

DURATION RISK: DO YOU KNOW YOUR NUMBERS?

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ABSTRACT

Most investors know that bond prices move inversely with interest rate fluctuations. These same investors, however, may not fully understand how to assess the interest rate risk of different fixed income investments. This is particularly timely given the improving U.S. economy and Federal Reserve's expectation of increasing interest rates in the near future. The simplest measure of interest rate risk for the average investor is something called duration. The Financial Industry Regulatory Authority (FINRA) stated that the one number a bond investor should know is duration. This article will explain what duration is, how to use this measure to evaluate risks, examples of how changes in interest rates will impact bonds or bond funds with different durations, and where investors can find this information.

JEL: M10, G10, M40

KEYWORDS: Duration, Interest Rate Risks, Bonds, Investments

INTRODUCTION

In light of FINRA's new emphasis on the importance of duration, a primer on Duration Risk is an important tool for investors who may not be well versed on the basics of this important tool for measuring investment risk. As baby boomers age, the traditional investment advice is to allocate a larger percentage of one's assets to fixed income as opposed to equities or other investments. Many investors erroneously believe that investments in high quality long-term bonds or bond funds have very minimal risks. This may be true if individual bonds are held to maturity or have a very short maturity. However, if interest rates increase, the value of their bonds or bond funds will decline. This is called interest rate risk and as this article will illustrate interest rate risk generally increases as the maturity of the bond or bond fund increases. Although there may be limited credit risk in a 20-year AA rated bond, investors should still be concerned about interest rate risk. A bond maturing in 20 years may be twice as sensitive (volatile) to changes in interest rates as a 10-year bond. This is a very important point to remember as individuals and some funds are "chasing higher yields" by investing in longer maturity bonds that generally pay a higher rate of interest. Some investors might initially believe that they will hold their bonds to maturity, but unforeseen circumstances may develop.

Even more likely, an investor may want to either sell their existing low yield investment to obtain a higher yield or to change their asset allocation to increase their equity exposure. A risk adverse conservative investor might be surprised to incur a significant loss on the sale of their "safe" investment. Current interest rates are at historically low levels and the realistic expectation is that they cannot go any lower and may increase in the near future. Hence, fixed income investments are not necessarily as safe and less volatile as they had been in the past. Bond funds also present a different challenge for investors than holding individual bonds. Individual bond investors can hold their bonds and receive the par amount at maturity assuming the bonds do not default. However, bond funds are typically structured as short-term, intermediate term (typically 5-7 year maturity) and long-term (typically ten or more years' maturity). The bond funds' managers will routinely purchase new bonds as existing ones mature in order to maintain the target average maturity so most bond funds never mature.

LITERATURE REVIEW

“Although duration has a long history, it is still an important and widely used tool in the risk management of portfolios of interest rate sensitive assets. Most papers studying duration focus on default-free (Treasury) bonds, but for the many portfolio managers also investing in defaultable (corporate) bonds it is important to understand the sensitivity of defaultable bonds to interest rate changes. The few existing papers address the duration of corporate bonds either derive durations from relatively simple firm-value based models or estimate the empirical relation between changes in the prices of corporate bonds and changes in interest rates.” (Kraft and Munk 2007). Macaulay developed the concept of duration in 1938 as an alternative to term-to-maturity. He proposed that it was a more precise way to measure the time period for a bond or a fixed income security. It has been utilized as a risk management tool since 1938 in various forms. Hicks focused on the mathematical aspects of duration by examining a series of cash flows (1939) and calculating the elasticity of this series with the discount factor, resulting in re-deriving Macaulay duration. He concluded that a small change in yield to maturity is proportional to duration. “Nonzero coupon bonds generate regularly scheduled payments before maturity. Thus maturity provides an incomplete description of the time pattern of all the payments of a bond.”

Fisher and Weil (1971) proposed that using yield to maturity to obtain duration implies that interest rates are the same for all maturities (a flat-term structure). Thus they felt that this flat term structure assumed by Redington (1952) and Macaulay were unrealistic. Fisher and Weil reformulated duration introducing an approximation of duration with convexity so that it could be used to immunize a bond portfolio against interest rate risk when hedging a bond position with respect to a parallel shift of the interest rate curve. They felt that this reformulation would address the increasing volatility of interest rates in the 1970s. “A bond’s duration increases with maturity but it is shorter than maturity unless the bond is a zero-coupon bond (in which case it is equal to maturity). The coupon rate also affects duration. This is because a bond with a higher coupon rate pays a greater percentage of its present value prior to maturity. Such a bond has greater weights on coupon payments, and hence a shorter duration.” Fisher & Weil (1971).

