RECENT ADVANCES IN APPLICATIONS OF MATHEMATICAL PROGRAMMING TO BUSINESS AND ECONOMIC PROBLEMS

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ABSTRACT

It is well known that Mathematical Programming techniques are well-developed and with widespread application. It would be useful for researchers and practitioners in the information systems field to have a categorization that would show the types of problems that have been solved using these techniques. This paper provides this categorization

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INTRODUCTION

Application of business, financial and economic problems to mathematical programming is of ongoing interest. Users include insurance companies, mutual funds, farmers and shipping companies. Many fields including the fields of micro-economics and finance have been impacted directly as both private business and governments have made use of Mathematical Programming, and in particular, Linear Programming. Its use in the production and allocation of resources cannot be underestimated. Linear Programming (LP) has been said to be one of the ten most important discoveries of the last century (Dongarra and Sullivan, 2000, and Fourcans and Hindelang, 1974).

Mathematical programming is used when an optimization decision has constraints that limit decisions. For example, a business may use mathematical programming to allocate scarce resources in an optimal way, an insurance company may be required to keep a small percentage of its assets in treasuries and another percentage in fixed income securities and a farmer's crops may require a minimum amount of fertilizer.

In the remainder of this paper we provide a literature review that shows the vast scope of business and economic applications of mathematical programs. We then develop the generic Linear Programming model and then apply the model to a specific application. Finally we conclude with and give a specific direction for future research that we believe would be useful for practitioners.

LITERATURE REVIEW

Applications of optimization problems extend over many fields. For example, in the fields of economics and finance, linear programming may be applied to production problems, shipping problems, asset allocation, crop growing, allocation of sewage, mortgage backed securities portfolios other similar problems.

In the area of crops and resource management, Myers and McIntosh et al (2008) apply optimization techniques to crop rotation for Idaho potatoes. Fritzsch and Wegener et al (2011) discuss linear programming applied to farm households in Europe. Lu and huang (2010) apply linear programming to water resource management. fan and huamg (2012) model Linear Programs in an environmental context. Hadani and Alwi (2010) apply integer linear programming to optimization of water networks and Zeng and Kang et al (2010) apply it to water management and additionally to agriculture and crop area planning. Similarly, Becker (1990) applies optimization to agriculture on a farm. April, Glover and Kelly (2002) discuss an application of optimization to portfolio optimization for capital investment products. Ho and presantat (2010) provide a literature review for supplier evaluation and selection.

Optimization is also big for financially related problems. Cagan, Carriero and Zenios (1993) apply optimization to pricing mortgage backed securities. We also discussed this topic in the context of the sub-prime mortgage problem (Yarmish, Fireworker and Nagel, 2008). Fourcans and Hindelang (1974) applied optimization to capital management for the multinational firm. Hiller and Schaack (1990) classified bond portfolio modeling techniques and Markowitz (1952, 1959) applies optimization to diversifying a portfolio. Schrage (1994) provides an overview of some financial optimization problems in the context of the LINDO optimization package and Cooper and Steinberg (1974) discuss general methods and applications.

In many applications the coefficients of the modal are not known with certainty. As an example, consider our production example further in this paper. In that example we used $70 and $80 as the profit for selling a bed and a chair respectively. This profit is based on an assumed selling price. In fact, many times a decision must be made now but the actual sale will occur later! Obviously the actual price is subject to supply and demand and using these prices in our model may be an assumption depending on the situation. To address this issue researchers have studied various techniques to deal with mathematical programming under uncertainty. Carino, Kent et al (1994), and Cariño and Ziemba (1998) discuss a model that was built for a large insurance company to balance financial obligation and investment within the context of the many laws governing insurance companies.

These laws were handled via the constraints of a mathematical program. Houck, Hedrich and Cohon (1978) apply stochastic LPs (under uncertainty) to the management of reservoir systems. Zenios (1991) address the uncertainty of mortgage backed security valuation by Monte Carlo simulations and Salmi (1974) focus on trade, production and financial flows in multinational firms. Tintner and Raghavan (1970) discuss a stochastic linear program application to dynamic planning in India. Yarmish, Nagel and Fireworker (2009) address the stochastic programming for asset allocation models. Geng and Chen (2009) apply stochastic programs for capacity planning to semiconductor wafer fabrication.

Many mathematical programs result in very large problems. This is common for the applications the must be solved under uncertainty. Research has focused on the use of parallel computers to solve these problems. Moriggia, Bertocchi and Dupaková (1998) discuss parallel computers in the context of dynamic bond portfolio management. Zenios (1991, 1994) discusses parallel computing application in general. Dantzig, (1988) the founder of linear programming, addresses mathematical programming under uncertainty with the idea of using parallel computers to help solve the large problems. Shu and Wu, (1993) discuss a parallel implementation of those applications the use the revised simplex method, Thomadakis and Liu (1996) discuss a variation for SIMD parallel machines and Yarmish and Van Slyke developed a distributed LP implementation for problems with dense matrices. The literature is vast and the number of applications continues to grow.

