THE STUDENT-MANAGED FUND: A CASE STUDY OF PORTFOLIO PROPERTIES

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CASE DESCRIPTION

This case provides students with an in-depth look at various risk measurements in portfolio management. The primary issues examined in this case are: 1) Review pertinent concepts of describing and summarizing a bath of numerical data in the context of identifying portfolio properties. Although these concepts have been covered in basic statistics courses, it is important enough to go over again so that students may be better prepared for discussions regarding various risk measurements in portfolio management; 2) A distinction between use of geometric and arithmetic return data; 3) How risk is measured in investments, and what some of the measures of risk are used. In particular, it is recommended that a spreadsheet model be used to compute these various risk measurements. Differentiate between different types of risk; namely, total risk, systematic risk, and nonsystematic risk; 4. Demonstrate that the true betas tend to move toward 1.0 over time. With more advanced students, it is recommended that they use the Excel spreadsheet, (or some other statistical software, i.e., SAS or Minitab), to run the single-index regression model and verify these beta estimates. This case has a difficulty level appropriate for senior or first year MBA students. It is designed to be taught in a single class period (60 to 80 minutes). With more advanced students, the case can be assigned as a team project. The team presents their findings and conclusions to the class. If the case is used as a team presentation project, approximately 2 to 3 hours of student preparation time should be adequate for most students depending on their computational ability.

JEL: G11; A29

KEYWORDS: Student-managed Fund, Portfolio Properties

CASE INFORMATION

Jackson Pettyjohn has been the faculty advisor of the Student-Managed Fund (*SMF*) at Lowell State University since its inception. Jackson was a member of Jim Sharpe's doctoral dissertation committee as well as his faculty advisor while Jim was in the doctoral program at Lowell State. Jim enrolled in the *SMF* class while attending Lowell State. After Jim received his Ph.D. in finance from Lowell State and subsequently became a faculty member at Wettown University, the two became friends and maintained contact.

The *SMF* class at Lowell State has an advisory committee, consisting of senior finance faculty members and finance professionals who assist in providing guidance. An implicit decision was made to invest primarily in equities because there is more to learn about the selection of common stocks, whether mid-cap, large-cap, growth or value securities, than from recognizing an AAA corporate bond or a Treasury security.

Since his enrollment in the *SMF* class at Lowell State, Jim has been of the opinion that a student-managed fund class is an outstanding way to provide practical "hands-on" education for students who are interested in investments. Wettown University does not have such a class at this point in time. Thus, the next best thing would be to work up a case that would be beneficial to students in the investment class.

In addition, Jim is of the opinion that the case should make use of spreadsheet software in making calculations and estimations of return and risk factors. Investment analysis is by its very nature quantitative, and spreadsheets are recommended for analyzing most of these assignments. Students can learn a great deal by going through the process of constructing a spreadsheet, seeing how it is structured, looking at the formulas and functions, and thinking about the implications of the spreadsheet model's output. As an example, data included in all the exhibits of this case are obtained by using a spreadsheet model.

Exhibit 1: Descriptive Measures of the Return Series from September 1997 to April 2006

Descriptive Measure	***The SMF***	***The S&P 500***
Arithmetic Mean	0.30%	0.59%
Mode	No Mode	No Mode
Count	104	104
Minimum	-12.24%	-14.46%
First Quartile	-2.31%	-1.89%
Median	0.70%	0.93%
Third Quartile	3.19%	3.80%
Maximum	10.22%	9.78%
Range	22.46%	24.24%
Variance	0.002250	0.002031
Standard Deviation	4.74%	4.51%
Geometric Mean	0.19%	0.49%

This exhibit shows the descriptive measures of the monthly return series of Lowell State University's SMF and that of S&P 500 from September 1997 to April 2006. Note: For all the estimates expressed in percentages, two digits after the decimal point are taken. For example, the standard deviation of the SMF fund is displayed as 4.74%, but the more accurate estimate obtained from the Excel spreadsheet is 4.74301379856173%. For the ease of exposition, it is rounded off to 4.74%.

For the outputs shown in Exhibit 2, the following regression equation was estimated.

$$r_{SMF,t} = \alpha_{SMF} + \beta_{SMF} \times r_{M,t} + \varepsilon_{SMF,t} \quad (1)$$

Equation (1) is called the single-index (market) model,
where:

$r_{SMF,t} =$ return for the *SMF* over month *t*.