Bierwag and Roberts (1990) conclude that when a change in interest rates occur in portfolios with higher durations, they found more price sensitivity. They determined that the volatility in their models was a function of the parameters of the stochastic process and of implied measures of duration. Fooladi and Roberts (1992) examined the effectiveness of immunization in practice using the actual prices of Canadian bonds over the period 1963-1986. They set a five-year investment horizon and rebalanced every six months. Their conclusion was that duration matching with hedges outperformed non-duration matched portfolios. They determined that this validated the use of duration in measuring risk and immunization. Thomas S.Y. Ho (1992) states in his paper entitled Key Rate Duration: Measures of Interest Rate Risks, that “identifying interest rate exposure is central to active and structured portfolio management.” Key rate duration measures the sensitivity of a portfolio to a 1% change in yield for a given maturity if all other maturities remain unchanged. T. Ho further states (2013) that “key rate duration gives us a measure to enable us to manage our yield curve risk”. Cox, Ingersoll and Ross (1979) examined duration as a measurement of basis risk. They found that Macaulay duration was a “valid measure of basis risk only under circumstances which are theoretically and empirically unrealistic.”

Their findings support modified or effective duration calculations. They proposed a general equilibrium model of duration and immunization within a continuous time and no arbitrage. Campbell and Mei (1993) investigated systematic risk resulting from variations in future returns in comparison to variations in expected cash flows. “...the survival of the corporate practice of discounting longer-term projects at higher rates is not irrational but is an intuitive response to correctly perceived risks. In fact, given the difficulties associated with estimating betas, the duration of a project may be one of the most accurate ways of assessing its systematic risk.” (Campbell and Mei 1993) Fooladi and Roberts (2000) surveyed the current applications of duration analysis in risk management. “Today both discrete and continuous time duration models are in

use.” “The realized rate of return has two components: interest accumulated from reinvestment of coupon income and the capital gain or loss at the end of the planning period when the portfolio is sold. The two components impact the realized rate of return in opposite directions. For example, if rates rise, coupon reinvestment income will increase, raising realized return. On the other hand, lower bond prices, associated with higher rates, will lead to a capital loss.” (Fooladi and Roberts, 2000).

Kraft and Monk (2007) examined the concept of duration to study differences in duration between corporate bonds and similar treasury bonds. “In a frequently used intensity-based model for corporate bond valuation we provide an example showing that, given the parameter estimates found in empirical studies, the duration of a corporate coupon bond may very well be larger than the duration of a similar Treasury bond.” (Kraft and Monk, 2007). Lajili and Rakotondratsimba (2012) proposed that the classical duration-convexity approximation of Fisher and Wield had several deficiencies including neglecting the passage of time, as well as the shift size of the yield curve and error approximation. They proposed an enhancement to the classical duration-convexity to avoid these deficiencies. They further maintain that “a parallel positive shift (as 100 basis points) remains also a standard and easily tractable approach in the perspective of stress-testing. The result we obtain in this paper enables the user to consider any large shifts lying inside a given range. This is particularly useful to grasp turmoil situations as we are faced frequently since the 2007 financial crisis.” Rakotondratsimba (2011) examined benchmark models introduced by Vasicek (V) and Cox-Ingersoll-Ross (CIR) which use only one variable, the instantaneous short-rate variable, to predict interest rate sensitivity. Empirical tests for bond immunization performance did not show superiority of the stochastic duration in comparison to simple classical duration. Rakotondratsimba introduces “suitable zero-coupon sensitivities with respect to the one factor shock related to the V/CIR model.”