MODEL DEVELOPMENT

A standard LP formulation is of the form:

$$\begin{array}{ll} \text{Maximize} & c_1x_1 + c_2x_2 + c_3x_3 \;\cdots\; + c_nx_n \\ \textit{Subject to} & a_{1,1}x_1 + a_{1,2}x_2 + a_{1,3}x_3 \cdots + a_{1,n}x_n \le b_1 \\ \textit{Constraints} & a_{2,1}x_1 + a_{2,2}x_2 + a_{2,3}x_3 \cdots + a_{2,n}x_n \le b_2 \\ & \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots\cdots \\ & a_{m,1}x_1 + a_{m,2}x_2 + a_{m,3}x_3 \cdots + a_{m,n}x_n \le b_m \\ & x_j \ge 0 \;\; \textit{for all } j \quad (j = 1,\ldots,n) \end{array} \qquad (1)$$

OR

$$\begin{array}{ll} \textit{Maximize} & \sum_{j=1..n} c_j x_j \\ \textit{subject to} & \sum_{j=1..n} a_{ij} x_j \le b_i \quad \textit{for all } i \quad (i = 1,\ldots,m) \\ & x_j \ge 0 \;\; \textit{for all } j \quad (j = 1,\ldots,n) \end{array} \qquad (2)$$

Where c_j, a_{ij} and b_j, are coefficients and x_j are the variables to be solved for j=1..n and i=1..m.

Definition: A constraint is a row of a linear program that restricts the variables to values that would not violate the expression.

Definition: A feasible solution x is a set of x_j, j=1..n that satisfy the constraints.

Definition: An optimal solution x is a set of x_j, j=1..n that both satisfy the constraints and maximize the function.

MODEL APPLICATION

The following describes the general Production Problem. Suppose a manufacturer manufactures j products where each product yields a known profit. Suppose further that there are i resources necessary in the production of all the products and there is a known limit on the amounts of each resource. The problem to be solved is: how many of each of the products should be produced to maximize profit. To make it specific, suppose a furniture company manufactures two items: wooden chairs and beds. The company employs workers skilled in carpentry, painting and upholstery. Suppose that the number of hours of skilled labor and profit per item are as defined in table 1.

Table 1: Production Example Coefficients

	Chair	Bed	Available labor
Carpentry	6	3	48
Painting	1	1	6
Upholstery	2	6	36
Profit	$80	$70	

This table shows the Constraints and Optimization Coefficients for the Production example. The rows show the three constraints on the available labor hours for employing carpenters, painters and upholstery experts. The columns show the profit per item for chairs and beds.

From the table one notes that 6 hours of carpentry work are needed per chair and 3 hours are needed per bed. There is a maximum of 48 carpentry work hours available. Interpretation of the other rows is analogous. We know too, that for each chair and bed manufactured one will earn $80 and $70 respectively.

This may be described as the LP problem:

$$
\begin{array}{llll}
\text{Maximize} & 80x_1 & +70x_2 & \\
\textit{Subject to} & 6x_1 & +3x_2 & \leq 48 \\
\textit{Constraints} & x_1 & +x_2 & \leq 6 \\
 & 2x_1 & +6x_2 & \leq 36 \\
 & x_1, x_2, \geq 0 & &
\end{array}
\qquad (3)
$$

The economic interpretation is straightforward. We wish to determine the number of chairs (x_1) and beds (x_2) to manufacture per day to enable maximization of profits without exceeding available resources. The company is constrained by the number of work-hours per skill to which it has access.

We examine table 1 on a row by row basis. We wish to know how many chairs (x_1) and beds (x_2) to produce in order to maximize the profit $80x_1+70x_2$. We must be careful not to violate the constraints designated by the rows:

(9 carpenter-hours)x_1 +(3 carpenter-hours)$x_2 \leq 48$

(1 painting-hour)x_1 +(1 painting-hour)$x_2 \leq 6$

(2 upholstery-hours)x_1 +(9 upholstery-hours)$x_2 \leq 36$

CONCLUDING COMMENTS

In this paper we tried to give the reader an appreciation of the vast array of business and economic applications to mathematical programming in general and to linear Programing in particular. These optimization techniques can be an important tool for the information system professional help businesses and organizations grow.

In our literature review we included a broad spectrum of applications for these optimization techniques. These included production problems, shipping problems, asset allocation, crop growing, allocation of sewage, mortgage backed securities portfolios and many others. Linear programs also occur frequently in such other important applications as wavelet decomposition, digital filter design, text categorization, image processing and relaxations of scheduling problems.

We then developed the linear programming model and followed up with its application to a specific problem.

We presented in this paper only a taste of the breadth of applications but the number of real applications is vast. It is well known that Mathematical Programming techniques are well-developed and with widespread application. It would be useful for researchers and practitioners in the information systems field to have a categorization that would show the types of problems that have been solved using these techniques. Research in this area would be very useful. Practitioners would then be able to use this categorization as a guide to applying proper optimization techniques to their own problems.

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