$\alpha_{SMF} =$ regression coefficient representing the intercept term for the *SMF*. It is the *SMF's* return component that is independent of the market's return.

$\beta_{SMF} =$ regression coefficient representing the slope of the regression line. It measures the expected change in the *SMF's* return given a change in the market's return.

$r_{M,t} =$ return on a selected market index (i.e., *S&P* 500) for month *t*.

$\varepsilon_{SMF,t} =$ error term of the regression for month *t*. It measures the deviation of the observed return from the return predicted by the regression and has an expected value of zero.

Ordinary Least Squares estimates were obtained. The results are presented in Exhibit 2.

Jim plans to include this case as a project in his investment class. In particular, Jim plans to cover the use of spreadsheets in working with various returns and risk measurements that are useful in portfolio management. Assume that you were a student in Jim's investment class at Wettown University, and he has given you the following assignments.

Exhibit 2: Selected Outputs from the Regression of the Single-index (Market) Model

Regression Statistics	
R-Square	89.90%
Adjusted *R*-Square	89.80%
The Standard Error of the	1.52%
The Coefficient of Correlation	0.9481
Observations	104

	Coefficients	***Standard***	***t Statistic***	***P-value***
Intercept	-0.0029	0.0015	-1.9180	0.0579
The Beta Estimate	0.9979	0.0331	30.1245	0.0000

This exhibit shows the regression results from the Ordinary Least Squares estimation using Equation (1). The monthly return series are those of Lowell State University's SMF and that of S&P 500 from September 1997 to April 2006, respectively.

QUESTIONS

1. Recall from your introductory business statistics course that three major properties that describe a batch of numerical data are (1) Central Tendency, (2) Dispersion, and (3) Shape. To describe the shape of a batch of data we need only compare the mean and the median. If these two measures are equal, we may generally consider the data to be symmetrical, i.e., *zero-skewed*. On the other hand, if the mean exceeds the median, the data may generally be described as *positive* or *right-skewed*. If the mean is exceeded by the median, those data can generally be called *negative* or *left-skewed*. With information presented in Exhibit 1, does either the *SMF* data or that of *S&P* 500 appear to be zero-skewed? Justify your answers.

2. Recall from your introductory business statistics course that when the distribution of a data set is skewed, the mean and standard deviation is not an adequate summary of the data. In this case, the five-number summary is a more complete summary of the data. Divide the original data into two sub-sets, one from September 1997 to December 2001 and another one from January 2002 to April 2006, respectively. Prepare a five-number summary for each of the two data sub-sets, and briefly describe your findings. Does either the *SMF* data or that of *S&P* 500 appear to be zero-skewed in either sub-period?

3. (a) Discuss the differences between the arithmetic mean and the geometric mean for each series. Relate your discussion to the difference in the standard deviations. (b) Compare the coefficient of variation of each series. By this relative measure of risk, does the data leave an impression concerning the relative risk of the *SMF* fund in comparison with the risk of *S&P* 500?

4. The data needed for answering this question is provided in the student-version of the *Excel* file accompanied with this case. Use *Excel* with the data provided to compute the covariance estimate between the two return series from September 1997 to April 2006. (Hint: The covariance estimate should be 0.002026.) Discuss the relationship between the covariance and the correlation coefficient. Compute the corresponding estimate of the correlation coefficient. Is your answer the same as the one shown in Exhibit 2?

5. Use the pertinent information in Exhibits 1 and 2. (a) What is the total risk estimate of the *SMF* fund? (b) What is its market (or systematic) risk? Use this measure of risk to discuss the riskiness of the *SMF* fund relative to that of the *S&P* 500. Is your answer different from that of Part (b) in Question 3? (c) What is its unique (or unsystematic) risk?

6. (Optional) The beta value of the *SMF* is obtained from running the single-index market model, and it is available in Exhibit 2 along with other selected outputs of the regression. Run the single-index

market model in *Excel*, and the data needed for this regression is provided in the student-version of the *Excel* file accompanied with this case. Verify that the beta value shown in Exhibit 2 is the same as the slope estimate obtained from your regression model.

