage of agents with the higher value generated will be the same as the Q_0 percentage of agents with the higher probability of generating value.

Assume the distribution of the agents' probability of generating value in period 1 is $f_0(P)$, and the cumulative density function is $F_0(P)$. Let P^* be such that $Q_0 = [1 - F_0(P^*)]$. That is, if an agent's probability of generating value greater than P^*, he will be promoted to the upper level. P^* is endogenous, but a single agent treats P^* as a parameter. Define the distribution of $(P^* - P)$ as $f(P^* - P)$, and the cumulative density function is $F(P^* - P)$. Then for an agent who has probability P of generating value, the probability of being promoted is:

$$Q = [1 - F(P^* - P)] \tag{37}$$

The firm's problem is

$$\max E(\pi) = \pi_1 + \pi_2 + \pi_3 + \ldots + \pi_H + \pi_{H+1} + \ldots \tag{38}$$

subject to

$$W_0 - C(e^g{}_1) + F(P^* - P^{g1}{}_1((e^g{}_1, 1))\textstyle\sum_{1<t\leq T}[W^l{}_t - C(e^{g1}{}_t)] + [1 - F(P^* - P^{g1}{}_1((e^g{}_1, 1))] \sum_{1<t\leq T} [W^u{}_t - C(e^{gm}{}_t)] \geq \sum_{t\leq T} \bar{u} \tag{39}$$

$$W_0 - C(e^s{}_1) + F(P^* - P^{s1}{}_1(e^s{}_1, 1)) \textstyle\sum_{1<t\leq T}[W^l{}_t - C(e^{s1}{}_t)] + [1 - F(P^* - P^{s1}{}_1(e^s{}_1, 1))] \sum_{1<t\leq T} [W^u{}_t - C(e^{sm}{}_t)] \geq \sum_{t\leq T} \bar{u} \tag{40}$$

$$\max_e E(U^g) = W_0 - C(e^g{}_1) + F(P^* - P^{g1}{}_1((e^g{}_1, 1))\textstyle\sum_{1<t\leq T}[W^l{}_t - C(e^{g1}{}_t)] + [1 - F(P^* - P^{g1}{}_1((e^g{}_1, 1))] \sum_{1<t\leq T}[W^u{}_t - C(e^{gm}{}_t)] \tag{41}$$

$$\max_e E(U^s) = W_0 - C(e^s{}_1) + F(P^* - P^{s1}{}_1(e^s{}_1, 1)) \textstyle\sum_{1<t\leq T}[W^l{}_t - C(e^{s1}{}_t)] + [1 - F(P^* - P^{s1}{}_1(e^s{}_1, 1))] \sum_{1<t\leq T}[W^u{}_t - C(e^{sm}{}_t)] \tag{42}$$

Eq. (39) and (40) are the agents' participation constraints, and eq. (41) and (42) are the agents' incentive compatibility constraints. Since the monotone likelihood ratio property holds under the assumptions, one can replace eq. (41) and (42) with their corresponding first-order conditions.

$$- C'(e^g{}_1) + F'(\bullet)P^{g1}{}_1{}'(e^g{}_1, 1) \textstyle\sum_{1<t<T} \{[W^u{}_t - C(e^{gm}{}_t)] - [W^l{}_t - C(e^{g1}{}_t)]\} = 0 \tag{43}$$

$$- C'(e^s{}_1) + F'(\bullet)P^{s1}{}_1{}'(e^s{}_1, 1) \textstyle\sum_{1<t<T} \{[W^u{}_t - C(e^{sm}{}_t)] - [W^l{}_t - C(e^{s1}{}_t)]\} = 0 \tag{44}$$

***Proposition 3**:* Under the assumption of types and productivities, given the contract under the gss form, the generalist has more incentive to exert higher effort in period 1 and thus is more likely to be promoted.

Proof: Because of the assumption of types and productivities, in period $t \geq 2$, being at the upper level and working with *m* assets is more favorable for a generalist. That is, in eq. (43) and (44) (the first-order conditions of incentive compatibility constraints), $\{[W^u{}_t - C(e^{gm}{}_t)] - [W^l{}_t - C(e^{g1}{}_t)]\}$ for a generalist is relatively larger than $\{[W^u{}_t - C(e^{sm}{}_t)] - [W^l{}_t - C(e^{s1}{}_t)]\}$ for a specialist. Therefore, because of the higher marginal benefit of effort, a generalist has more incentive to exert effort and thus can have higher probability of generating value in period 1, and thus, he is more likely to be promoted in period 2.

qed.

CONCLUSION

This paper analyzes optimal hierarchy using a T-period model. Agents' types are unobservable at hiring, and their effort levels are unobservable, so there are hidden information problem and hidden action problem. Firm wants to choose the optimal hierarchy form to maximize profit and assign proper types of agents to realize the optimal hierarchy. Because the specialist can accumulate specialty of working with individual asset after period 1 and the generalist is more productive on coordinating multiple asset after period 1, in the optimal hierarchy the specialist should work with individual asset and the generalist should work with multiple assets. The optimal hierarchy form depends on the externality of working with multiple assets. If the externality is large, the generalist at the top is desirable; if it is small, the specialist at the top may be desirable. How many agents should be at the top depends on the elasticity of the externality of working with multiple assets. Given the pyramid form, if the externality of working with multiple assets is very sensitive to the number of the assets, the span of control should be large, and there are fewer generalists at the top; otherwise, more agents should be at the top. Finally, the T-period model with promotion can give the agents who should be at the top more incentive to get promoted.

The model has exogenous levels of hierarchy, and I only analyze the case where there are two hierarchy levels. Future work can extend the model to endogenize the layers of the hierarchy. If there are many hierarchy levels, then the lowest-level agents can get incentive from promotion opportunity; the highest-level agents get incentive from evaluation of performance; and any intermediate agents get incentive from both promotion opportunity and evaluation of performance.

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