The Financial Industry Regulatory Authority (FINRA) in 2013 issued an investor alert cautioning investors who own bonds or bond funds that the one number an investor should know is duration. FINRA was established in 2007 by Congress to protect investors. On February 14, 2013 FINRA issued a new investor alert called “Duration: What an Interest Rate Hike Could Do To Your Bond Portfolio” which warned investors that duration could cause their bond or bond portfolios to decline in value if interest rates were to rise. “The higher a bond’s duration, the greater its sensitivity to interest rates changes. This means fluctuations in price, whether positive or negative, will be more pronounced. (FINRA, 2013).

How Can an Investor Determine Their Interest Rate Risk?

The most widely accepted measure of interest rate risk for the average investor is something called duration, which is a measure of the sensitivity of bond prices to changes in interest rates. Duration is a number that at a particular point in time will tell an investor approximately what percentage a non-callable bond will change in value given a 1 percent change in interest rates. With a known bond or bond fund’s duration, an investor can estimate, all else being equal, how the investment will react to a change in interest rates and make an informed decision based on his expectation of future interest rates. The time to maturity for a given bond may provide a general sense of its interest rate sensitivity, but duration is a much more accurate measure. An example is Vanguard’s Intermediate Term Treasury Bond Fund (VBILX), which has an “effective maturity” of 7.2 years but an “average duration” of 6.5 years. Generally, the higher the duration number of a bond, the more sensitive the bond is to interest rate changes. Investors who are expecting higher interest rates might wish to invest in bonds with shorter durations or maturities to limit their potential losses.

How is Duration Calculated?

Duration is the number of years it would take an investor to recoup the full cost of their bond or bond fund considering the present value of all interest and principal payments to be received. Hence, a bond’s cash flow payments for both interest and principal are discounted at current market interest rates divided by the current market value of the bond. In general, the greater the duration of the bond, the more sensitive that

bond will be to interest rate changes. It should be noted though that duration is generally accurate for small changes in interest rates. Utilization of duration for large interest rate changes requires taking into account a concept known as convexity which measures the change in duration as interest rate changes become more significant. The market value of a bond will generally decline less for larger interest rate increases. Convexity depending on the characteristics of the bond or bond fund can be both negative and positive, which means that the duration can actually decrease or increase based on the degree of interest rate change. Thus the correlation between a change in interest rates and their effect on bond prices is not necessarily symmetric. This paper will not address convexity however; but investors should be aware that it may impact their investment decisions if there are large changes in interest rates.

An Example of how Duration Can Be Applied

Assume an investor puts \$10,000 into a bond fund which has an average duration of 5. Generally, a 2 percent across the board increase in interest rates will result in approximately a 10 percent (5 duration x 2%) or a \$1,000 decline in the fund’s value to \$9,000, all else being equal. However, if that same \$10,000 had been invested in a fund with duration of 10 with this same 2 percent interest rate increase, the investment would have decreased 20 percent (10 duration x 2%) or \$2,000 and now be worth \$8,000. Conversely, a 2% decrease in interest rates could result in the fund increasing in value by 20%.

Figure 1: Illustration of how A 2% Increase in Interest Rates Reduces A \$10,000 Bond's Value Based on Its Duration

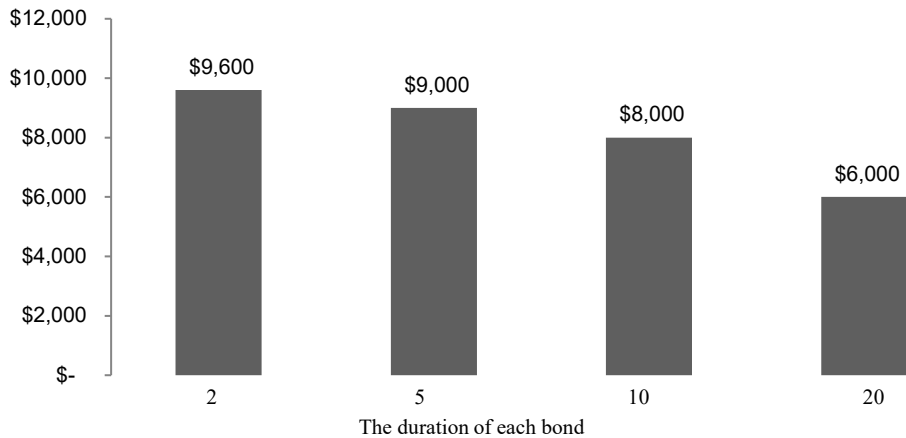


Figure illustrates the theoretical impact of a two (2%) interest rate increase for bonds with different durations. Note that a 2% increase in interest rates with a duration of 2 would result in a \$400 loss for a \$10,000 bond. A 2% increase in interest rates in a \$10,000 bond with a duration of 5 would result in a \$1,000 loss whereas a 2% increase in a \$10,000 bond with a duration of 10 would result in a \$2,000 loss on the investment.