7. Refer to Exhibits 1 and 2. (a) Construct an equally weighted portfolio (that is, each of the two portfolios, the *SMF* fund and the *S&P* 500, is weighted by 50 percent), and compute the resultant portfolio's average return and its standard deviation. (b) Compute the *weighted-average* standard deviation, that is, 0.50 × the standard deviation of the *SMF* fund + 0.50 × the standard deviation of the *S&P* 500. (c) What is the difference between the portfolio's standard deviation from Part (a) and the *weighted-average* standard deviation from Part (b)? What explains this difference?

8. The following tables contain beta estimates of the *SMF* fund in the two sub-periods, respectively.

Exhibit 3: Output from the Single-index Regression Model: September 1997 to December 2001 Data

Regression Statistics				
R-Square	88.00%			
Adjusted *R*-Square	87.76%			
The Standard Error of the Estimate	1.94%			
The Coefficient of Correlation	0.9381			
Observations	52			
	Coefficients	***Standard Error***	***t Statistic***	***P-value***
Intercept	-0.0028	0.0027	-1.0272	0.3093
The Beta Estimate	1.0076	0.0526	19.1452	0.0000

This exhibit shows the regression results from the Ordinary Least Squares estimation using Equation (1). The monthly return series are those of Lowell State University's *SMF* and that of *S&P* 500 from September 1997 to December 2001, respectively.

Exhibit 4: Output from the Single-index Regression Model: January 2002 to April 2006 Data

Regression Statistics				
R Square	93.95%			
Adjusted R Square	93.83%			
The Standard Error of the	0.95%			
The Coefficient of Correlation	0.9693			
Observations	52			
	Coefficients	**Standard Error**	**t Statistic**	**P-value**
Intercept	-0.0029	0.0013	-2.2165	0.0312
The Beta Estimate	0.9796	0.0352	27.8640	0.0000

This exhibit shows the regression results from the Ordinary Least Squares estimation using Equation (1). The monthly return series are those of Lowell State University's SMF and that of S&P 500 from January 2002 to April 2006, respectively.

Compare these two beta estimates. What could explain the difference?

APPENDIX 1

Note: The pertinent Excel files along with the data used in this case are available from the Institute for Business and Finance Research or the authors of the case.

Appendix 2: Data Used in the Analysis

Month	The SMF	The S&P
Sep-97	0.020500	0.054770
Oct-97	-0.009230	-0.033400
Nov-97	0.020840	0.046290
Dec-97	0.009890	0.017170
Jan-98	0.031650	0.011060
Feb-98	0.025220	0.072120
Mar-98	0.045260	0.051210
Apr-98	0.029030	0.010060
May-98	-0.032140	-0.017190
Jun-98	0.022222	0.040620
Jul-98	-0.026033	-0.010650
Aug-98	-0.122445	-0.144580
Sep-98	0.041340	0.064060
Oct-98	0.082391	0.081340
Nov-98	0.043546	0.060610
Dec-98	0.055454	0.057620
Jan-99	0.005117	0.041820
Feb-99	-0.024592	-0.031080
Mar-99	0.038025	0.040010
Apr-99	0.034210	0.038730
May-99	-0.024231	-0.023610
Jun-99	0.049547	0.055500
Jul-99	-0.046383	-0.031220
Aug-99	-0.020796	-0.004950
Sep-99	-0.062593	-0.027410
Oct-99	0.088335	0.063280
Nov-99	0.045984	0.020330
Dec-99	0.091666	0.058900
Jan-00	-0.056397	-0.050240
Feb-00	0.027519	-0.018930
Mar-00	0.100554	0.097830
Apr-00	-0.022784	-0.030090
May-00	-0.031698	-0.020520
Jun-00	0.050456	0.024650
Jul-00	-0.021080	-0.015630
Aug-00	0.066648	0.062110