An example of a bond fund with a high duration is the Schroder Long Duration Investment Grade Bond Fund (STWLX) which has a duration of 14.79 as of September 2015. “To the extent that changes in prices reflect the degree of interest rate risk assumed, duration represents a good first-approximation measure of risk because it is proportional to the price change. For example, the price of the 5-year CD will change five times as much for a given change in interest rates as the price of the one-year CD. This makes it five times as risky, which is reflected in a duration five times as great.” (Kaufman, 1984).

How Does Duration Impact Investment Decisions?

The level of interest rate risk that an investor may take is a function of their overall risk tolerance, time horizon and their expectation of future interest rate changes. The key is that knowing the duration of a fixed income investment allows the investor to quantify their interest rate risk and make an informed decision. An example of such an evaluation process was expressed by the brokerage firm, Charles Schwab, on their

website in September 2015. Their advice to clients who might be tempted to purchase longer-term bonds for a slightly higher yield was that the increased duration may not be worth the risk.

“We see limited value and higher risks in long-term funds today compared to intermediate-term funds. The benefits of a slightly higher rate aren’t well-balanced with the increased interest-rate risk, in our view, for funds with average maturities much greater than 10 years. An exception might be if you’re focused on income and income alone and won’t need to sell, or if you believe that interest rates will fall. While we believe rates could stay lower longer than many investors expect, they will rise eventually. Also, pay attention to the fund’s duration. Duration is a measure of interest rate sensitivity, but it can also be thought of as a measure of how long it takes to recover your initial investment. Funds with shorter durations will typically be less sensitive to increases in interest rates and you’ll generally recover your initial investment sooner if interest rates rise as compared to funds with longer durations. However, funds with shorter durations typically have lower yields.” C Schwab 2015.

Where Does an Investor Find Duration Information?

The duration for an individual bond may be obtained by contacting the bond issuer, a broker or by using an online bond duration calculator, which can compute a specific bond’s duration. With regard to bond funds (versus individual bonds), most fund issuers provide “fund facts” on their websites, which disclose their average duration as well as other information including yield, expense ratio and average maturity. Another excellent source of information on the duration of many bond funds can be found at finance.yahoo. By inputting the fund’s ticker symbol and then selecting “holdings”, the duration, average maturity and credit quality are disclosed and can be compared to the relevant category averages.

Several Methods Are Utilized to Calculate Duration

$$\text{Macaulay Duration} = \frac{\sum_{t=1}^n \frac{t * C}{(1+i)^t} + \frac{n * M}{(1+i)^n}}{P}$$

n = number of cash flows

t = time to maturity

C = cash flow

i = required yield

M = maturity (par) value

P = bond price

$$\text{Modified Duration} = \left[\frac{\text{Macaulay Duration}}{\left(1 + \frac{\text{Yield to maturity}}{\text{Number of coupon periods per year}}\right)} \right]$$

The Macaulay duration calculation is a commonly used method. It measures a bond’s sensitivity to interest rate changes by calculating the weighted average number of years the investor must hold a bond until the present value of the bond’s cash flows equals the amount paid for the bond. It is a pure discounting of the present value of a bond’s cash flows so tends to be theoretical. (Cox et al). Modified duration is a modified Macaulay computation that directly measures price sensitivity. Effective duration, on the other hand, is often the calculation cited for bonds that may have special redemption features. Unlike Macaulay duration, effective duration takes into account the potential changes in cash flows which can occur from redemptions. Redemptions can occur from prepayments and the exercise of call and put options. Understanding the formula calculation is not as important though as understanding that duration is a measure of risk in relation to interest rate fluctuations. However, some bond funds, such as Blackrock, have their own proprietary method for calculating duration and in those instances financial websites may not show duration for those funds and direct you to the fund’s actual web site.

There are also some special circumstances that could affect duration. Duration assumes that for every movement in interest rates, there is an equal change in bond prices in the opposite direction. However, this is not always the case. For example, when interest rates drop, a residential mortgage-backed security (a bond backed by home loans) might not see an equal increase in the bond’s price because it might prompt homeowners to refinance their mortgages. This in turn may limit increases in the bond’s price as the underlying mortgages are being refinanced at lower interest rates.

Changes in the Slope of the Yield Curve

The yield curve refers to the graphical representation of interest rates over a specified time period. Generally, interest rates increase as the time to maturity increases. However, there have been rare times when the yield curve has been “inverted”, meaning that shorter-term rates were higher than long-term rates. More importantly, the application of duration assumes that a given 1% change in interest rates occurs uniformly across the yield curve. The reality is that interest rate changes may not be the same for all maturities. Consequently, the change in value of a bond or a bond fund is directly affected by its position on the yield curve and where interest rates are changing on that curve. Figure 2 illustrates both the 2008 and 2014 yield curves. One can see that interest rates may not stay at their current historically low levels and might revert to a more normal level in future years. This figure also visually depicts that the actual shifts in the yield curve are not usually perfectly parallel when interest rates rise across the board.

Figure 2: Yield Curves for US Treasuries

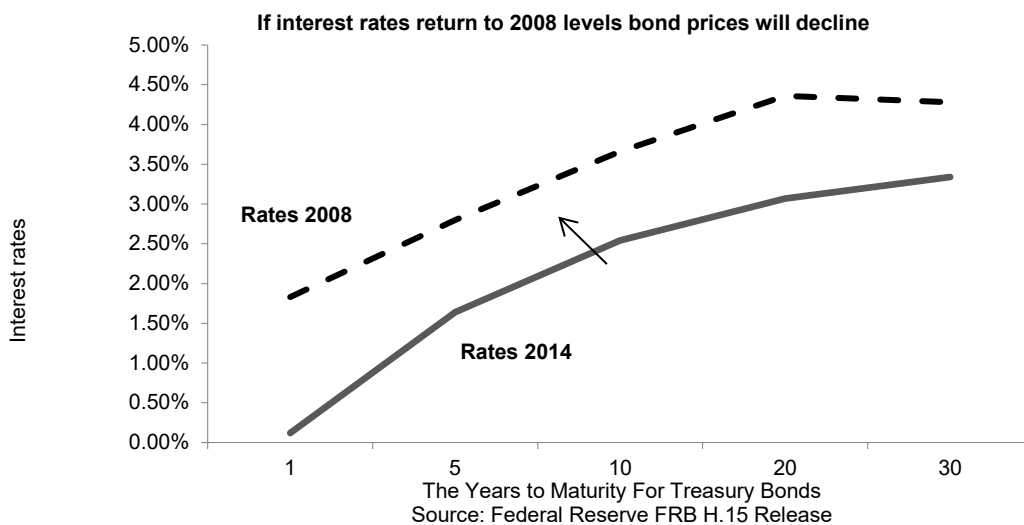


Figure 2 illustrates the interest rate yield curve for 2014 from the Federal Reserve Board’s statistical release. It also illustrates the interest rate yield curve as it was in 2008 from the Federal Reserve. One can see that the interest rate yield curve for US Treasuries is historically lower than it was in 2008.

A Bond Fund’s Name Is Not Always an Accurate Gauge of Its Duration Level

It is not always possible to judge a bond fund by its name. U.S. News and World Report regularly publish a listing of “Best Mutual Funds” which includes a list of “Intermediate-Term Bond” funds. U.S. News does appropriately note that their list of intermediate-term bond funds has durations from 3.5 to 6 years, which represents a significant range of interest rate sensitivity. Investors cannot just rely on the name of bond funds to accurately judge their interest rate risk. A case in point are two well respected funds on this intermediate-term bond fund listing that have very different durations as of October 2015. The Loomis Sayles Intermediate Duration Bond Fund (LSDIX) has a duration of 3.87 which is below the category average of 4.93 for Intermediate-term bond funds and therefore is less sensitive to interest rate changes than