Month	The SMF	The S&P
Sep-00	-0.076770	-0.052790
Oct-00	-0.001633	-0.004230
Nov-00	-0.088034	-0.078840
Dec-00	0.005365	0.004890
Jan-01	0.040820	0.035480
Feb-01	-0.121966	-0.091180
Mar-01	-0.088636	-0.063350
Apr-01	0.095197	0.077710
May-01	0.006465	0.006700
Jun-01	-0.030008	-0.024340
Jul-01	-0.010210	-0.009840
Aug-01	-0.077836	-0.062600
Sep-01	-0.073001	-0.080750
Oct-01	0.022674	0.019070
Nov-01	0.087691	0.076710
Dec-01	0.014334	0.008760
Jan-02	-0.033532	-0.014590
Feb-02	-0.033717	-0.019280
Mar-02	0.032799	0.037610
Apr-02	-0.062621	-0.060630
May-02	-0.015330	-0.007370
Jun-02	-0.088511	-0.071240
Jul-02	-0.069693	-0.077950
Aug-02	0.020488	0.006570
Sep-02	-0.110535	-0.108680
Oct-02	0.102168	0.088020
Nov-02	0.040080	0.058860
Dec-02	-0.053664	-0.058750
Jan-03	-0.029881	-0.026200
Feb-03	-0.020994	-0.015000
Mar-03	0.023200	0.009710
Apr-03	0.080493	0.082370
May-03	0.045638	0.052690
Jun-03	0.017090	0.012760
Jul-03	0.013981	0.017630
Aug-03	0.017495	0.019500

Month	The SMF	The S&P
Sep-03	-0.001824	-0.010620
Oct-03	0.050170	0.056570
Nov-03	0.017139	0.008800
Dec-03	0.028311	0.052440
Jan-04	0.025052	0.018360
Feb-04	0.015466	0.013900
Mar-04	-0.024924	-0.015090
Apr-04	-0.012571	-0.015700
May-04	0.003768	0.013720
Jun-04	-0.002713	0.019440
Jul-04	-0.042004	-0.033100
Aug-04	0.003686	0.004040
Sep-04	0.002257	0.010830
Oct-04	0.009196	0.015280
Nov-04	0.047501	0.040460
Dec-04	0.037804	0.034030
Jan-05	-0.009034	-0.024370
Feb-05	0.013640	0.021040
Mar-05	-0.014130	-0.017710
Apr-05	-0.012823	-0.018970
May-05	0.028183	0.031820
Jun-05	0.000671	0.001420
Jul-05	0.025148	0.037190
Aug-05	-0.020460	-0.009120
Sep-05	-0.001709	0.008100
Oct-05	-0.019144	-0.016670
Nov-05	0.037343	0.037820
Dec-05	-0.000491	0.000340
Jan-06	0.007516	0.026480
Feb-06	-0.000203	0.002710
Mar-06	0.011515	0.012450
Apr-06	0.010409	0.013430

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TEACHING NOTES

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CASE DESCRIPTION

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QUESTIONS

Question 1: Recall from your introductory business statistics course that three major properties that describe a batch of numerical data are (1) Central Tendency, (2) Dispersion, and (3) Shape. To describe the shape of a batch of data we need only compare the mean and the median. If these two measures are equal, we may generally consider the data to be symmetrical, i.e., *zero-skewed*. On the other hand, if the mean exceeds the median, the data may generally be described as *positive* or *right-skewed*. If the mean is exceeded by the median, those data can generally be called *negative* or *left-skewed*. With information presented in Exhibit 1, does either the *SMF* data or that of *S&P* 500 appear to be zero-skewed? Justify your answers.

Solution 1: It is indicated by the pertinent information given in Exhibit 1 that the distribution of both return series is left-skewed. The reason is that the mean is smaller than the median for both series.

	The SMF **From September 1997 to April 2006**	**The S&P 500** **From September 1997 to April 2006**
Mean	0.30%	0.59%
Median	0.70%	0.93%

Question 2: Recall from your introductory business statistics course that when the distribution of a data set is skewed, the mean and standard deviation is not an adequate summary of the data. In this case, the five-number summary is a more complete summary of the data. Divide the original data into two sub-sets, one from September 1997 to December 2001 and another one from January 2002 to April 2006,

respectively. Prepare a five-number summary for each of the two data sub-sets, and briefly describe your findings. Does either the *SMF* data or that of *S&P* 500 appear to be zero-skewed in either sub-period?

Solution 2: The five-number summary along with the mean and the standard deviation estimates for each data sub-set is given below.

	The SMF From September 1997 to December 2001	**The S&P 500** From September 1997 to December 2001
Mean	0.004413	0.007154
Minimum	-0.122445	-0.144580
First Quartile	-0.027027	-0.025108
Median	0.012112	0.009410
Third Quartile	0.041892	0.052100
Maximum	0.100554	0.097830
Standard Deviation	0.055539	0.051708

Since the mean is smaller than the median, both series are left-skewed in the first sub-period.