the general category. At the other end of the spectrum on that same list is the Vanguard Intermediate-Term Bond Index (VBIIX) which has a duration of 6.49. This is 68% greater than the aforementioned Loomis Sayles Fund, and is also 32% higher than other funds in this category average. Consequentially, Vanguard's fund has a significantly higher interest rate risk. This does not mean that the Vanguard Fund is inferior to the Loomis Sayles Fund. In fact, the Vanguard fund has very high credit quality and half the annual expense ratio of the Loomis Sayles Fund. The two funds just have very different interest rate risk characteristics. During the last year twelve months when interest rates declined slightly the Vanguard fund reported a one-year return of 4.07% versus a return of only 2.42% for the lower duration Loomis Sayles Fund. This is an excellent example for investors that higher duration (all else being equal) means higher returns in a declining interest rate environment and lower relative returns in an increasing interest rate scenario. In addition to duration, investors should consider the type of bonds, their credit quality, fund managers and the expense ratios charged. FINRA warned investors to be cautious of duration but that also other factors including inflation risk, call risk and default risk could affect the value of a bond or bond fund. Investors should always check a bond fund's prospectus for these risk factors.

Lifecycle or Target Date Funds Bear a Closer Examination for Interest Rate Risk

Many baby boomers who may currently be or approaching 60 years old and hoping to retire by age 65 may be in an age based life cycle fund such as TIAA-CREF's Lifecycle 2020 Fund. The purpose of an age based lifecycle fund is that it regularly changes the asset allocation reducing the individual's equity exposure and increasing their fixed income exposure under the premise that fixed income investments are less risky than equities. However, in our current economic climate in which interest rates are at abnormally low levels, fixed income investments of any significant duration may not be significantly less risky than equities. There is a risk for older investors in lifecycle target date funds since they are more heavily invested in fixed income. Consider the example of a baby boomer with \$1 million in the TIAA/CREF's 2015 Lifecycle fund (TCNIX). At September 30, 2015, 34.1% of the fund's assets were invested in CREF bonds. (ttaa-cref.org). This means that if interest rates increase 2% across the board (parallel shift of the yield curve), the bond fund portion of the portfolio would decline by approximately 10.8% (5.4 duration x 2%). 34.1 percent of the \$1 million investment is \$341,000, and a 10.8% decline would result in a loss of \$36,828. (TIAA/CREF has received multiple awards including the 2013 and 2014 Lipper Fund Award "as the best large fund management company overall". (de Aenlle). Most investors consider TIAA/CREF to be one of the safest options for retirement portfolios and thus might never expect *any* loss.)

How to Lower Your Interest Rate Risk (Duration) Without Giving Up Yield

As previously discussed, during this time of extremely low interest rates, many investors and money managers seek to increase the yield on their fixed income investments by buying longer maturity bonds, "going further out on the yield curve". Longer maturities will typically result in a higher duration/interest rate risk. However, there is a little known strategy used primarily by institutional investors of keeping the higher yield on longer dated bonds but reducing their duration. Consider an example of two 20-year bonds which both have an effective yield of 2%. Bond A is a bond trading at par (value at maturity) which has a stated interest rate of 2%. Since the bond is trading at par, the effective yield is also 2%. Bond B trades at a large premium (price over par) because the stated interest rate (coupon) on the bond is 5%. However, the effect of paying a premium for the bond reduces the effective yield on Bond B to 2%, equivalent to Bond A. While the term and yield on these two bonds are identical, the durations are very different. Using one of the many online duration calculators (such as WolframAlpha), Bond A with the 2% coupon has a 16.41 modified duration (16.58 Macaulay), while Bond B with the 5% coupon has a 14.13 modified duration (14.27 Macaulay). How is that possible? While the overall return (yield) to investors is the same, the higher stated/coupon interest payment the investor receives from Bond B (typically semi-annually) effectively returns the investment slightly faster back to the holder of Bond B and consequently reduces the duration of the bond. How important is this difference of 2.28 in duration between Bond A and B? If interest rates

increase 1%, Bond A's price will theoretically decline 2.36% MORE than Bond B's price. Besides the reduced interest rate risk, institutional money managers can also report a lower duration for their mutual fund or portfolio by using this tactic. Hence, an observant investor might notice that bond issuers prefer to sell more of their bonds at a premium. In an efficient market, the buyer of Bond B who paid a premium could potentially lose some or all of that premium paid over the face value of the bond if the issuer calls that bond before maturity. Consequently, it is important for an investor to understand the call or redemption provisions before buying a bond at a premium. A low duration does not mean that a bond or bond fund is risk-free. (FINRA 2013) Another option recommended by Bohlin and Strickland (2004) is to build a portfolio of bonds with staggered maturities so that bonds mature every year as a way to minimize portfolio risk. This stream of cash flows as bonds mature can be reinvested into new less expensive higher-yield bonds.

CONCLUSION

The importance of bond duration for most investors is that it predicts the amount of change in the market price of a bond as a result of changes in interest rates. We are currently experiencing historically low interest rates. Consequently, some investors are tempted to invest in longer maturities for their bonds. If interest rates were to rise, the price of a bond would fall by an amount equal to approximately the change in the interest rates times the duration of the bond. This primer on duration was to inform investors who might have forgotten or might not know duration basics including what duration is, how it is calculated and where an investor can find the duration number for a bond. Duration is so important that FINRA issued an investor alert in 2013 that all investors should know their duration numbers. This paper also serves to caution an investor to look beyond the title of a bond fund. The duration in an intermediate-term bond fund can range from 3.5 to 6 years in the U.S. World and News Report listing of "Best Intermediate-Term Bond Funds". "There are two main reasons why duration is a basic concept in bond analysis and management. It provides useful information on the bond's riskiness and it is essential to the procedure of protection against unforeseen changes in interest rates." (Pianca, 2005). It is interesting to note also that because of the Eurozone debt crisis, duration has risen steadily in Europe. "The duration of the iBoxx euro sovereign index, which tracks Eurozone government bonds, has risen to 7.2 years, according to financial data firm Markit. That is up from less than five years in 2006 and about six years until as recently as 2011." (M. Bird, 2016). "The fact that two bonds have the same term to maturity does not mean that they have the same interest-rate risk. A long-term discount bond with ten years to maturity, a so-called zero-coupon bond, makes all of its payments at the end of the ten years, whereas a 10% coupon bond with ten years to maturity makes substantial cash payments before the maturity date. Since the coupon bond makes payments earlier than the zero-coupon bond, we might intuitively guess that the coupon bond's effective maturity, the term to maturity that accurately measures interest-rate risk, is shorter than it is for the zero-coupon discount bond." (Mishkin 2012). The bond which makes substantial cash payments before maturity would have an expected lower duration even though both bonds have the same term to maturity.

Investors do not have to know how to calculate duration. A savvy investor should not only know their duration numbers but also be aware of what percentage of their life cycle holdings are held in bonds and the duration number for those bonds. Duration is a good estimate of what effect an interest rate increase or decrease will have on bond investments or bond funds. If interest rates were to rise by 3 percent over the next 10 years, a bond or bond might potentially decline in value by approximately 3 percent times a bond's duration number. Conservative investors who seek safety may be unpleasantly surprised to realize that they may be experiencing losses in their "safe" investment portfolios if there is a reversion to the mean and more normal interest rates. Convexity is not addressed in this paper. Although large interest rate changes are not expected in this current economy, investors should be aware that it may impact duration and their investment decisions if there are large changes in interest rates.