	The SMF From January 2002 to April 2006	**The S&P 500** From January 2002 to April 2006
Mean	0.001687	0.004718
Minimum	-0.110535	-0.108680
First Quartile	-0.016284	-0.015243
Median	0.003727	0.009255
Third Quartile	0.023663	0.022400
Maximum	0.102168	0.088020
Standard Deviation	0.038144	0.037740

Since the mean is smaller than the median, both series are left-skewed in the second sub-period, too.

Question 3: (a) Discuss the differences between the arithmetic mean and the geometric mean for each series. Relate your discussion to the difference in the standard deviations. (b) Compare the coefficient of variation of each series. By this relative measure of risk, does the data leave an impression concerning the relative risk of the *SMF* fund in comparison with the risk of *S&P* 500?

	The SMF	**The S&P 500**
Arithmetic Mean	0.30%	0.59%
Geometric Mean	0.19%	0.49%
Standard Deviation	4.74%	4.51%
Coefficient of	15.55	7.59

Solution 3: The values of the arithmetic means, the geometric means, and the standard deviations are obtained using the pertinent formulas and functions in the *Excel* spreadsheet model. However, the built-in function of a geometric mean in *Excel*, "*=GEOMEAN(number1, number2, ...)*", cannot be performed directly on the holding period yield (*HPY*) series. It is routine to construct the corresponding holding period return (*HPR*) series (or sometimes called the *return relatives*) in *Excel* and compute the geometric mean from its definition or apply the geometric mean function to the *HPR* series. The geometric mean is equal to,

$$\text{The Geometric Mean} = (\text{return relative } 1 \times \text{return relative } 2 \times \cdots \times \text{return relative } n)^{1/n} - 1.$$

The formula for computing the coefficient of variations is:

$$The\ Coefficient\ of\ Variation = \frac{The\ Standard\ Deviation}{The\ Arithmetic\ Mean}.$$

(a) If the rates of return vary over time, the geometric mean of the return series will always be lower than its arithmetic mean. The larger the standard deviation, the larger the difference. Only if the rates of return are the same in each period, will the geometric mean equal the arithmetic mean. Otherwise, the geometric mean should be smaller than the arithmetic mean.

(b) The coefficient of the variation (*CV*) equals the ratio of the standard deviation over the arithmetic mean, and it measures the risk per unit of return. The *CV* of the *SMF* fund is much larger because its average return is lower while it has more volatility in its return series. Using this relative measure of risk, the returns of the SMF fund appear to be much more volatile than the returns of the S&P 500, the proxy for the market portfolio in the case.

Question 4: The data needed for answering this question is provided in the student-version of the *Excel* file accompanied with this case. Use *Excel* with the data provided to compute the covariance estimate between the two return series from September 1997 to April 2006. (Hint: The covariance estimate should be 0.002026.) Discuss the relationship between the covariance and the correlation coefficient. Compute the corresponding estimate of the correlation coefficient. Is your answer the same as the one shown in Exhibit 2?

Solution 4:

	The SMF	**The S&P 500**
The Standard Deviation	4.74%	4.51%
The Covariance Estimate	0.002026	
The Correlation	0.9481	

The standard deviation estimates are available from Exhibit 1 and the correlation estimate is obtained from Exhibit 2. The covariance estimate is obtained from the instructor-version of the *Excel* file. In class, using the monthly *HPY*s of the *SMF* and the *S&P* 500, students are shown step-by-step procedures in *Excel* of how to compute the covariance estimate from its definition. The definition of a covariance between two random variables *X* and *Y* is,

$$\text{The Covariance} = \sum \left\{[X_i - E(X)] \times [Y_i - E(Y)]\right\}, \text{ where } i = 1, 2, \cdots, n.$$

Alternatively, the covariance between two random variables can be computed as the correlation between the two random variables times the product of their standard deviations. The correlation coefficient rescales (or standardizes) the covariance to facilitate comparison with corresponding values for other pairs of random variables; correlation coefficients always lie between *-1* and *+1*. The correlation estimate in Exhibit 2 is obtained from an *Excel* output. If students compute the correlation estimate from the covariance and the standard deviations listed in the case, the answer is 0.9477. Ignore the rounding error, and these two estimates are the same. In class, it is demonstrated that in the single-index regression model, the positive square root of the R^2, the coefficient of determination of the regression, is the absolute value of the correlation estimate, and it takes the sign of the beta (slope) estimate. The *SMF* fund's returns are highly correlated with the returns of the market and it explains why its beta estimate is very close to one.