REFERENCES

- Bird, M. (2016). Duration Risk: The Bomb Ticking Inside Today's Bond Market. *Wall Street Journal*. April 14, 2016.
- Bierwig, G.O., Roberts, G.S. (1990). Single-Factor Duration Models: Canadian Tests. *The Journal of Financial Research*. Volume 13 (1), Pages 23-38. Retrieved June 3, 2016. DOI: 10.1111/j.1475-6803.1990.tb00533.x
- Bohlin, S., Strickland, G. (2004), Climbing the Ladder: How to Manage Risk in Your Bond Portfolio. American Association of Individual Investors. July 2004. Retrieved June 1, 2016.
- Campbell, J. Y., and J. Mei. 1993. Where Do Betas Come From? Asset Price Dynamics and the Sources of Systematic Risk. *Review of Financial Studies* 6:567–92.
- Cooper, I.A. (1977). Asset Values, Interest-Rate Changes, and Duration. *Journal of Financial and Quantitative Analysis*, 12, 701-723. Retrieved June 12, 2016. doi:10.2307/2330252.
- Cox, J.C, Ingersoll, J.E, Ross, S.A. (1979), "Duration and Measurement of Basis Risk." *The Journal of Business*, Volume 52(1), 51-61.
- de Aenlle, C. (2014). LIPPER AWARDS-Why TIAA-CREF stands out as a fund manager. Retrieved November 14, 2015. <http://www.reuters.com/article/2014/03/24/lipper-awards-tiaa-idUSL2N0M81RI20140324#OrgcWhdjvXX2swh8.99>
- Federal Reserve HRB H.15 Release. Retrieved November 6, 2015. <http://www.federalreserve.gov/releases/h15/data.htm>
- FINRA Investor Alert (2013). Duration--What an Interest Rate Risk Could Do to Your Portfolio. Retrieved September 6, 2015. <https://www.finra.org/investors/alerts/duration-what-interest-rate-hike-could-do-your-bond-portfolio>
- Fisher L., & Weil R. (1971). Coping with the risk of interest rate fluctuations: Returns to Bondholders from a Naive and Optimal Strategy. *Journal of Business*, 44 (3), 408-31. Retrieved June 1, 2016. <http://dx.doi.org/10.1086/295402>
- Fooladi, I.J., Roberts, G.S. (2000). Risk Management with Duration Analysis. *Managerial Finance*, Volume 26 (3). Retrieved June 2, 2016. <http://dx.doi.org/10.1108/03074350010766558>
- Fitch Ratings (2013). Timing Is Everything for Potential "Bond Bubble". Retrieved September 16, 2015. https://www.fitchratings.com/gws/en/fitchwire/fitchwirearticle/Timing-Is-Everything?pr_id=779231
- Fitch Ratings (2012). The "Bond Bubble": Risks and Mitigants. Retrieved September 14, 2015. http://www.sifma.org/uploadedfiles/for_members/thought_leader_library/2012/fitch-bond-bubble.pdf?n=34035
- Hawawini, G. On the relationship between Macaulay's Bond Duration and the Term to Maturity. *Economics Letters* 16 (1984) 331-337
- Ho, Thomas (2013). Yield Curve Movements and Key Rate Durations. Retrieved November 1, 2015. <http://www.thomasho.com/space/viewBlog.asp?blog=33>

- Kaufman, G. (1984). Measuring and Managing Interest Rate Risk: A Primer. *Economic Perspectives*, Federal Reserve Bank of Chicago. Retrieved June 3, 2016. <https://www.chicagofed.org/.../ep-jan-feb1984-part2>.
- Kraft, H., Monk C. (2007). Bond durations: Corporates vs. Treasuries. *Journal of Banking & Finance*. (31) 12, 3720-3741.
- Macaulay, F.R. (1938). Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields and Stock Prices in the United States since 1856. National Bureau of Economic Research, 44-53.
- Mishkin, F. (2012). The Economics of Money, Banking and Financial Markets (*Pearson*) 10/e (Global edition) 2012.
- Pianca, P. (2005). Maximum Duration of Below Par Bonds: A Closed-form Formula. Retrieved June 2, 2016. <http://dx.doi.org/10.2139/ssrn.738445>
- Rakotondratsimba Y. (2011). Interest Rate Sensitivities Under the Vasicek and Cox-Ingersoll-Ross Models. Retrieved June 2, 2016 <http://dx.doi.org/10.2139/ssrn.1977902>
- Rakotondratsimba Y., Lajili, S. (2012). Enhancement of the Bond Duration-Convexity Approximation. *International Journal of Economics and Finance*. Volume 4 (3). Retrieved June 3, 2016. <http://ccsenet.org/journal/index.php/ijef/article/view/15154>
- Redington, F. (1952). Review of the Principle of Life-Office Valuations. *Journal of the Institute of Actuaries* 78, 316-340.
- U.S. News and World Report. Best Long-Term Bond. Retrieved October 26, 2015. <http://money.usnews.com/funds/mutual-funds/rankings/long-term-bond>
- U.S. News and World Report (2010). Best Fit Intermediate-Term Bond. Retrieved October 26, 2015. <http://money.usnews.com/funds/mutual-funds/rankings/intermediate-term-bond>
- Vasicek, O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics* 5, 177-188.

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