With more advanced students, they are asked to verify the covariance estimate using *Excel* with the data provided in the student-version of the file.

Question 5: Use the pertinent information in Exhibits 1 and 2. (a) What is the total risk estimate of the *SMF* fund? (b) What is its market (or systematic) risk? Use this measure of risk to discuss the riskiness of the *SMF* fund relative to that of the *S&P* 500. Is your answer different from that of Part (b) in Question 3? (c) What is its unique (or unsystematic) risk?

Solution 5:

	The SMF	*The S&P 500*
The Variance Estimate	0.002250	0.002031
The Covariance Estimate	0.002026	
The Beta Estimate	0.9979	
The Total Risk Estimate	0.002250	
The Systematic Risk Estimate	0.002022	
The Unsystematic Risk Estimate	0.000228	

The variance estimates are available from Exhibit 1. The covariance estimate is obtained from the instructor-version of the *Excel* file.

$$\text{The beta estimate} = \frac{The\ Covariance_{\ (the\ SMF,\ The\ S\&P\ 500)}}{The\ Variance\ of\ the\ Market} \cong 0.9979\,.$$

Using the *Excel* spreadsheet, students are shown that the beta estimate obtained from the single-index regression is exactly the same as the one computed from the formula,

$$\frac{The\ Covariance_{\ (the\ SMF,\ The\ S\&P\ 500)}}{The\ Variance\ of\ the\ Market}\,.$$

The variance estimate and the covariance estimate displayed in the case are rounded values; therefore, students are reminded that if one computes the beta value using these rounded values, i.e., 0.002026/0.002031 = 0.9975 then his/her answer will not be the same as the one computed from the *Excel* spreadsheet which contains more accurate inputs.

a) In class, the *Excel* spreadsheet is used to demonstrate computation of different types of risks.

   The Total Risk = σ^2_{SMF} =0.002250 (obtained from Exhibit 1). Alternatively, Total Risk = Total Systematic Risk + Total Unsystematic Risk.

$$\begin{aligned}\sigma^2_{SMF} &= \beta^2_{SMF} \times \sigma^2_{market} + \sigma^2_{\varepsilon} \\ &= 0.002022 + 0.000228, \\ &= 0.002250\end{aligned}$$

   The *SMF's total systematic risk* is estimated to be 0.002022, as shown above, and its *unsystematic risk*, $\sigma^2_{\varepsilon} = \sigma^2_{SMF} - \beta^2_{SMF} \times \sigma^2_{market}$, is estimated at 0.000228.

b) As shown in 5a) above, the total systematic risk of the SMF's return is estimated to be 0.002022. Thus, it is quite close to the total risk estimate of the *S&P 500* in the amount of 0.002031. The difference is inconsequential. However, in 3b) the comparison relates to the coefficient of variation and now the comparison is with total systematic risk only.

c) The total unsystematic risk, the variance of the error term, is estimated to be 0.000228. Use of the spreadsheet would provide more accurate data.

Question 6: (Optional) The beta value of the *SMF* is obtained from running the single-index market model, and it is available in Exhibit 2 along with other selected outputs of the regression. Run the single-index market model in *Excel*, and the data needed for this regression is provided in the student-version of the *Excel* file accompanied with this case. Verify that the beta value shown in Exhibit 2 is the same as the slope estimate obtained from your regression model.

Solution 6: With more advanced students, the class is shown how to run a simple regression in *Excel*. Then, the students are asked to run a regression using the data provide in the student-version of the *Excel* file accompanied with the case. In turn, they are asked to verify that the beta estimate given in Exhibit 2 is the same as the one from their own regression results.

Question 7: Refer to Exhibits 1 and 2. (a) Construct an equally weighted portfolio (that is, each of the two portfolios, the *SMF* fund and the *S&P* 500, is weighted by 50 percent), and compute the resultant portfolio's average return and its standard deviation. (b) Compute the *weighted-average* standard deviation, that is, 0.50 × the standard deviation of the *SMF* fund + 0.50 × the standard deviation of the *S&P* 500. (c) What is the difference between the portfolio's standard deviation from Part (a) and the *weighted-average* standard deviation from Part (b)? What explains this difference?

Solution 7:

	The SMF	The S&P 500
Average Return	0.30%	0.59%
Standard Deviation	4.74%	4.51%
The Covariance Estimate	0.002026	
The Correlation Estimate	0.9481	
The Weights	0.50	0.50
The Portfolio's Expected Return	0.45%	
The Portfolio's Standard Deviation	4.56%	
The Weighted-average Standard Deviation of	4.63%	

a) The portfolio's expected return is obtained as follows,

b)

$$E(R_p) = 0.50 \times 0.30\% + 0.50 \times 0.59\% = 0.45\%$$

c) The portfolio's standard deviation is obtained as follows,

$$\sigma_p = \sqrt{0.50^2 \times (4.74\%)^2 + 0.50^2 \times (4.51\%)^2 + 2 \times 0.50 \times 0.50 \times 0.002026} = 4.56\%.$$

The *weighted-average* standard deviation of this equally weighted portfolio is obtained as follows,

$$\sigma_{weighted-average} = 0.50 \times 4.74\% + 0.50 \times 4.51\% = 4.63\%.$$

By forming a portfolio, the portfolio's expected return is a linear combination of individual asset's average (expected) returns. Through this assignment, it demonstrates that the portfolio mean return is seen to be simply the weighted average of returns on individual securities, where the weights are the percentage invested in those securities. However, the portfolio variance is not the weighted average of the variances of individual securities. Rather, the portfolio variance is the sum of the variances of the individual securities multiplied by the square of their weights plus a third term, which includes the covariance. The covariance is an extremely important concept because it is the appropriate measure of the contribution of a single asset to portfolio risk. The real importance of the covariance is the correlation coefficient component. If the correlation is positive, the risk is increased by the covariance term, but, if the correlation is negative, it will reduce the risk, with no change in the return.

Question 8: The following tables contain beta estimates of the *SMF* fund in the two sub-periods, respectively.

Exhibit 3: Output from the Single-index Regression Model: September 1997 to December 2001 Data

Regression Statistics	
R-Square	88.00%
Adjusted *R*-Square	87.76%
The Standard Error of the Estimate	1.94%
The Coefficient of Correlation	0.9381
Observations	52

	Coefficients	***Standard Error***	***t Statistic***	***P-value***
Intercept	-0.0028	0.0027	-1.0272	0.3093
The Beta Estimate	1.0076	0.0526	19.1452	0.0000

This exhibit shows the regression results from the Ordinary Least Squares estimation using Equation (1). The monthly return series are those of Lowell State University's SMF and that of S&P 500 from September 1997 to December 2001, respectively.

Exhibit 4: Output from the Single-index Regression Model: January 2002 to April 2006 Data

Regression Statistics	
R Square	93.95%
Adjusted R Square	93.83%
The Standard Error of the	0.95%
The Coefficient of Correlation	0.9693
Observations	52

	Coefficients	**Standard Error**	**t Statistic**	**P-value**
Intercept	-0.0029	0.0013	-2.2165	0.0312
The Beta Estimate	0.9796	0.0352	27.8640	0.0000

This exhibit shows the regression results from the Ordinary Least Squares estimation using Equation (1). The monthly return series are those of Lowell State University's SMF and that of S&P 500 from January 2002 to April 2006, respectively.

Compare these two beta estimates. What could explain the difference?

Solution 8: A statement was made pertaining to the issue of regression tendency. Blume (1975) showed that beta estimates in the single-index regression model tend to move toward the mean over time. The tendency for betas to regress toward its mean value implies that a security (or portfolio) with either an extremely high (β_i>1) or low (β_i<1) beta value during one estimation period will tend to have a less extreme beta value in the next estimation period. We use the two beta estimates of the *SMF* fund shown in this assignment to demonstrate this issue.

With more advanced students, a request is made to verify the beta estimates by running two separate single-index market model regressions with the data provided in the student-version of the *Excel* file.

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